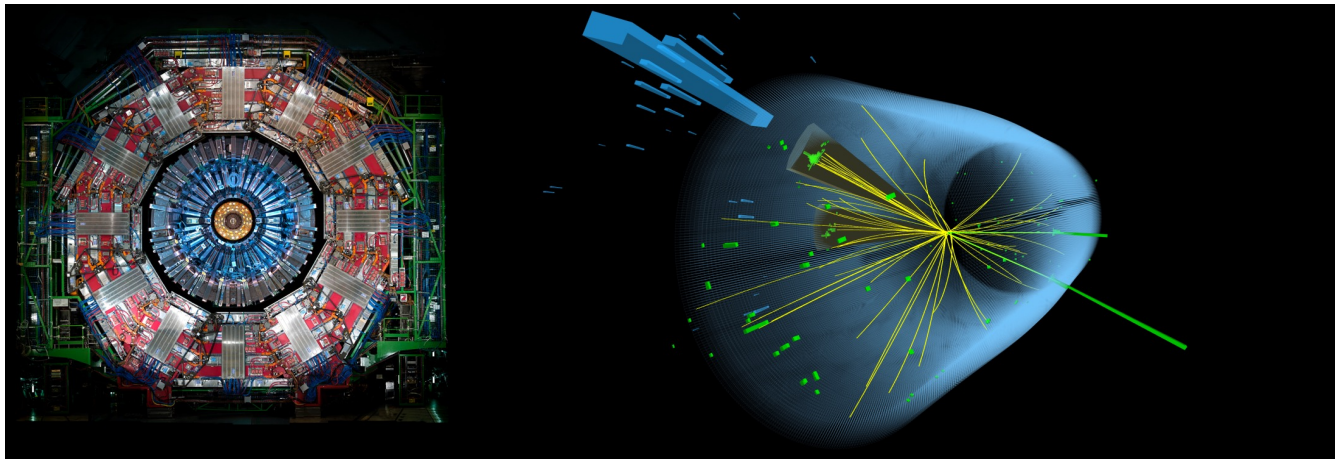


Top quark physics (part2)

Kim, Tae Jeong
Hanyang University

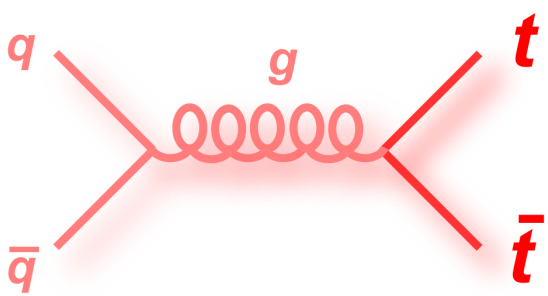
Spring, 2022

Top quark (reminder)

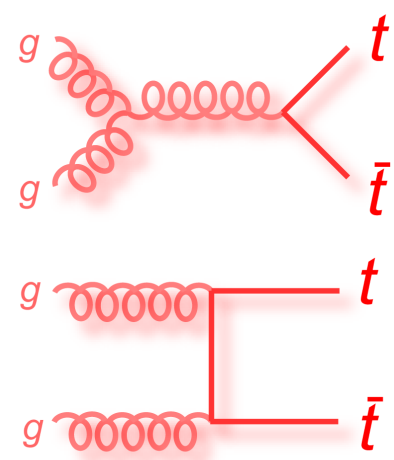
- Predicted as isospin partner of bottom quark (bottom quark was discovered in 1977)
- From virtual loop corrections and precise LEP data can predict the top quark mass
- The top quark is the most massive of the quarks ~ 173 GeV in the standard model : m(t) > m(H) > m(Z) > m(W)
- Because of its mass, it was not observed directly at the LEP.
- In 1995 top quark is observed at the Tevatron, proton anti-proton collider at Fermilab.
- Owing to its large mass, the lifetime is very short

$$\tau = \frac{1}{\Gamma} = 4\times10^{-25}s, \Gamma = 1.5\ \mathrm{GeV}.$$

- It decays before it can form strongly interacting bound states (hadronization).
- The top quark properties can be studied without non-perturbative effects.



Quark annihilation



Gluon-gluon fusion

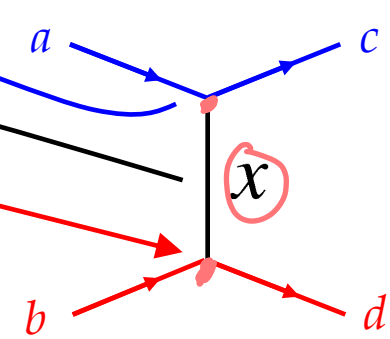
Decay rate and Cross section

$$\frac{1}{\tau} = \Gamma = \frac{|\vec{p}^*|}{32\pi^2 m_i^2} \int |M_{fi}|^2 d\Omega$$

$$\sigma = \frac{1}{64\pi^2 s} \frac{|\vec{p}_f^*|}{|\vec{p}_i^*|} \int |M_{fi}|^2 d\Omega^*$$

$$M = \langle \psi_c | V | \psi_a \rangle \frac{1}{q^2 - m_x^2} \langle \psi_d | V | \psi_b \rangle$$

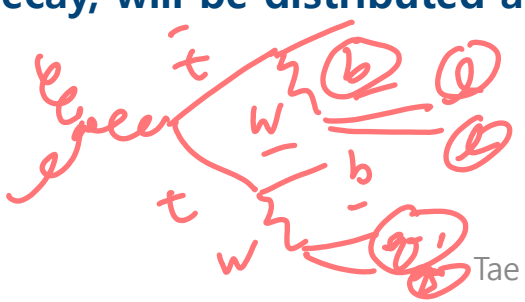
$$\langle \psi_c | V | \psi_a \rangle = g_a \qquad \langle \psi_d | V | \psi_b \rangle = g_b$$



- **The presence of the propagators for the two virtual top quarks implies that**

$$|M|^2 \propto \frac{1}{(q_1^2 - m_t)^2 + m_t^2 \Gamma_t^2} \times \frac{1}{(q_2^2 - m_t^2)^2 + m_t^2 \Gamma_t^2}$$

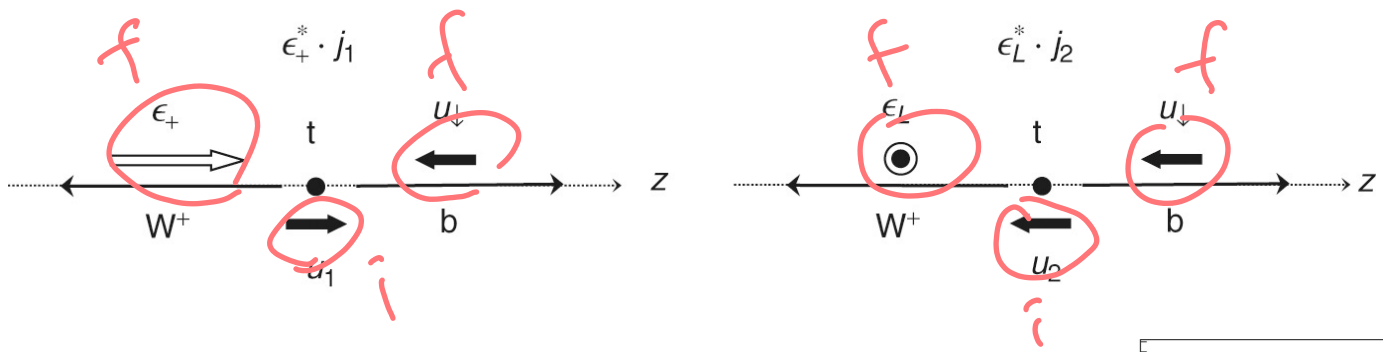
- **The invariant masses of each of W^+b and $W^-\bar{b}$ systems, produced in the top decay, will be distributed according to Lorentzian centered on m_t with width Γ_t**



Top quark polarization

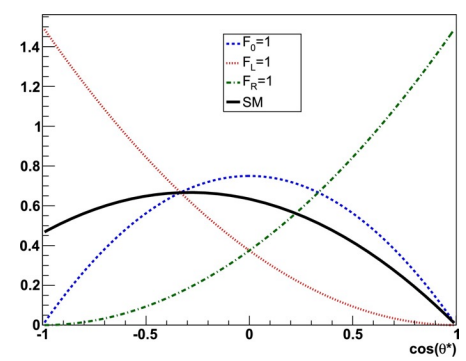
- Two possible spin states of the top quark at rest
- The b-quark can only be produced in a left-handed helicity $h = \vec{s} \cdot \vec{p}$
- There are three possible polariation states of the W boson
  - $S_z = \pm 1$, longitudial polarization states

- Only two combinations are allowed (see below)

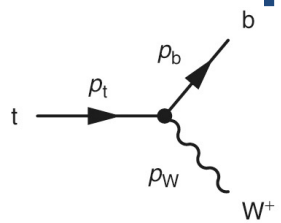


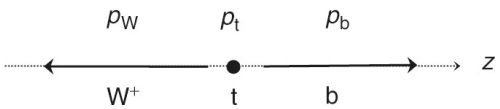
- Experimental measurement

- θ^* is the angle between the direction of either the down-type quark or the charged lepton from the W boson and the reversedd direction of the top quark in the rest frame of the W boson



Top quark decay





$$p_t = (m_t, 0, 0, 0),$$
$$p_b = (p^*, 0, 0, p^*)$$
$$p_W = (E^*, 0, 0, -p^*)$$

- **W boson can be treated as a real on shell final-state particle**

$$-iM = \left[\bar{u}(p_b)\frac{-ig_W}{\sqrt{2}}\gamma^\mu\frac{1}{2}(1-\gamma^5)u(p_t)\right] \times \epsilon^*_\mu(p_W)$$

$$M = \frac{ig_W}{\sqrt{2}}\epsilon^*_\mu(p_W)\left[\bar{u}(p_b)\gamma^\mu\frac{1}{2}(1-\gamma^5)u(p_t)\right]$$

- **In the case of $p^* \gg m_b$, the chiral = helicity states and, consequently, the b-quark can only be produced in a left-handed helicity state**

$$M = \frac{g_W}{\sqrt{2}}\epsilon^*_\mu(p_W)\,\bar{u}_\downarrow(p_b)\gamma^\mu u(p_t)$$

- **LH helicity spinor for the b-quark is**

$$\bar{u}_\downarrow(p_b) \approx \sqrt{p^*}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}$$

- **Two possible spin states of the top quark can be written using the u_1 and u_2 spinors, which for a top quark at rest are**

$$u_1(p_t) = \sqrt{2m_t}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \text{ and } u_2(p_t) = \sqrt{2m_t}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

- **Four-vector current for the two possible spin states**

$$j_1^\mu = \bar{u}_\downarrow(p_b)\gamma^\mu u_1(p_t) = \sqrt{2m_t p^*}(0,-1,-i,0)$$
$$j_2^\mu = \bar{u}_\downarrow(p_b)\gamma^\mu u_2(p_t) = \sqrt{2m_t p^*}(1,0,0,1)$$

Top quark decay

- **Three possible polarization states of the W boson**

$$\epsilon_+^*(p_W) = -\frac{1}{\sqrt{2}}(0,1,-i,0), \qquad \epsilon_-^*(p_W) = \frac{1}{\sqrt{2}}(0,+1,+i,0), \qquad \epsilon_L^*(p_W) = \frac{1}{m_W}(-p^*,0,0,E^*)$$

- **For a particular top quark spin and W polarization state, the matrix element is given by the four-vector scalar product**

$$M = \frac{g_W}{\sqrt{2}} j_i \cdot \epsilon_\lambda^*$$

- **The matrix elements for these two allowed spin configurations are**

$$M_1 = \frac{g_W}{\sqrt{2}}\epsilon_+^* \cdot j_1 = -\frac{g_W}{\sqrt{2}}\sqrt{m_t p^*}(0,1,i,0)\cdot(0,-1,-i,0) = -g_W\sqrt{2m_t p^*}$$

$$M_2 = \frac{g_W}{\sqrt{2}}\epsilon_L^* \cdot j_2 = \frac{g_W}{m_W}\sqrt{m_t p^*}(-p^*,0,0,E^*)\cdot(1,00,1) = -\frac{g_W}{m_W}\sqrt{m_t p^*}(E^* + p^*)$$

- **The spin-averaged matrix element squared for the decay $t \to bW^+$ is**

$$< |M^2| >= \frac{1}{2}\left(|M_1^2| + |M_2^2|\right) = \frac{1}{2} g_W^2 m_t p^* (2 + \frac{m_t^2}{m_W^2})$$

$$\Gamma(t \to bW^+) = \frac{p^*}{8\pi m_t^2} < |M^2| > = \frac{g_W^2 p^{*2}}{16\pi m_t}\left(2 + \frac{m_t^2}{m_W^2}\right)$$

- **Top quark decay width**

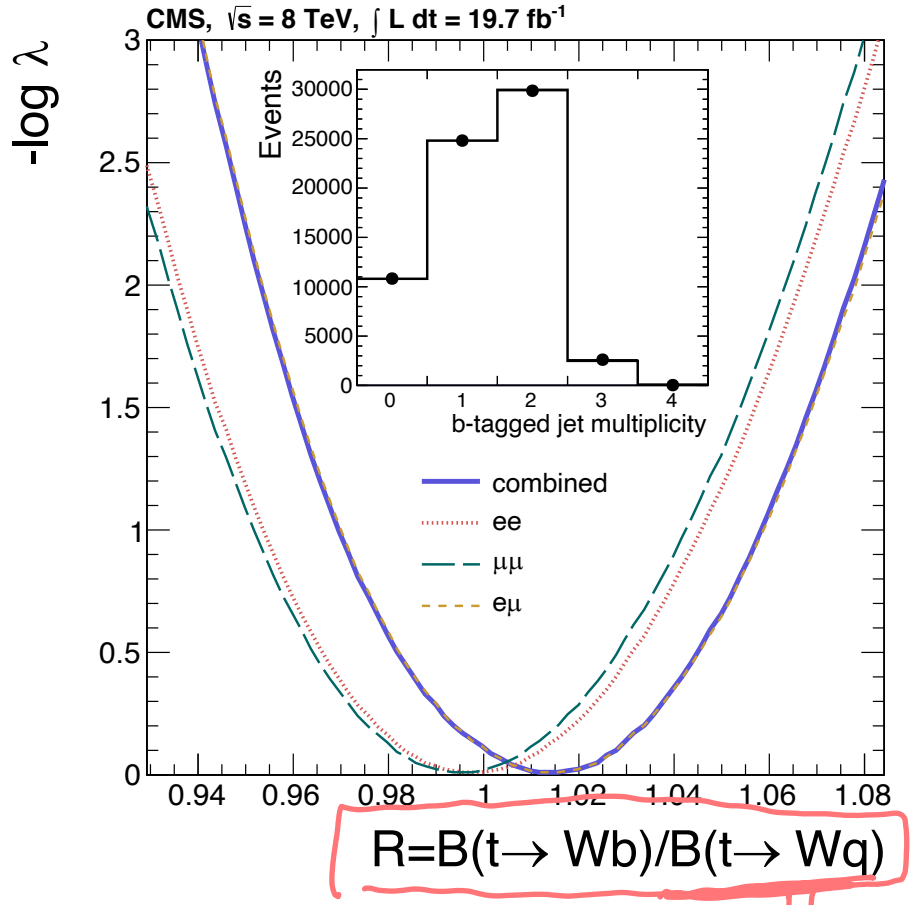
$$\Gamma(t \to bW^+) = \boxed{\frac{G_F m_t^3}{8\sqrt{2}\pi}\left(1 - \frac{m_W^2}{m_t^2}\right)^2 (1 + \frac{2m_W^2}{m_t^2})}$$

$$\Gamma_t = 1.5 \text{ GeV}$$

- **The top quark lifetime is of order $\tau_t = \frac{1}{\Gamma_t} \approx 5\times10^{-25} s$**

- **Distance of order 10^{-16} m**
- **This is small compared to the typical length scale for the hadronization process.**

Width measurement



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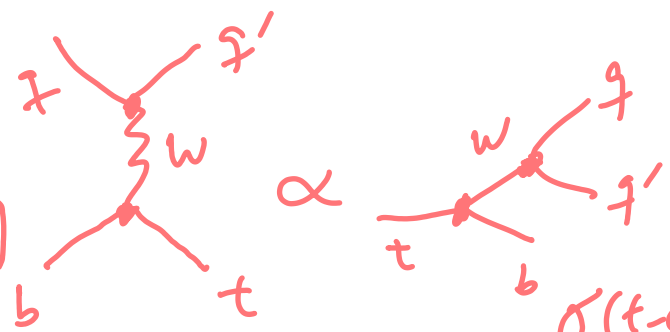
$\Gamma_t = 1.36 \pm 0.02^{+0.14}_{-0.11}$ GeV

indirectly measured Γ_t

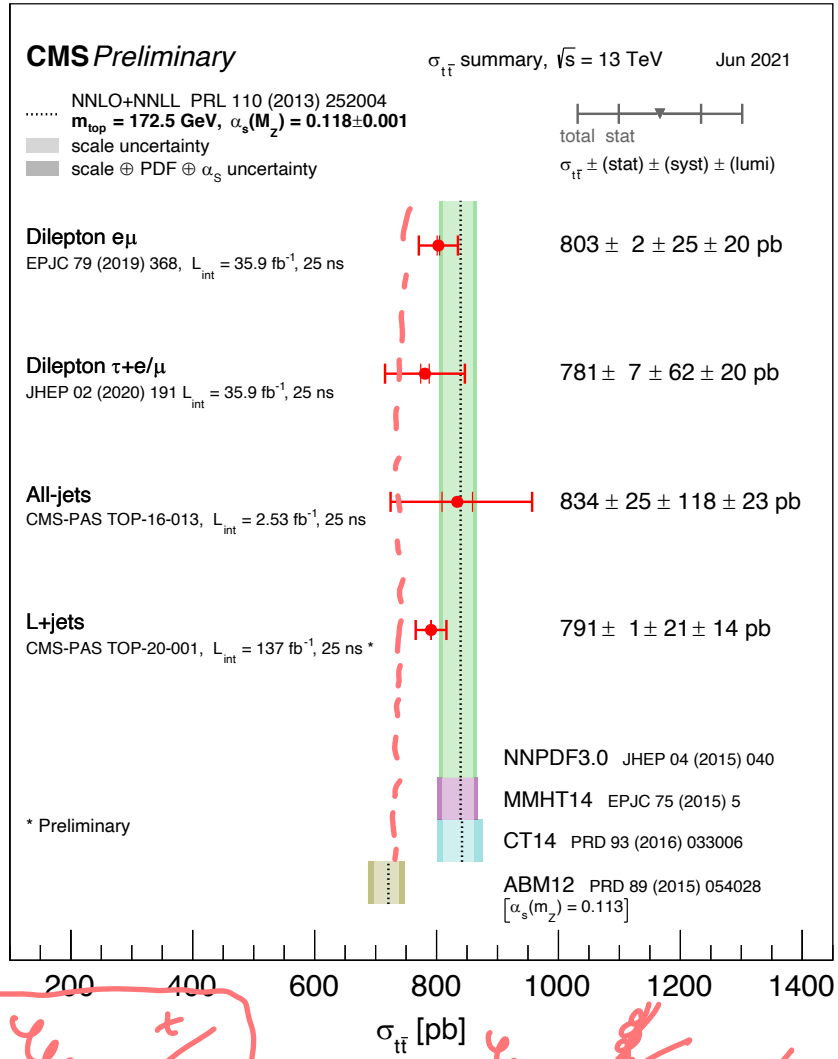
$B(t \to Wb) = \frac{\Gamma(t \to Wb)}{\Gamma_t}$ = ?

$\Gamma(t \to Wb) = \Gamma(t \to Wb)_{theory} \frac{\sigma(t\text{-ch})}{\sigma(t\text{-ch})_{theory}}$



$\Gamma_t = \frac{\sigma(t\text{-ch})}{B(t \to Wb)} \frac{\Gamma(t \to Wb)_{theory}}{\sigma(t\text{-ch})_{theory}}$

Cross section measurement



$\frac{1}{137}$

$N_{t\bar{t}} = \sigma_{t\bar{t}} \cdot L$, $\sigma_{t\bar{t}} = \frac{N_{t\bar{t}}}{L}$

$\text{acceptance} = \frac{N^{Acc}_{t\bar{t}}}{N_{t\bar{t}}}$

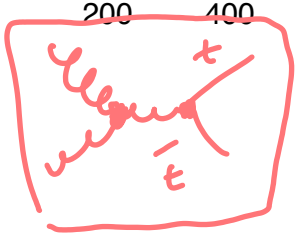
$\text{efficiency} = \frac{N^{evt}_{t\bar{t}}}{N^{Acc}_{t\bar{t}}}$

$N^{evt}_{t\bar{t}} = \sigma_{t\bar{t}} \cdot L \cdot A \cdot \varepsilon$

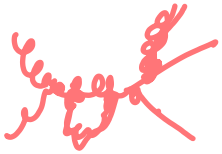
$\sigma_{t\bar{t}} = \frac{N^{evt}_{t\bar{t}}}{L \cdot A \cdot \varepsilon} = \frac{N_{data} - N^{non-t\bar{t}}_{bg}}{L \cdot A \cdot \varepsilon}$

ML data-driven

$\sigma_{t\bar{t}} = \frac{N_{data} - N^{bg}_{t\bar{t}\text{-dilepton}}}{L\, A \cdot \varepsilon \cdot Br}$

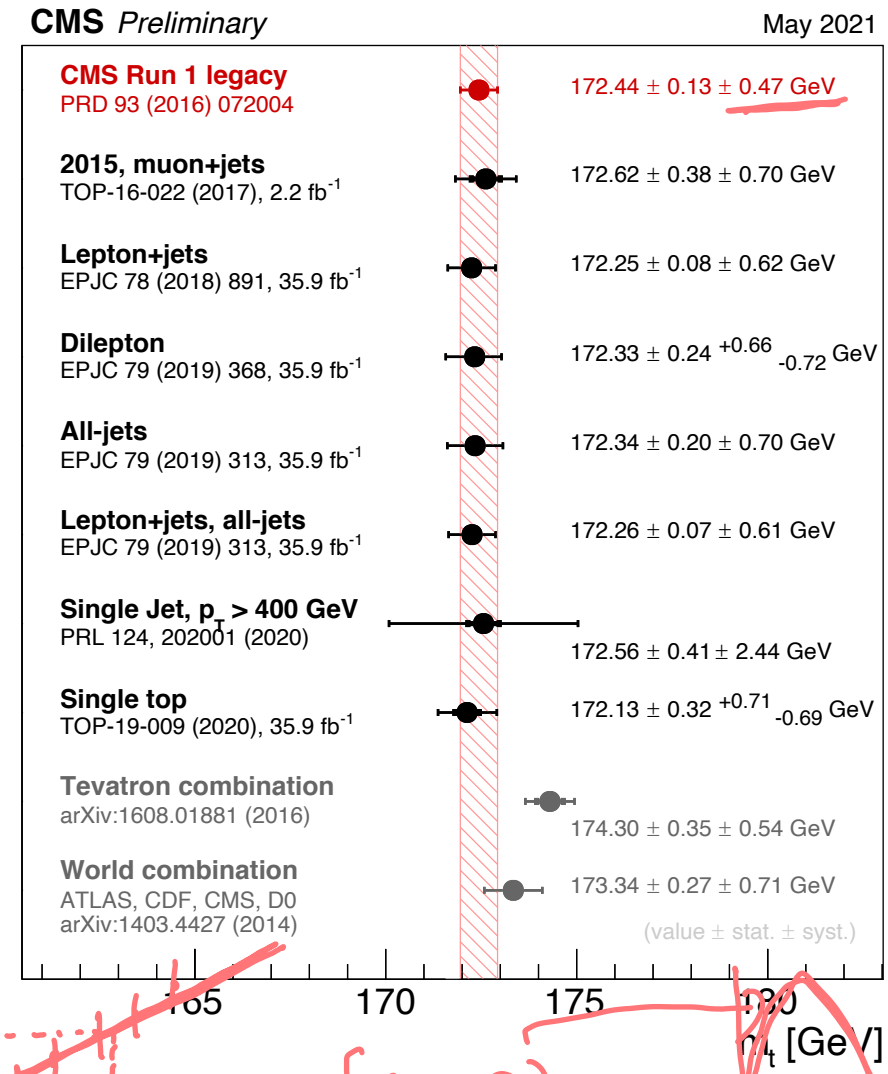


$\sigma \simeq \alpha^2$



$\sigma \approx \alpha^2 + \alpha^3 + \alpha^4 + \ldots$

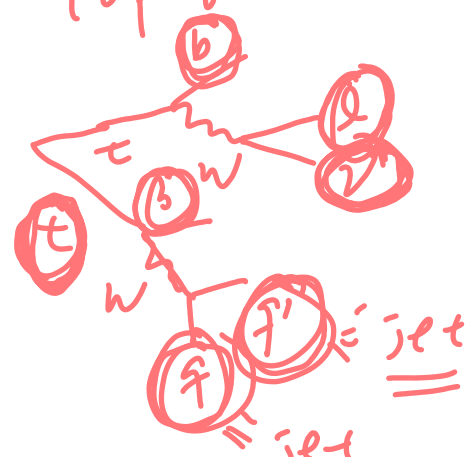
Top quark mass measurement



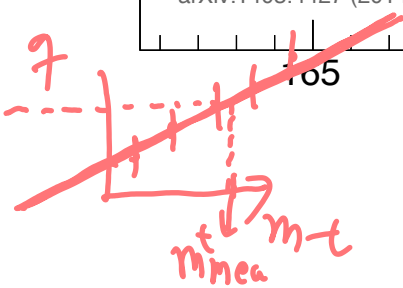
Z boson mass ?

$p = p_{\mu^+} + p_{\mu^-}$

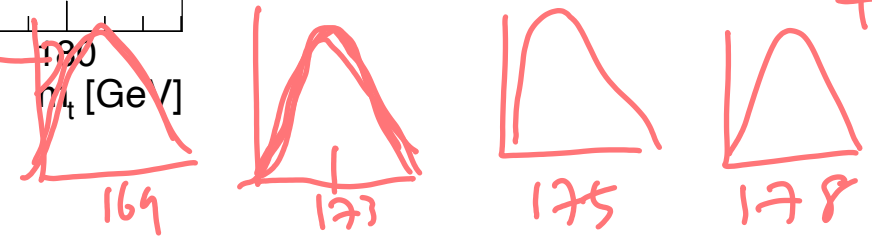
top quark mass ?



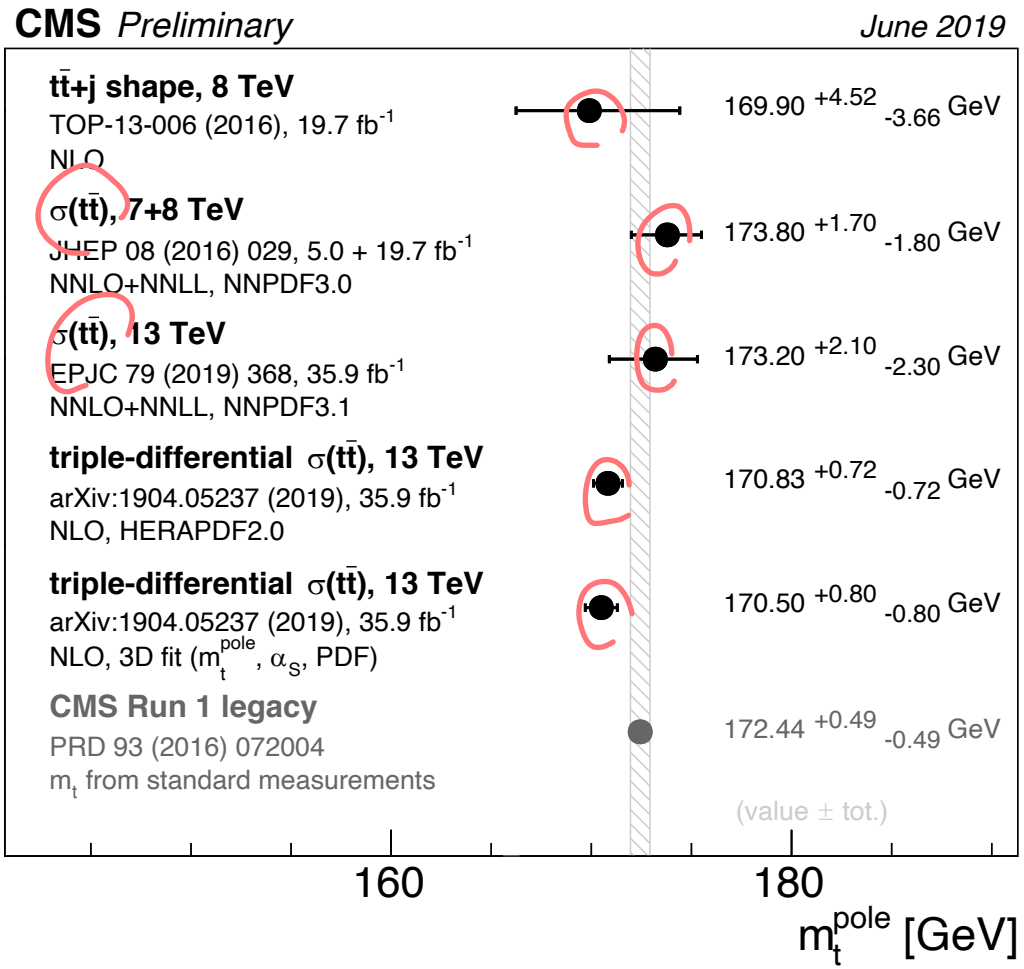
$$\chi^2 = \frac{(m_{bqq'} - m_t)^2}{\sigma_{m_t}} + \frac{(m_{qq'} - m_W)^2}{\sigma_{m_W}} + \cdots$$

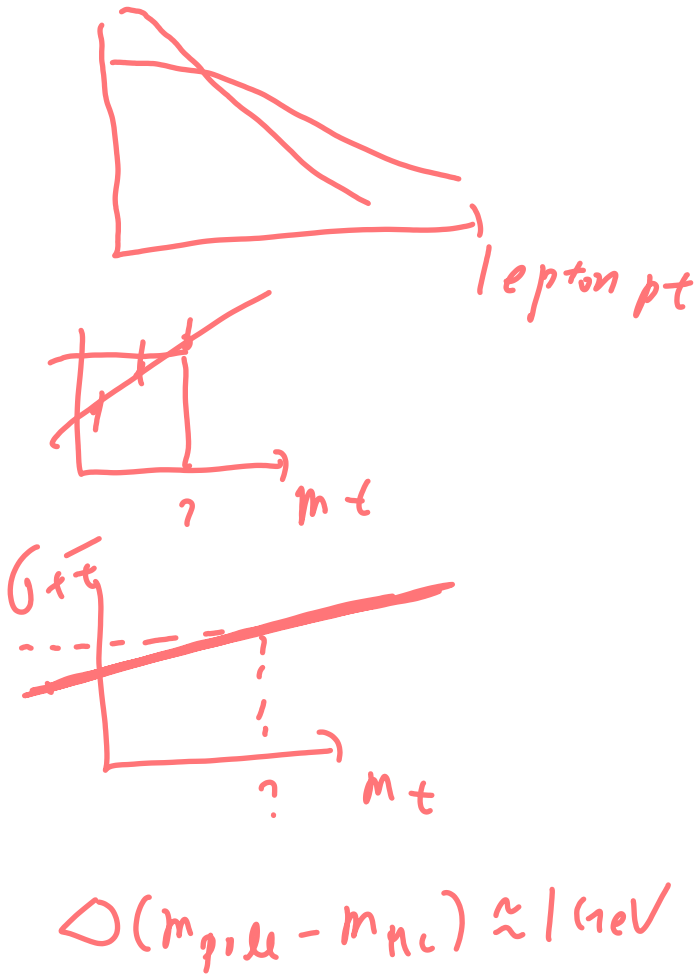


$f(q, \mu)$

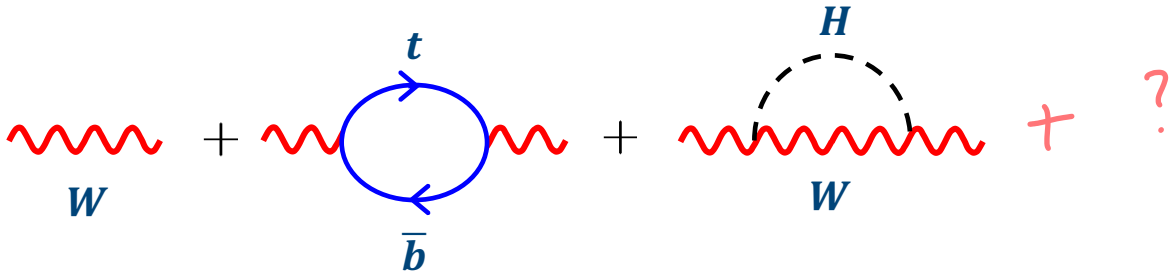


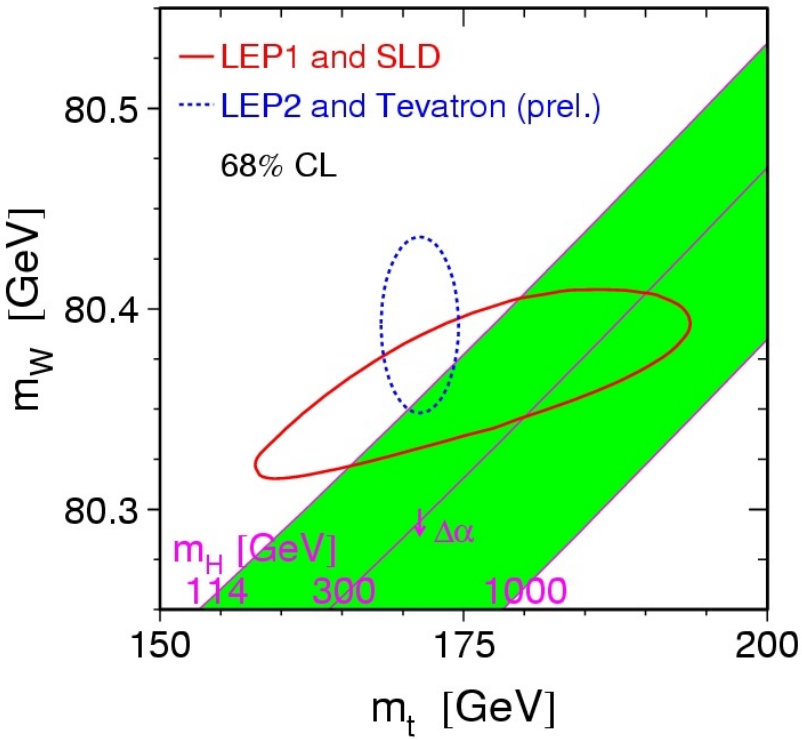
Pole mass measurement

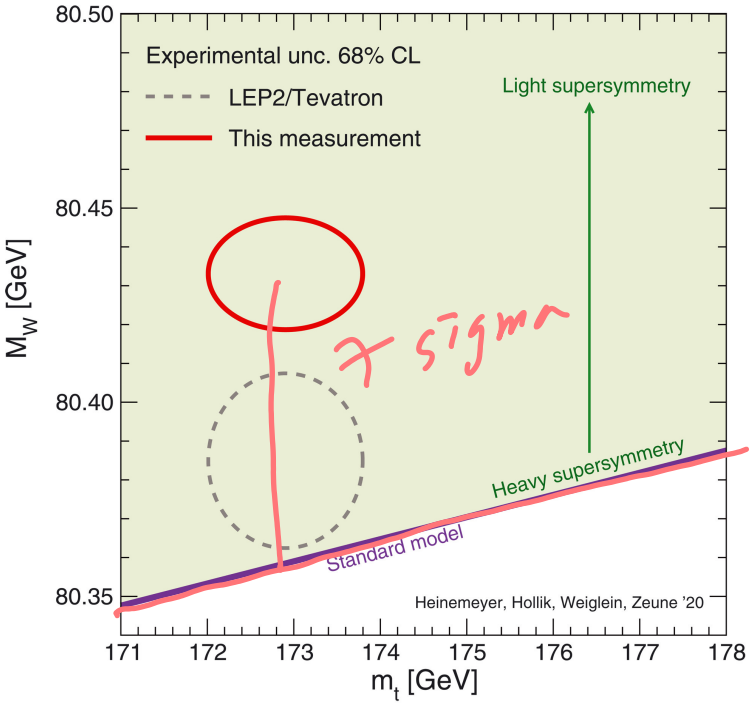




W boson mass depens on the top and Higgs

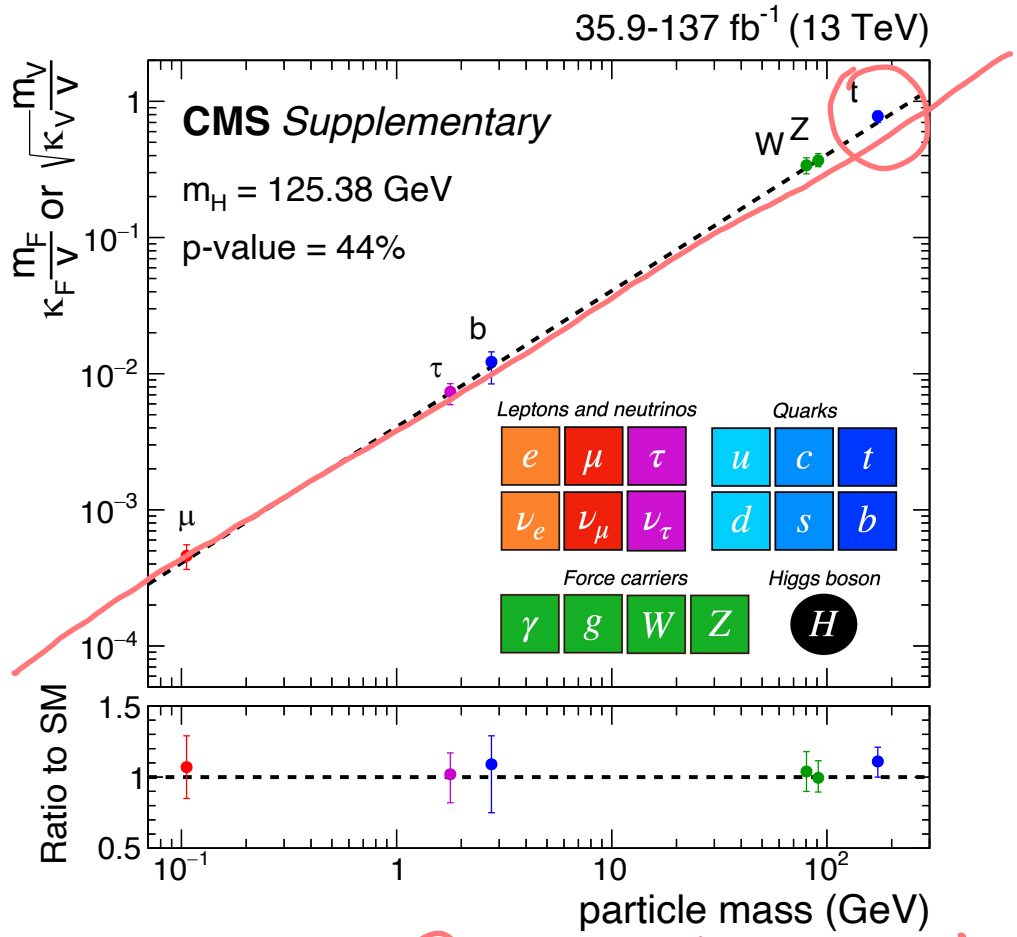


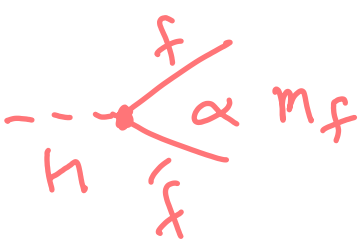


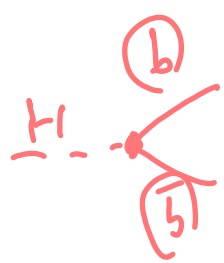


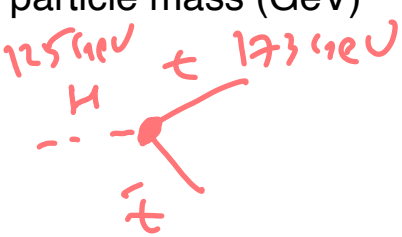
Latest CDF result

How about top coupling with the Higgs?



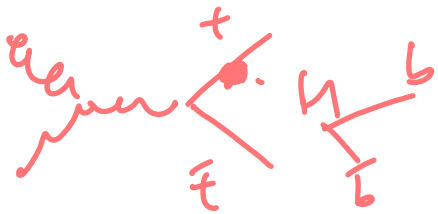


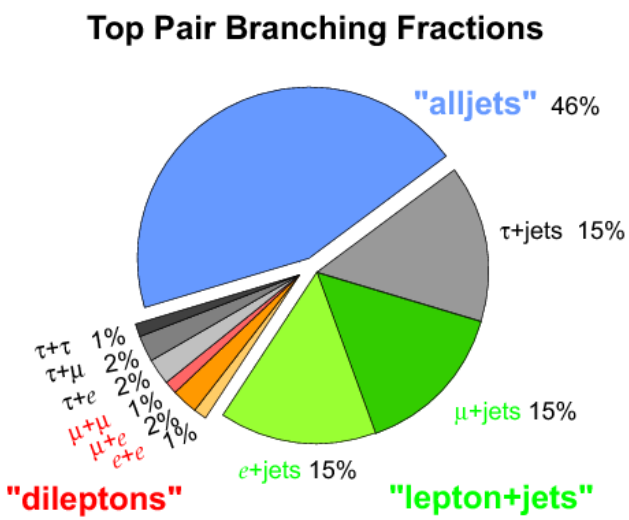


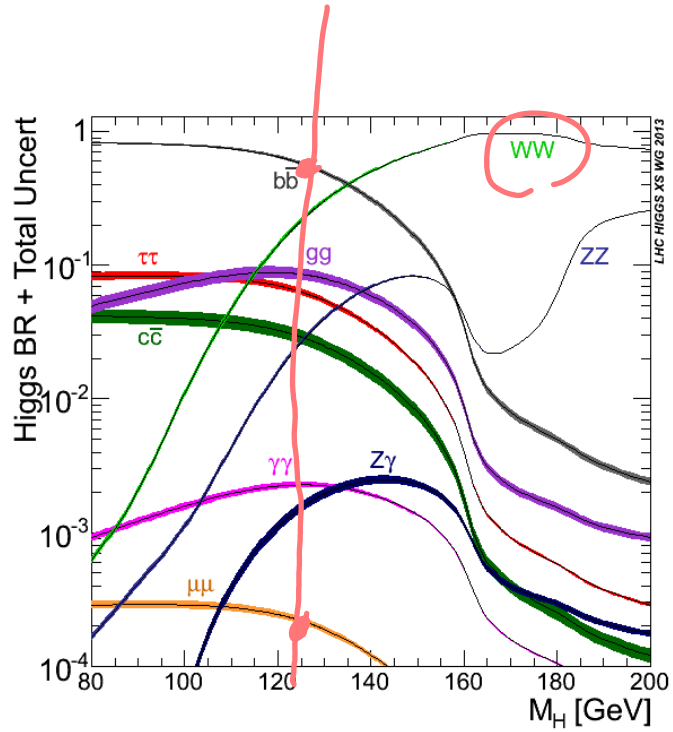


How to measure top coupling with the Higgs

- Higgs to ttbar is not kinematically allowed at m(H)=125 GeV
- Top quark pair production associated with H is the main mode
- Should consider all decay channels of top and Higgs to increase sensitivity



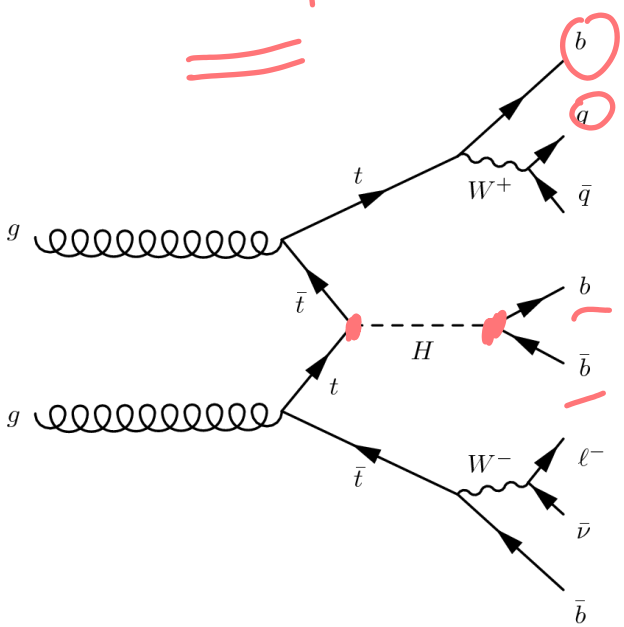


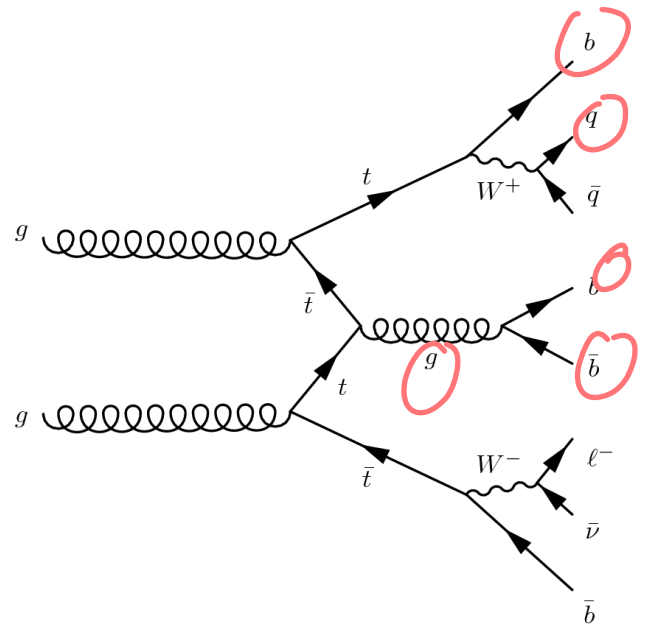


New discovery requires precision measurement

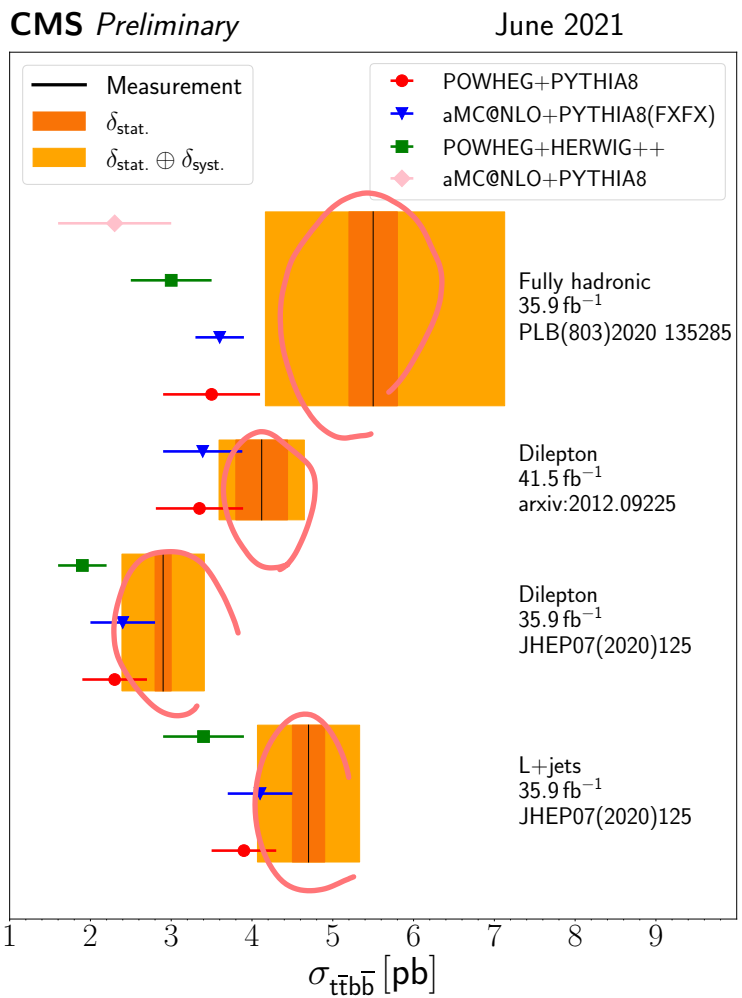
- **We need understand the main background process**
- **e.g. ttbar+bbbar is the main background for the ttbarH(bbbar) process**

$t\bar{t}H(b\bar{b})$?





ttbar+bbbar process measurement



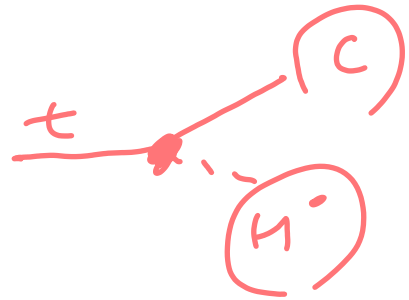
4 tops

4개

$\boxed{t\bar{t}b\bar{b}}$

4개

Rare process



SM ≈ 10^{-14}

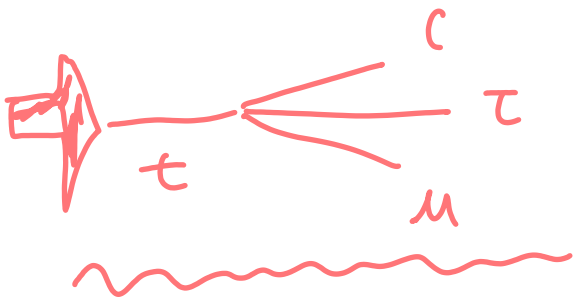
"BSM" 10^{-4}

" Flavor - changing - Neutral - Current "
= FCNC

$$R_D = \frac{Br(B \to D\tau\nu)}{Br(B \to D\ell\nu)}$$

$R_D^{SM} = 0.258 \pm 0.005$

$R_D = 0.297 \pm 0.015$

3σ

LFU in top quark