Progress in high-energy atmospheric neutrino flux calculations

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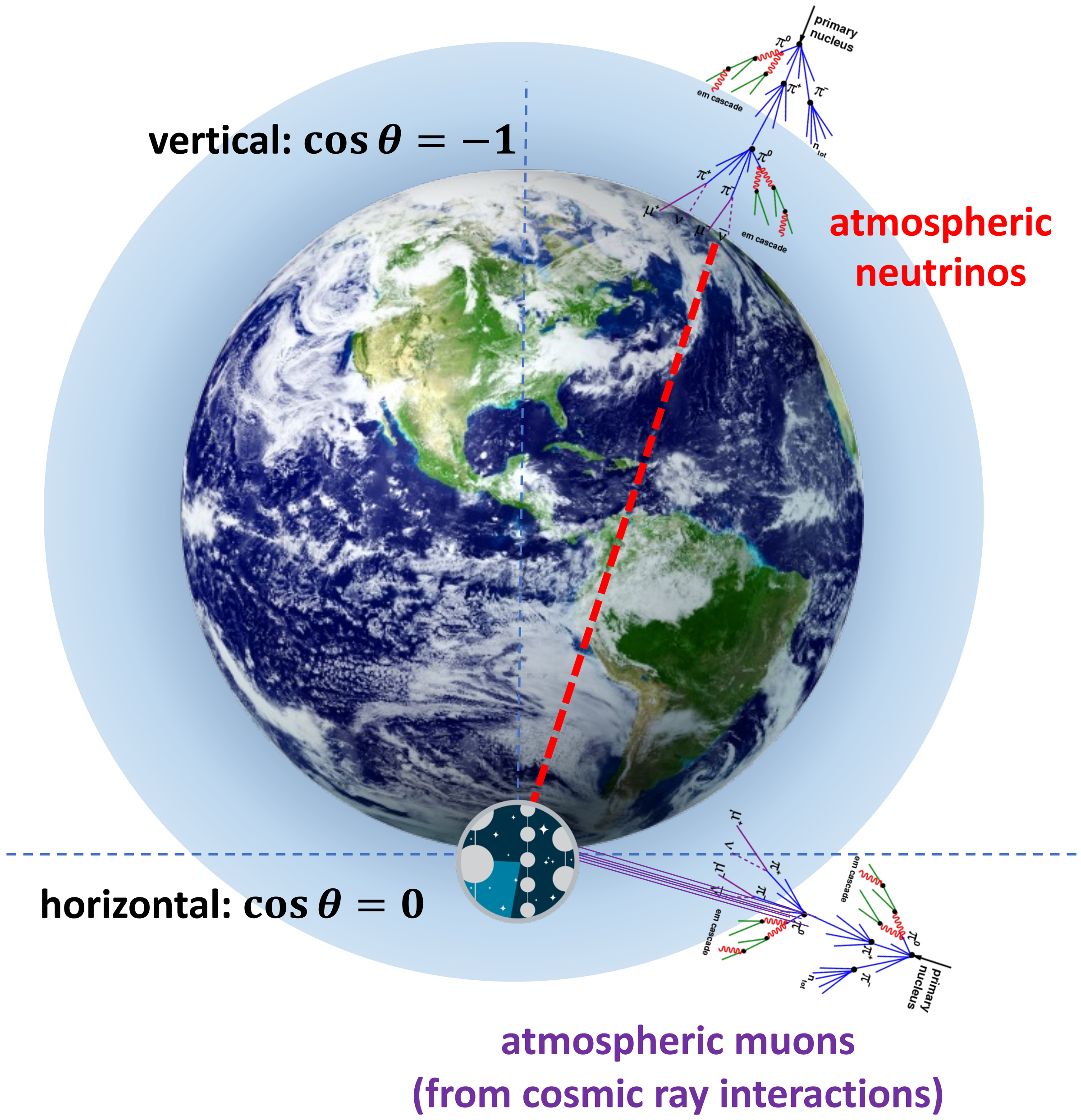
NEUTRINO 2022, Virtual Seoul, 2022/06/02

Big losses for the field

Morihiro Honda

Tom Gaisser

High-energy atmospheric neutrinos

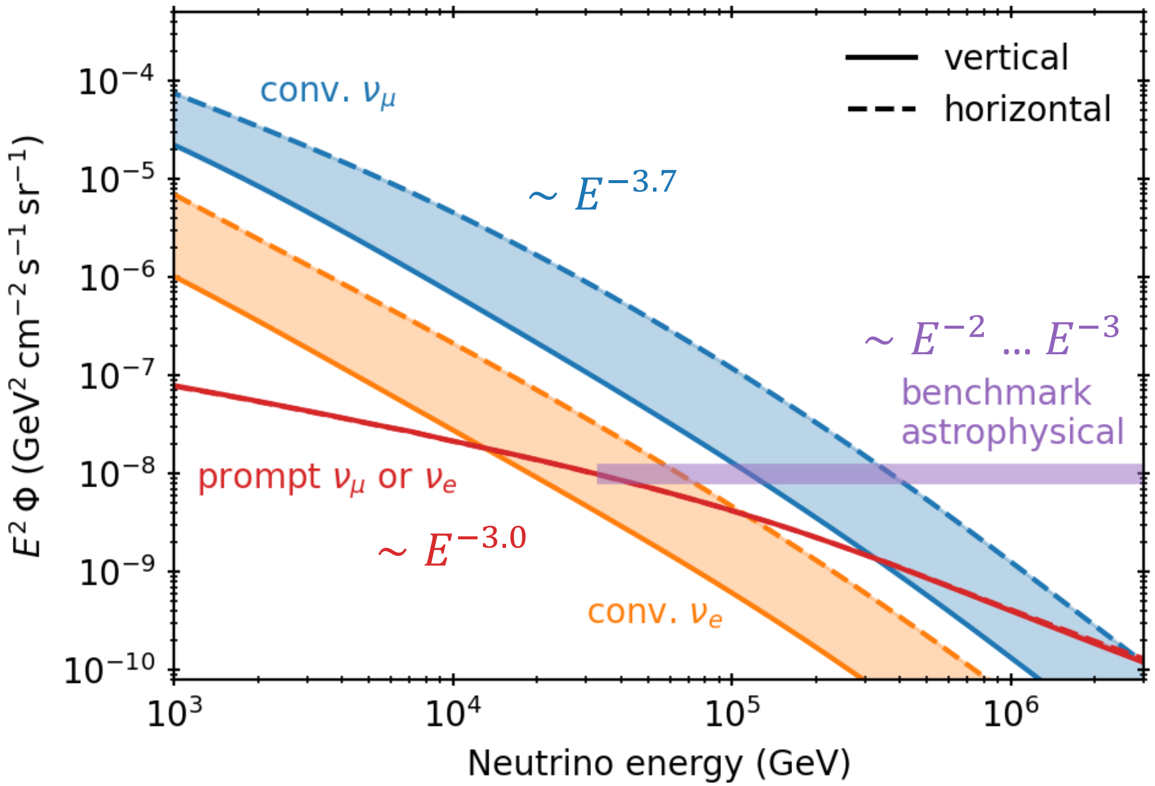


- Foreground for studying astrophysical neutrinos
- "Beam" for atmospheric oscillations
- …and searches for sterile neutrinos
- Probes of very forward particle production phase space (non-perturbative QCD & heavy flavor)

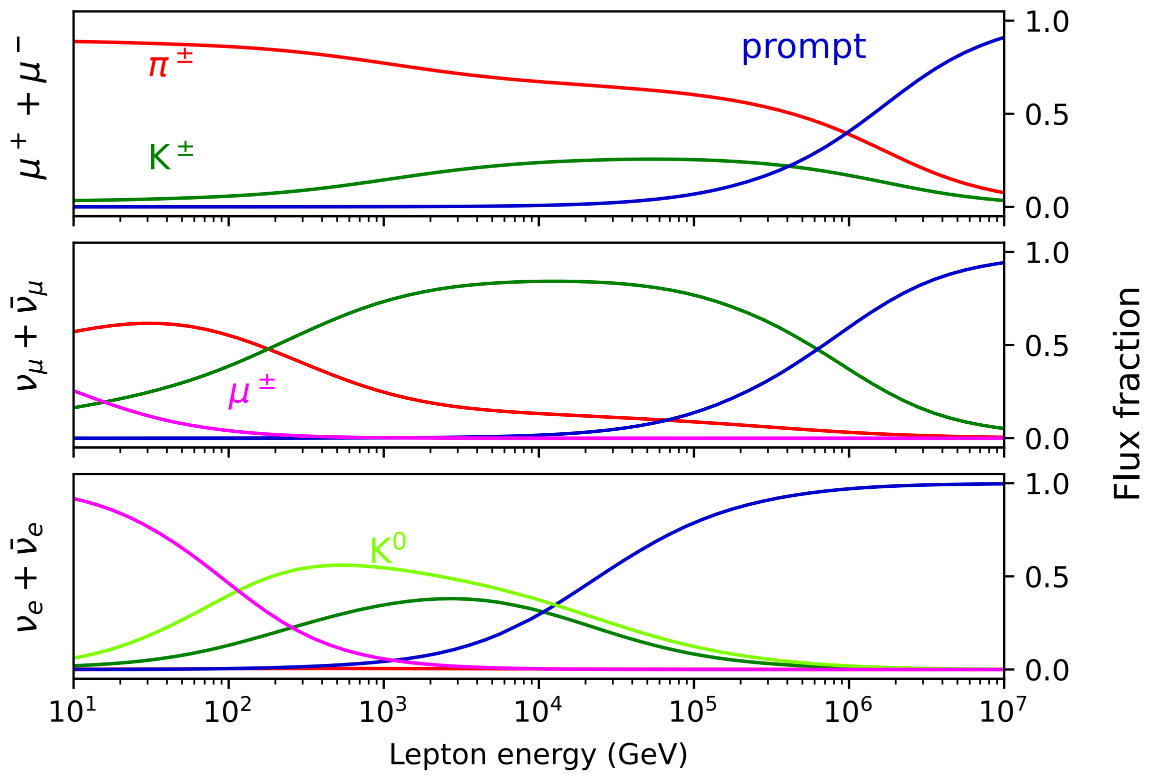
Models entering flux calculations

Cosmic ray spectrum + Atmosphere & geometry + Hadronic interactions

High-energy spectrum



AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, PRD 100 2019



Bands (zenith-enhancement):

- Lower boundary $\cos\theta = 1$, vertical
- Upper boundary $\cos\theta = 0$, horizontal

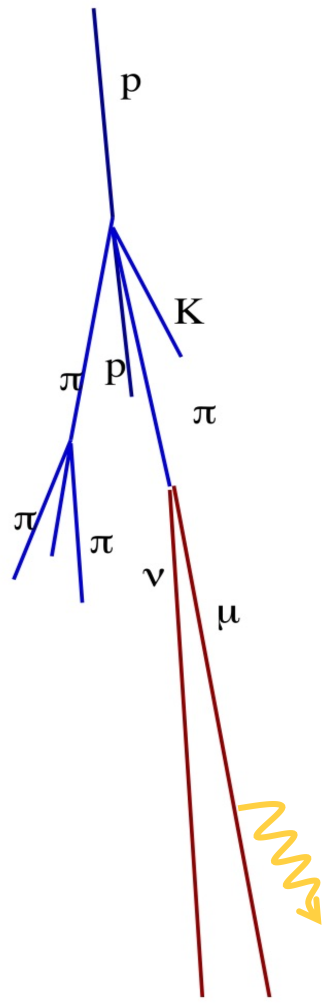
Different hadronic contributing, due to:

- Hadron production cross sections
- Branching ratio & decay kinematics

Calculation techniques

1D particle cascade Monte Carlo:

- CORSIKA 7: AF, Becker Tjus, Desiati, PRD86 114024 (2012)
- High-energy part of HKKMS and Bartol calculations
  M. Honda et al., PRD **92**, 023004 (2015), Barr et al. PRD 70, 023006 (2004)
- FLUKA: G. Battistoni et al. Astroparticle Physics **12**, 315 (1999)



Approximate semi-analytical solutions of cascade equations:

Gaisser, Engel, Resconi book (2016) or e.g.,
AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, PRD 100 2019

$$\Phi_\ell(E) = \frac{\phi_{\rm N}(E)}{1 - Z_{\rm NN}} \sum_{\substack{h=\pi,\\ {\rm K},{\rm K}^0_{\rm L},\ldots}} \frac{Z_{{\rm N}h,\gamma} Z_{h\to\ell,\gamma}}{1 + B_h E \cos\theta / \varepsilon_h}$$

$\phi_N(E)$: cosmic ray flux
Z_{Nh}: particle production yields
B_h and $Z_{h\to l}$: kinematic factors

Matrix Cascade Equations (MCEq)

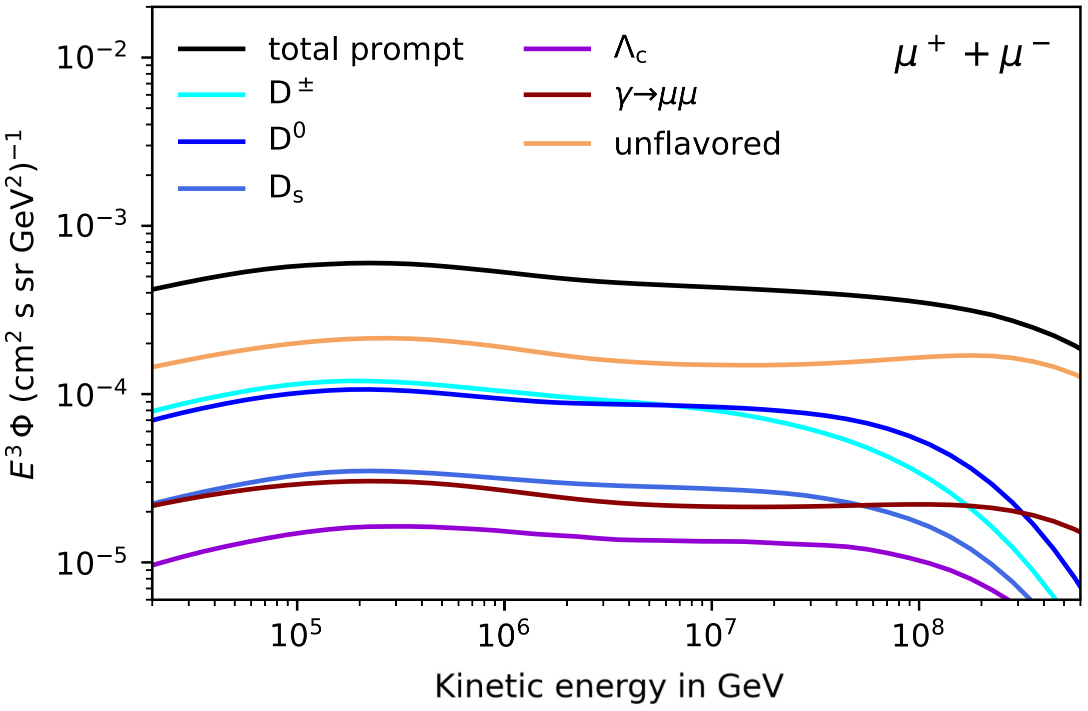
AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, PRD 100 2019

- Iterative solution of coupled cascade equations
- Very fast and accurate
- Open source https://github.com/afedynitch/MCEq
- Soon in 2D!! (see Poster by Tania Kozynets)

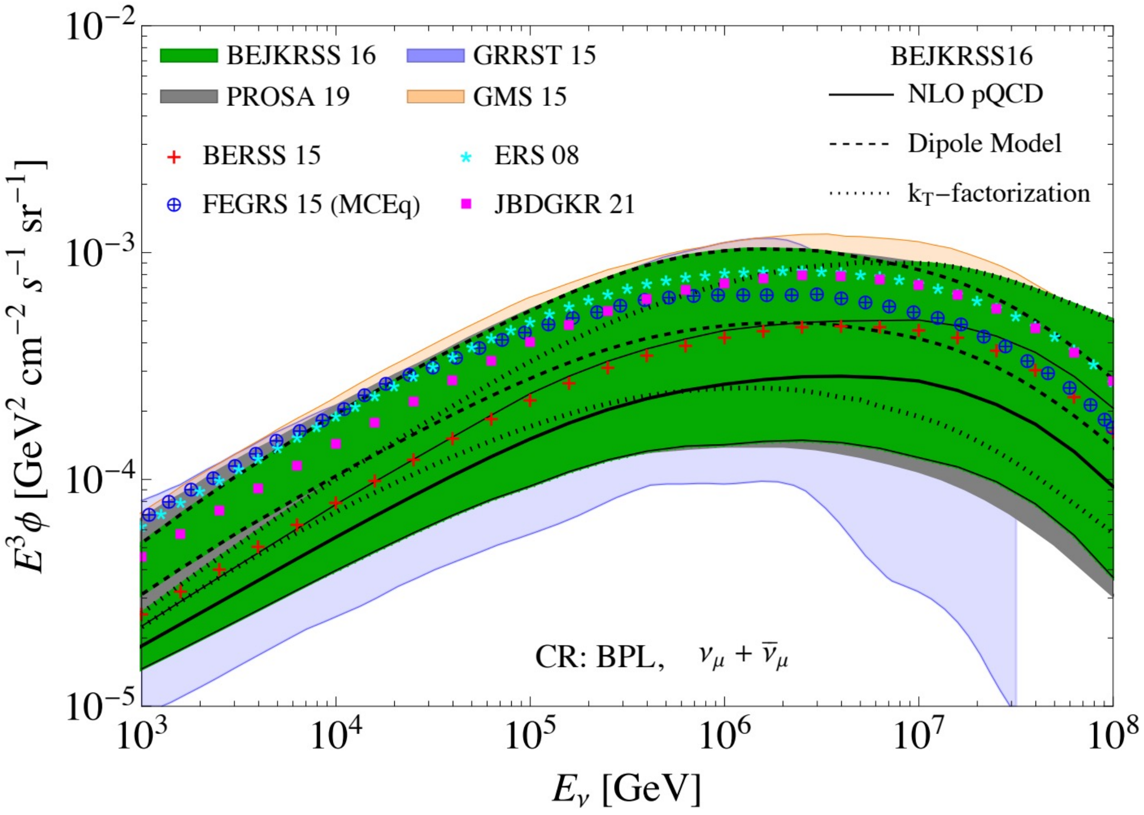
$$\frac{\mathrm{d}}{\mathrm{d}X}\vec{\Phi} = -\vec{\nabla}_E(\mathrm{diag}(\vec{\mu})\vec{\Phi}) + (-\mathbf{1} + \mathbf{C})\mathbf{\Lambda}_{\rm int}\vec{\Phi} + \frac{1}{\rho(X)}(-\mathbf{1} + \mathbf{D})\mathbf{\Lambda}_{\rm dec}\vec{\Phi}$$

Above 100 TeV: territory of the (undiscovered) prompt muons and neutrinos

AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, PRD 100 2019



Forward Physics Facility Snowmass arXiv: 2203.05090

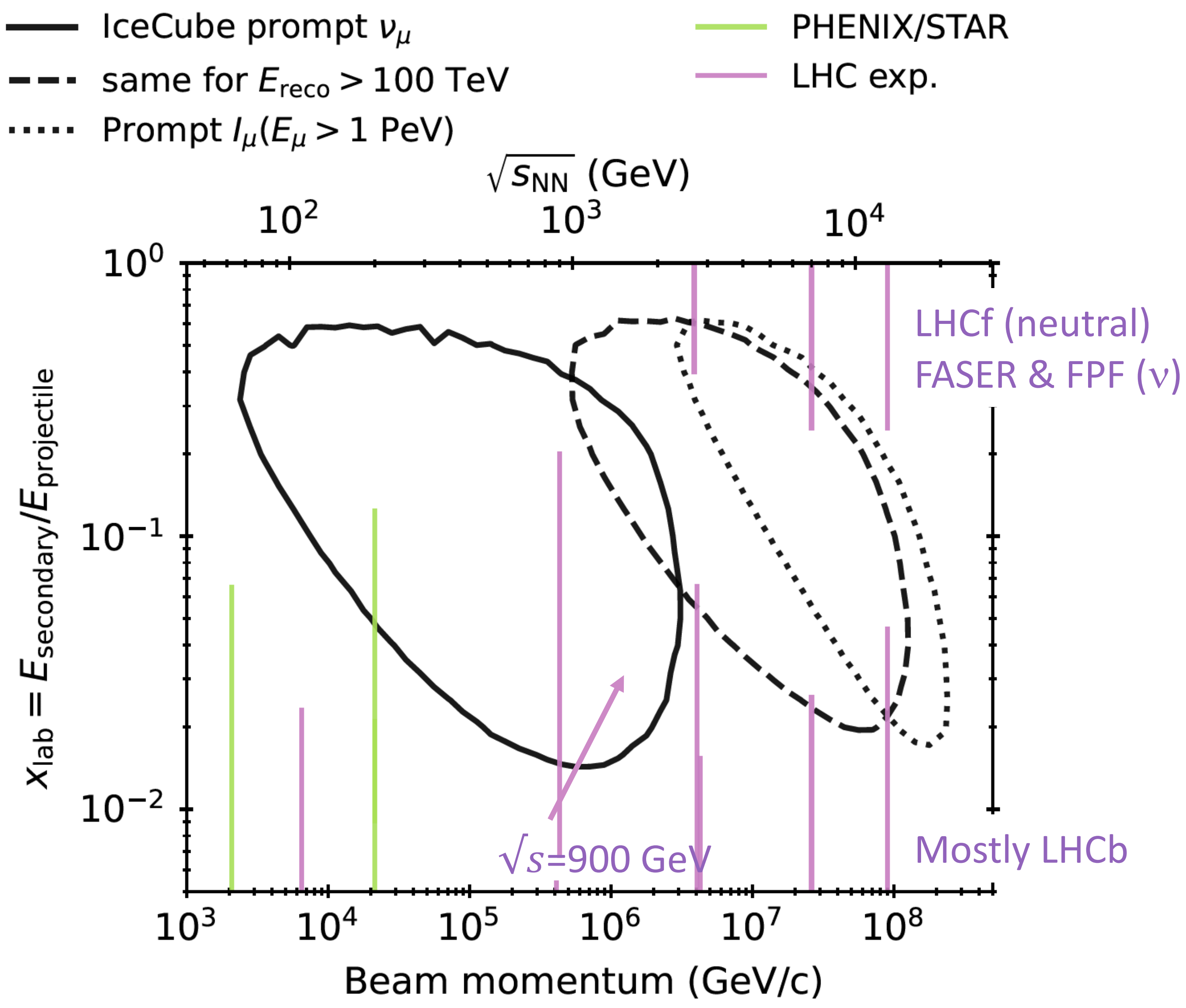


Prompt muons more production channels than prompt neutrinos:

- Rare decays of unflavored mesons *e.g.*, $\eta \rightarrow \mu^+\mu^-$
- EM pair production $\gamma \rightarrow \mu^+\mu^-$

- Large uncertainties from pQCD
- pQCD might be incomplete (intrinsic charm)
- The fragmentation (c $\rightarrow D$) function is a choice

Charm production cross section inaccessible to present-day colliders



- Each line represents a collider running at fixed $\sqrt{s}$
- Gap in x between LHC coverage is due to the beam pipe
- Detectors need particle ID capability & sufficient luminosity
- Indirect constraints from new forward detectors like FASER and the proposed FPF (see 2203.05090)
- New insights expected from proton-oxygen collisions in Run3

AF & M. Huber, arXiv:2205.14766

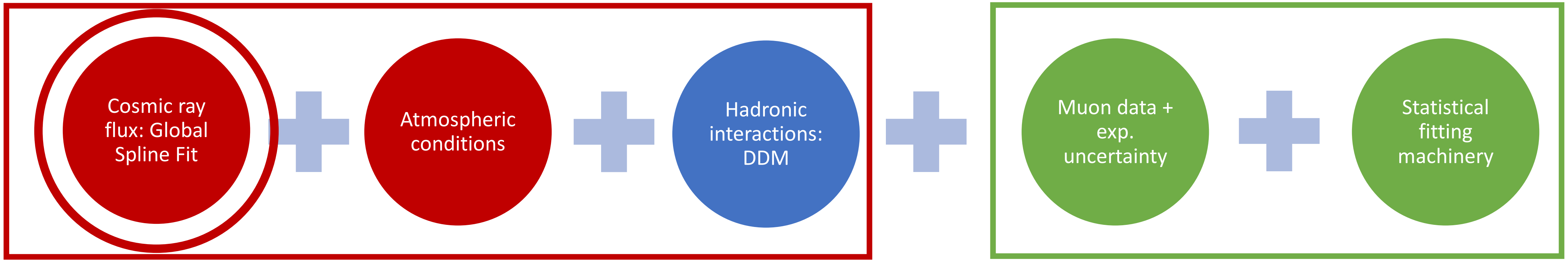
Neutrino contours: 90% events in IceCube

Muon contours: 90% of integral flux

Present challenges for GeV-PeV atmospheric neutrinos

- Modern tools, like MCEq, solve accurately the equations, but flux predictions only as good as the choice of models
- Very difficult to quantify theoretical error
- “Data-driven” models parameterize **data and uncertainty** without imposing a physics-motivated functional form

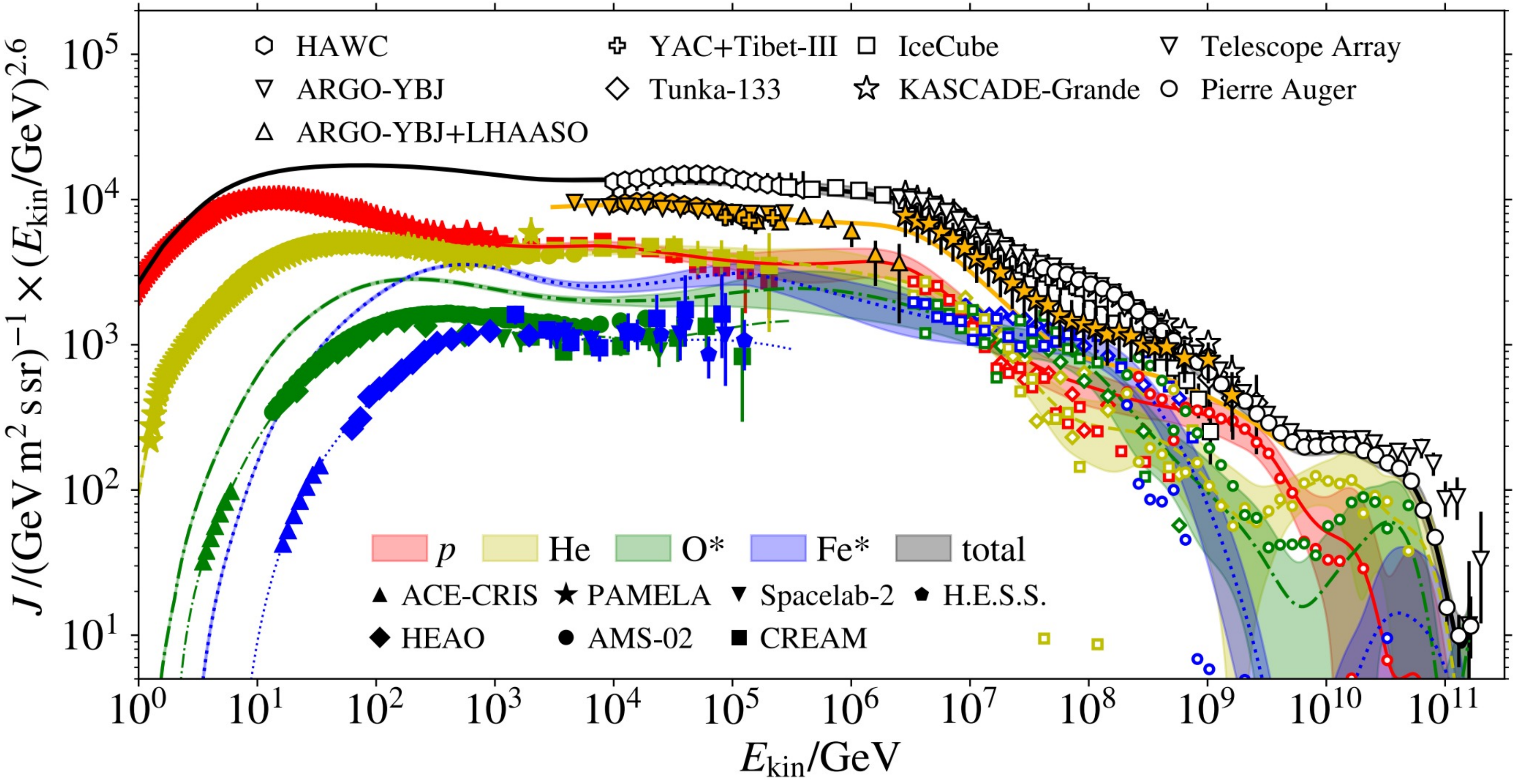
”Flexible” flux model with uncertainty priors from data



Cross-calibration with atmospheric muons

…~5 more bullets for the low-energy fluxes…

The Global Spline Fit (parameterization of CR fluxes)



Pros:

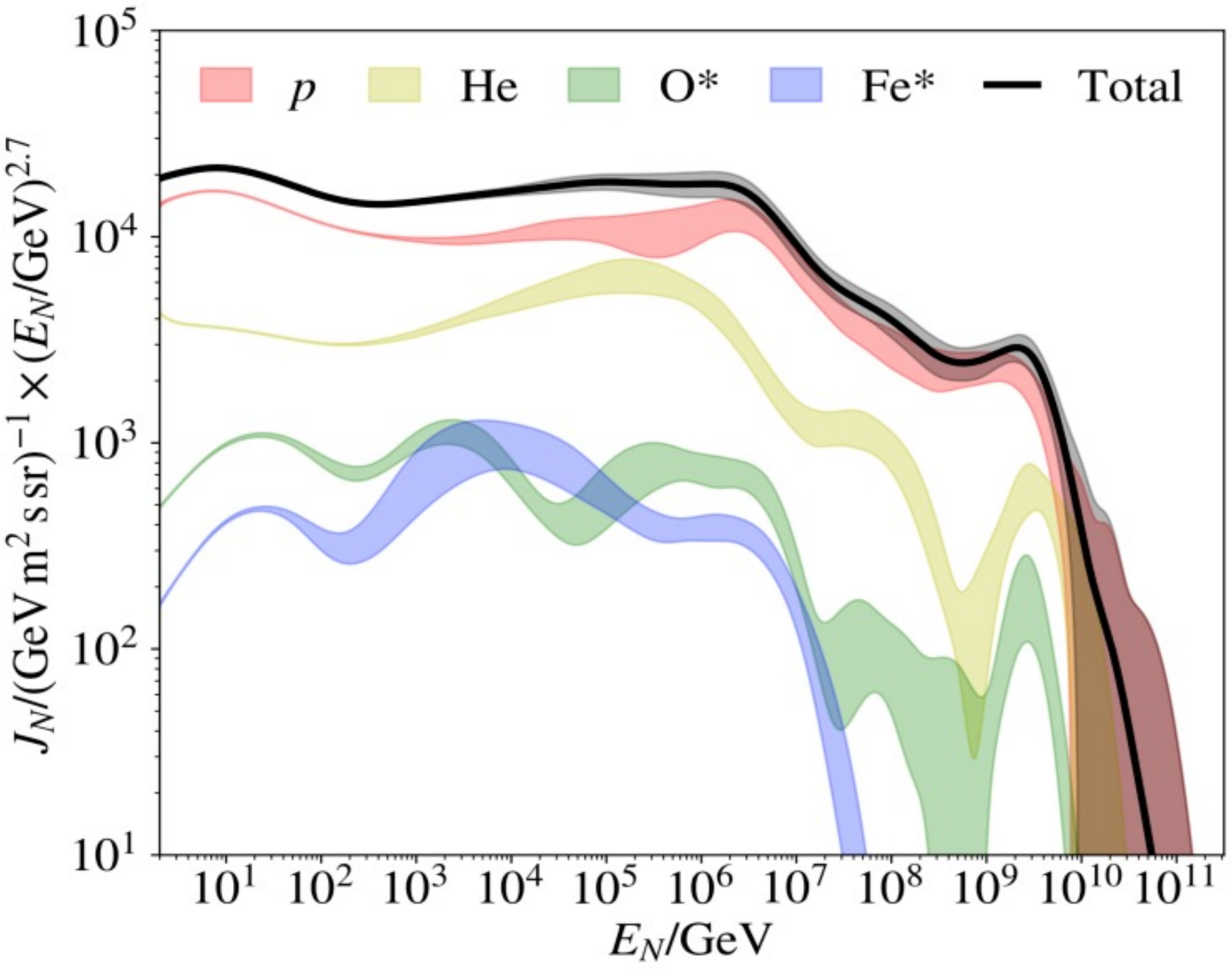
- Parameterizes data
- AND uncertainty
- AND covariance matrix
- Can be updated “easily”

Cons:

- Many parameters
- ~5 * 20 ☺
- Not all equally important for ν fluxes
- Splines somewhat sensitive choice

Dembinski, AF, Gaisser, ICRC 2017 & H. Dembinski 2019

The Global Spline Fit – nucleon fluxes (MCEq input)

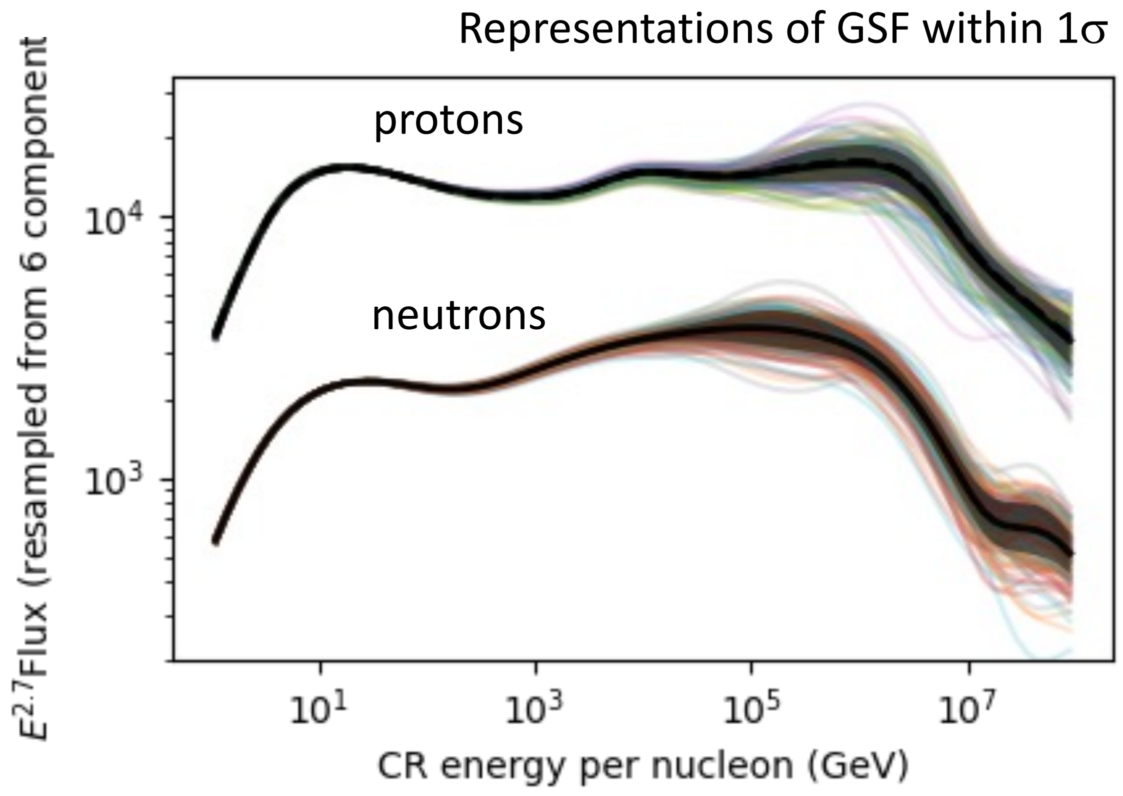


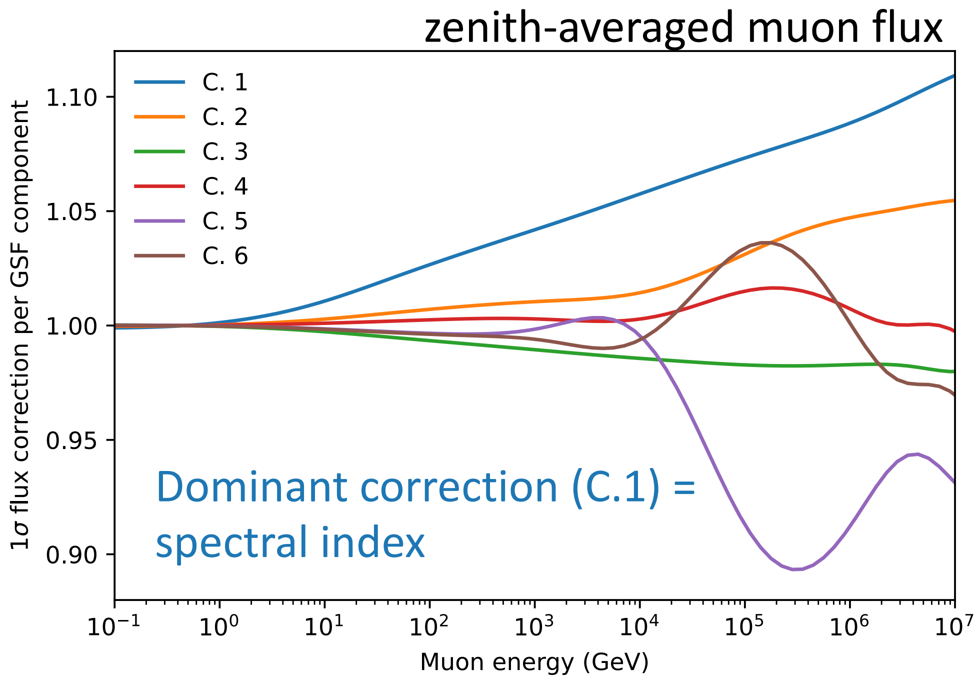
- Most contribution from proton and helium flux
- Correlations between H and He affect
  - CR neutron fraction
  - Muon charge ratio
  - Neutrino/Antineutrino ratio

→ Need to model two correlated components

AF, Dembinski, Engel, Riehn, Gaisser, Stanev ICRC 2017

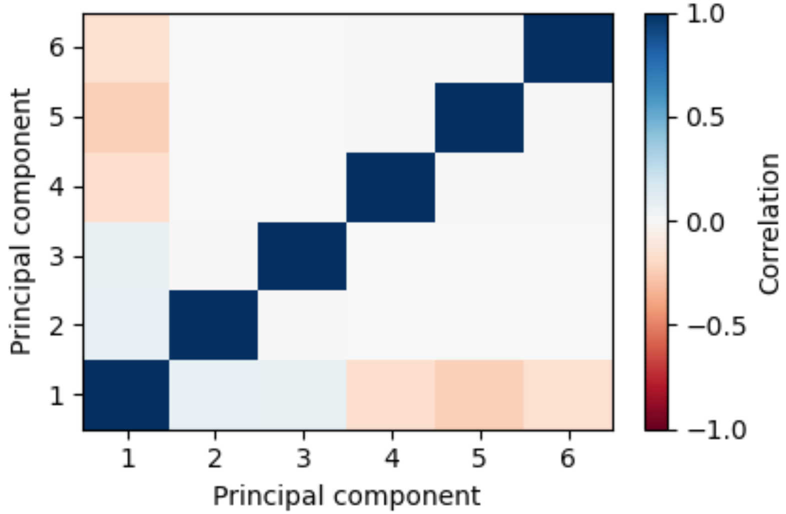
Dimensionality reduction to ~6 parameters





90% of the GSF variance can be explained by 6 components (from PCA)

→ CR nucleon flux represented by weighted sum of 6 weakly base vectors



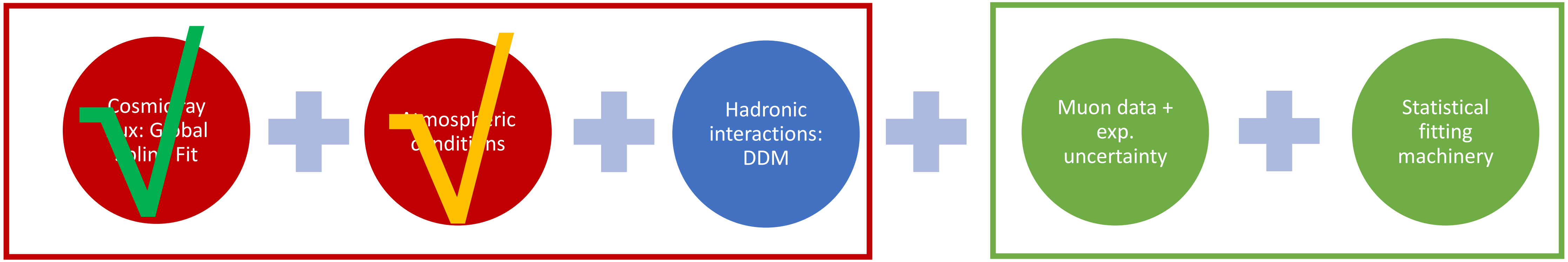
Summary nucleon flux model

→ Data-motivated/-constrained nuissance parameters

→ GSF can be dynamically updated if new data comes in

→ **Optimal CR nucleon flux model for neutrino flux calculations**

”Flexible” flux model with uncertainty priors from data

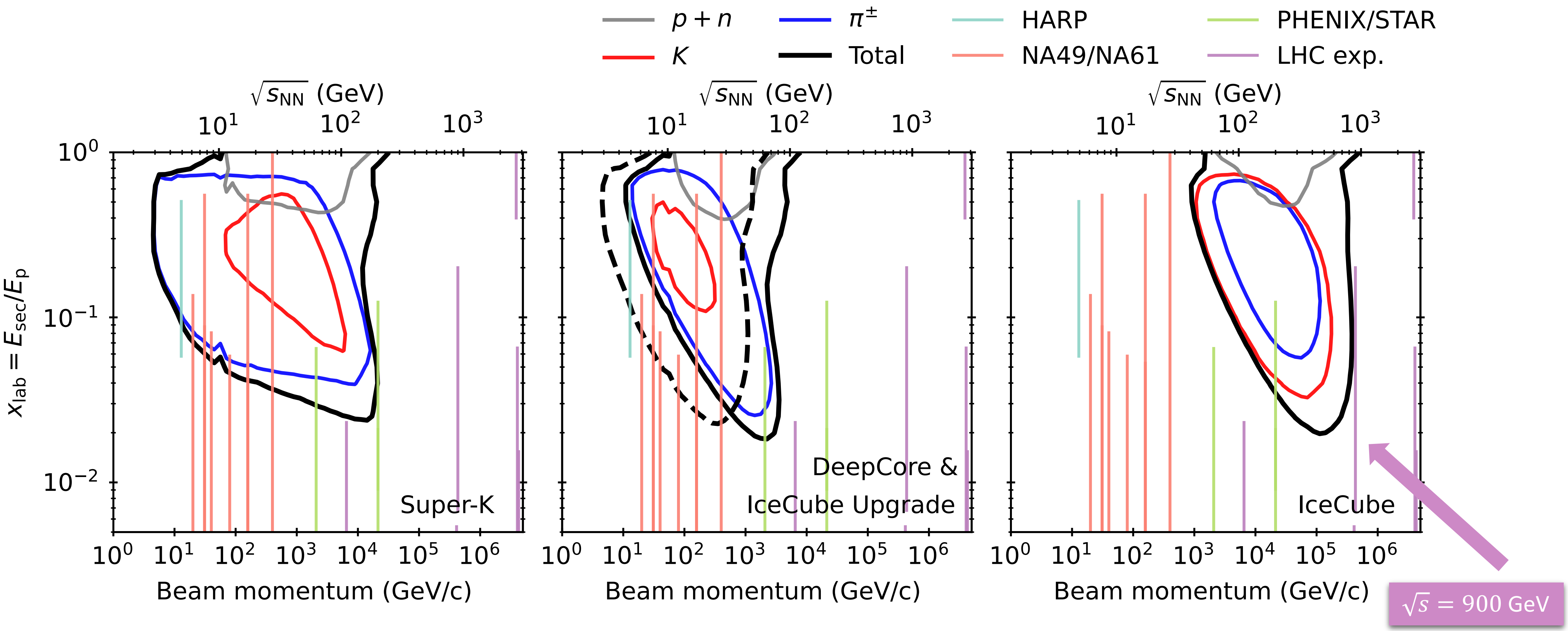


Cross-calibration with atmospheric muons

See also Albrecht et al., Muon Puzzle review, 2105.06148

Hadron production phase space seen by neutrino detectors

AF & M. Huber, arXiv:2205.14766



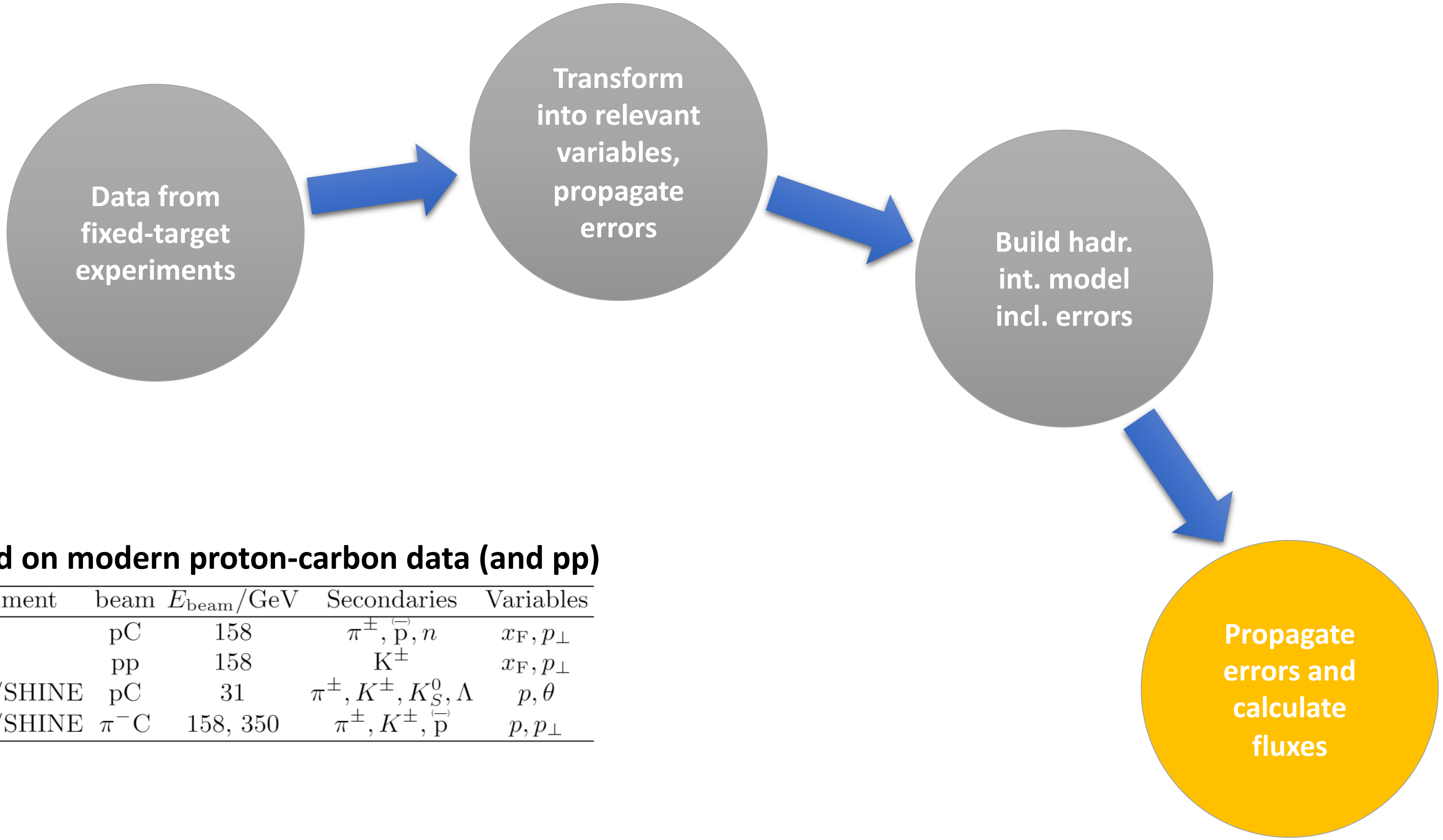
SK: sum of ν_μ FC and PC

DeepCore :
tracks, E_{reco} < 60 GeV (osc.)

IceCube Northern Tracks

Contours = 90% of ν_μ events

Data-Driven Hadronic Interaction Model



Based on modern proton-carbon data (and pp)

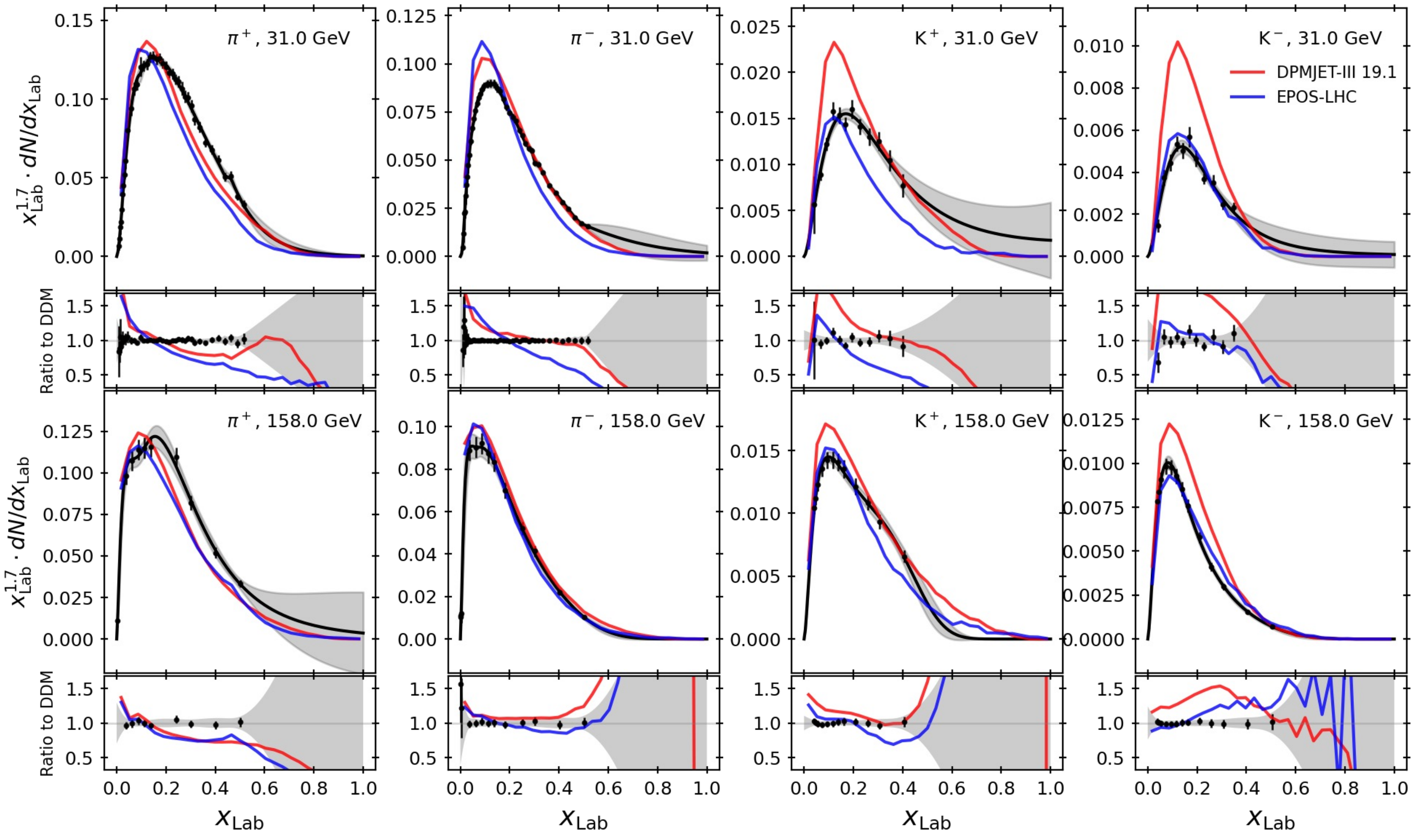
Experiment	beam	$E_{\mathrm{beam}}/\mathrm{GeV}$	Secondaries	Variables
NA49	pC	158	$\pi^{\pm}, \overset{(-)}{\mathrm{p}}, n$	$x_{\mathrm{F}}, p_{\perp}$
NA49	pp	158	$\mathrm{K}^{\pm}$	$x_{\mathrm{F}}, p_{\perp}$
NA61/SHINE	pC	31	$\pi^{\pm}, K^{\pm}, K^0_S, \Lambda$	p, θ
NA61/SHINE	π^-C	158, 350	$\pi^{\pm}, K^{\pm}, \overset{(-)}{\mathrm{p}}$	$p, p_{\perp}$

AF & M. Huber, arXiv:2205.14766

Fits to proton-carbon data and uncertainties

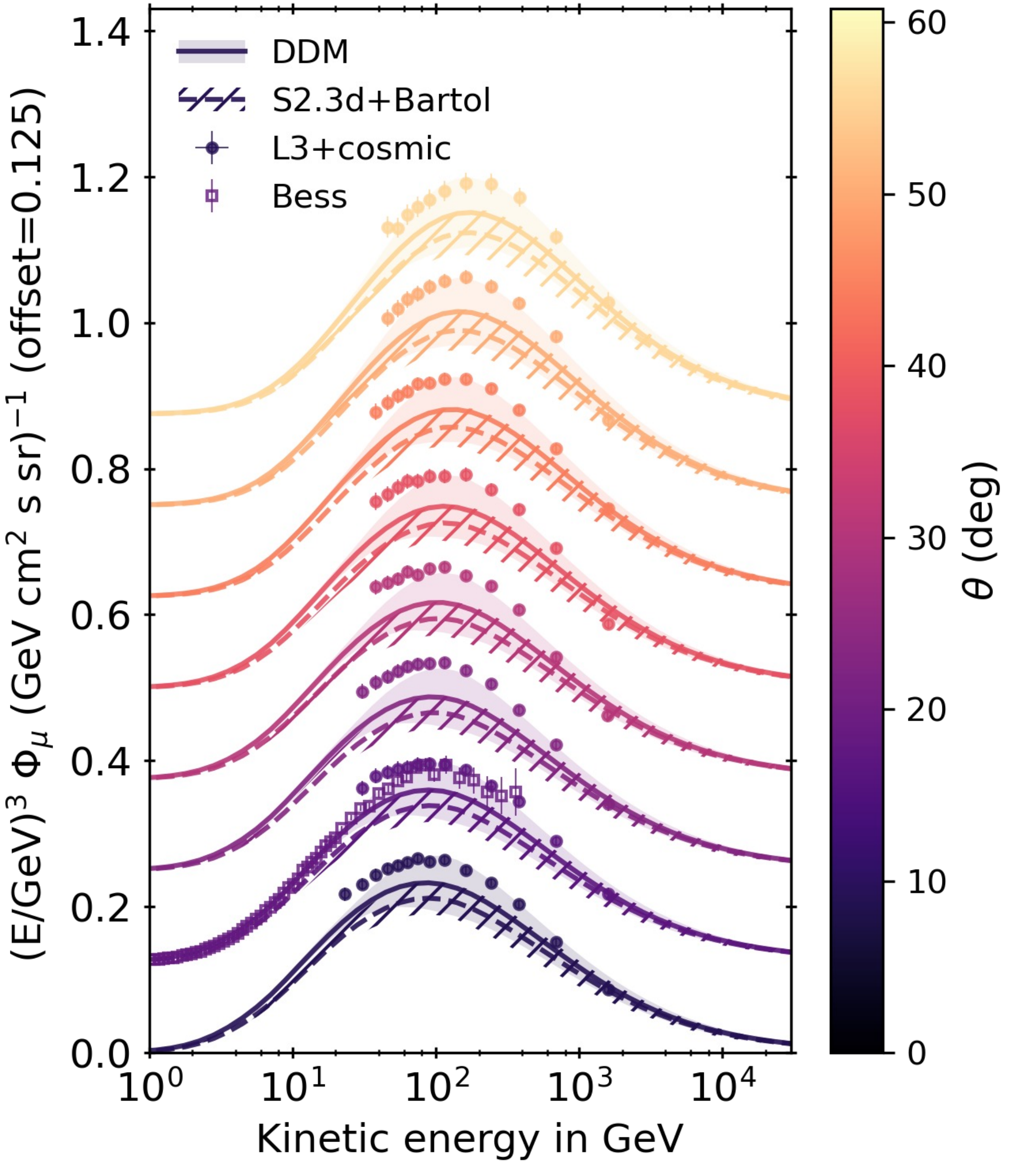
NA49 & NA61 proton-carbon

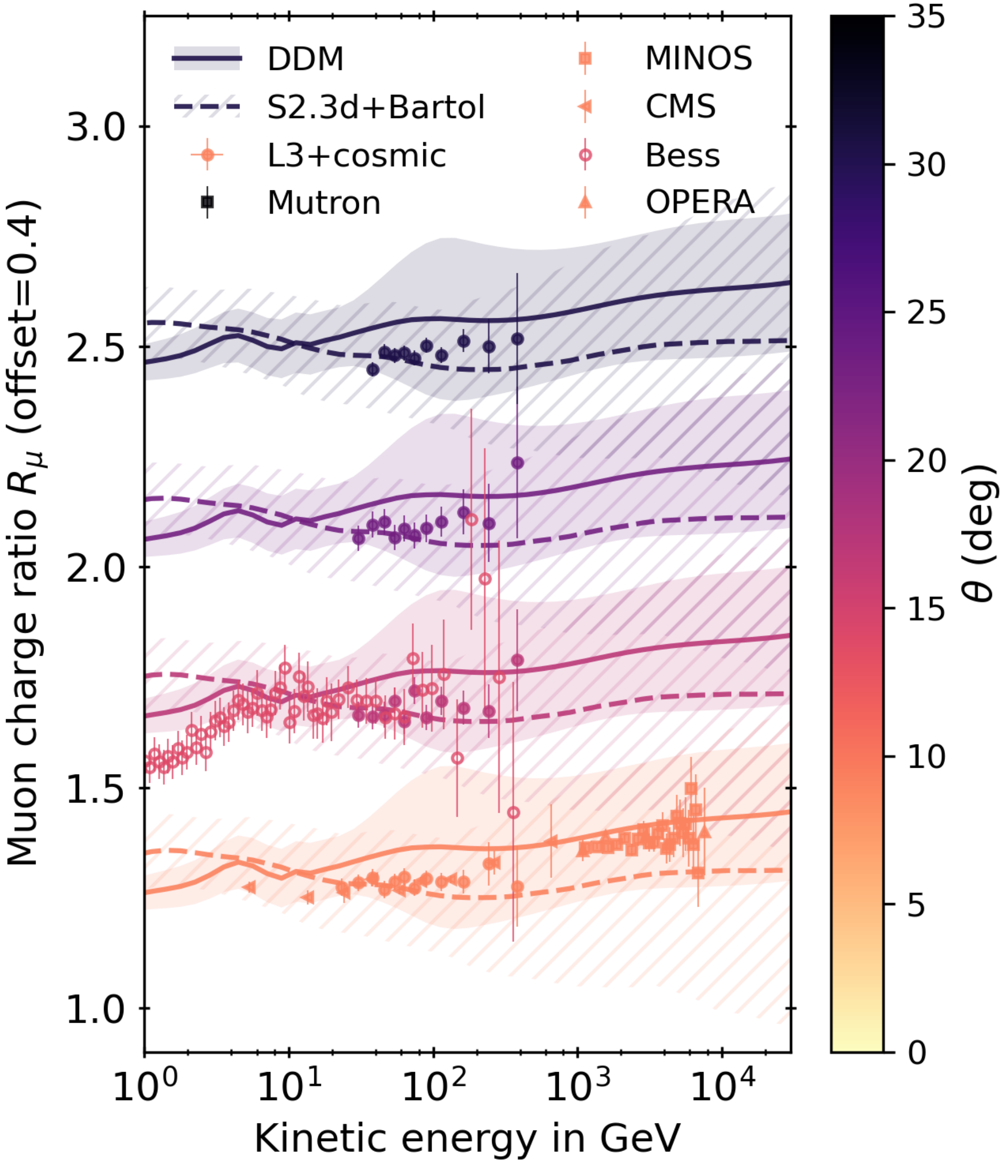
AF & M. Huber, arXiv:2205.14766



- **Uncertainties conservatively scale up** in absence of forward data
- K^{+-} data at 158 GeV extrapolated from pp→pC
  - → + 5-7% error from MC
- Carbon to air correction < 1%
- + proton and neutron secondaries (not shown)
- Neutron and π^+ projectiles via isospin relations
- K^0 via isospin

Description of muon data with DDM + GSF



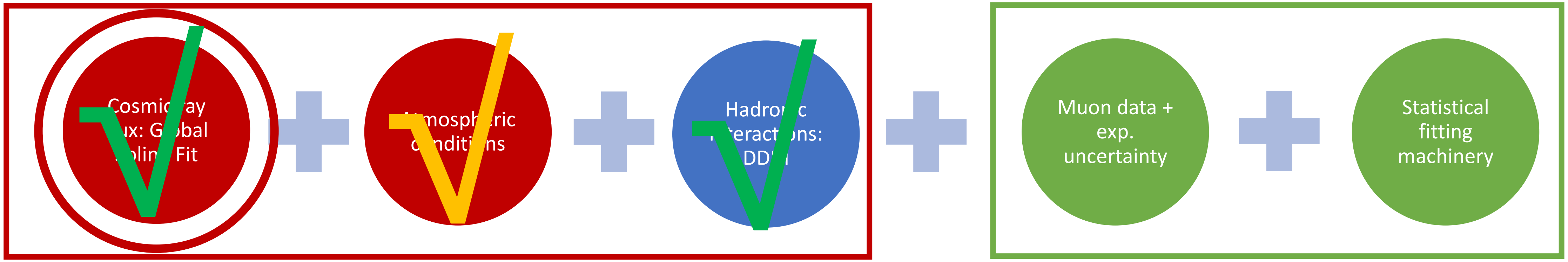


Summary hadronic interaction model

→ Intermediate energy range (100 MeV – 200 GeV) well constrained by NA49 (+61) data

→ More directly usable data available but requires some extrapolation from *e.g.,* p-Be to p-C

”Flexible” flux model with uncertainty priors from data

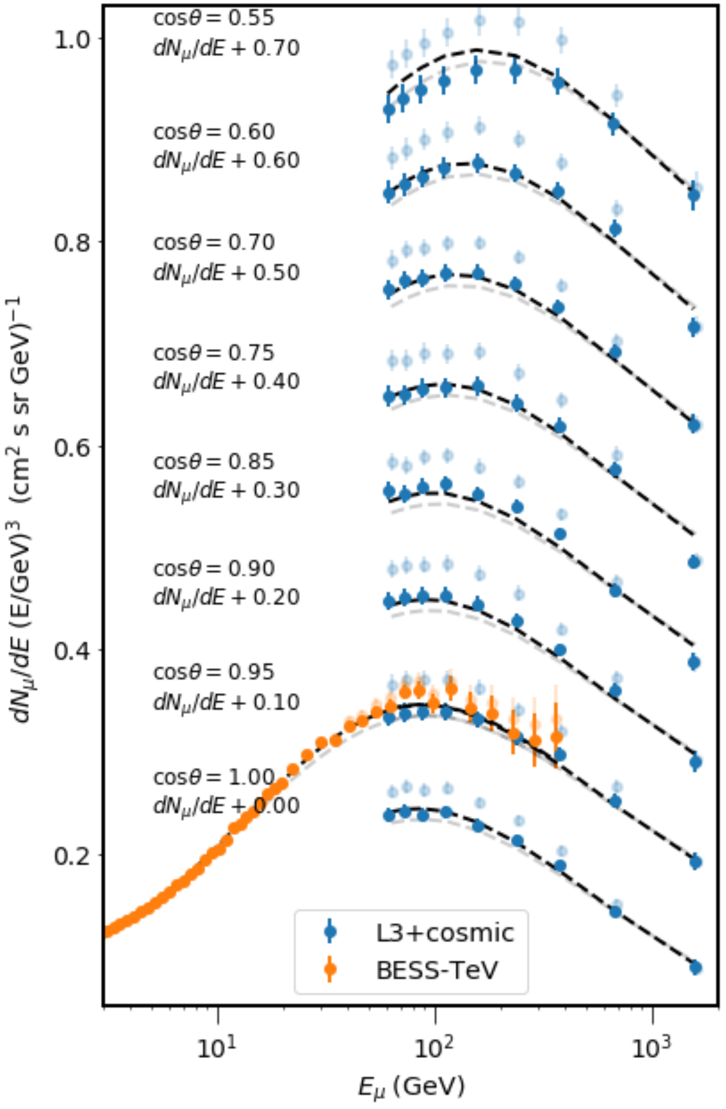


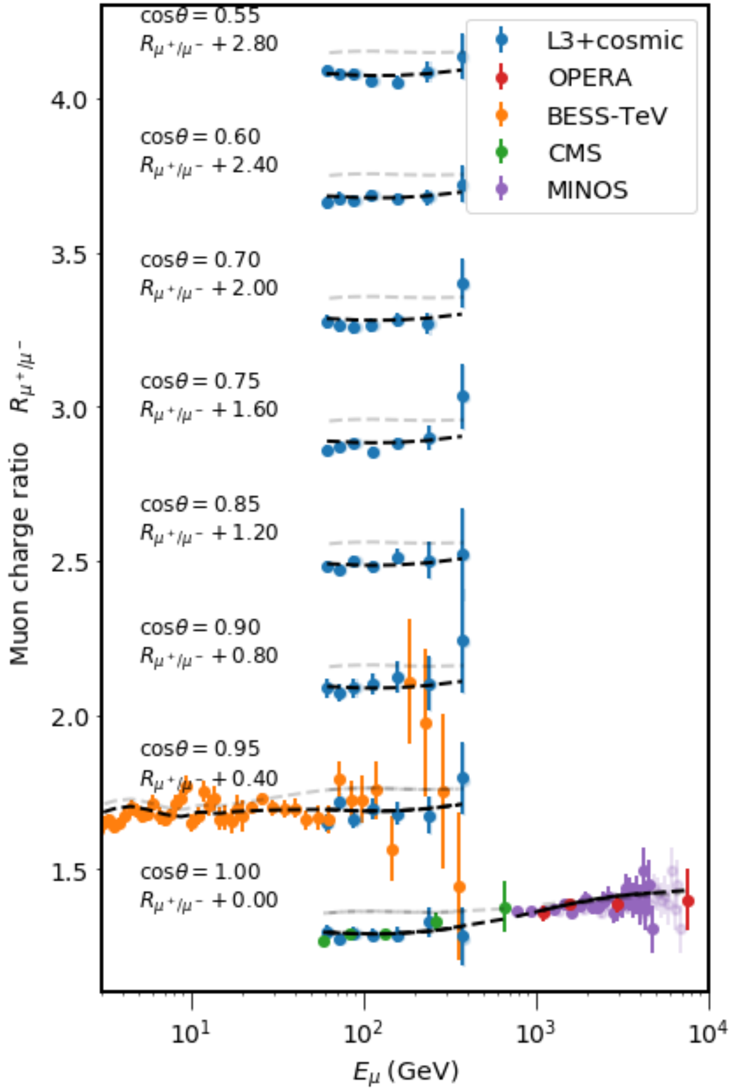
Cross-calibration with atmospheric muons

Low-energy extension for numerical flux calculations

Calibration with surface muon data

J. P. Yanez & AF, ICRC 2019, in prep.





Piece of the correlation matrix:
Total 16 fit parameters, 36 incl. exp nuissance parameters

	le_pi+	le_pi-	p	n	GSF_2	he_K-	le_K+	le_K-	he_K+	he_pi-	GSF_5	GSF_6	GSF_4	GSF_1	he_pi+	GSF_3
le_pi+	1.00	-0.11	-0.22	-0.17	0.00	0.00	-0.77	0.06	-0.01	0.21	-0.02	0.01	0.01	-0.02	0.23	0.00
le_pi-	-0.11	1.00	-0.18	-0.43	0.01	0.01	0.35	-0.59	-0.08	0.04	-0.05	0.02	0.02	0.24	-0.05	0.01
p	-0.22	-0.18	1.00	-0.37	-0.01	-0.00	-0.14	0.01	0.01	-0.12	0.04	-0.00	-0.01	0.01	-0.35	-0.00
n	-0.17	-0.43	-0.37	1.00	-0.01	0.03	-0.08	-0.01	-0.06	-0.40	0.09	-0.02	-0.02	-0.16	-0.03	-0.00
GSF_2	0.00	0.01	-0.01	-0.01	1.00	0.02	0.00	0.00	-0.06	-0.01	0.03	-0.01	-0.01	-0.43	0.00	-0.00
he_K-	0.00	0.01	-0.00	0.03	0.02	1.00	-0.03	0.00	0.51	-0.14	-0.13	0.05	0.02	-0.03	-0.07	0.01
le_K+	-0.77	0.35	-0.14	-0.08	0.00	-0.03	1.00	0.06	0.06	0.13	-0.03	0.02	0.01	0.00	0.06	0.00
le_K-	0.06	-0.59	0.01	-0.01	0.00	0.00	0.06	1.00	-0.01	-0.00	0.00	-0.00	0.00	0.00	0.01	0.00
he_K+	-0.01	-0.08	0.01	-0.06	-0.06	0.51	0.06	-0.01	1.00	0.32	0.37	-0.14	-0.06	-0.13	0.04	-0.02
he_pi-	0.21	0.04	-0.12	-0.40	-0.01	-0.14	0.13	-0.00	0.32	1.00	0.02	0.03	-0.02	-0.47	0.82	-0.00
GSF_5	-0.02	-0.05	0.04	0.09	0.03	-0.13	-0.03	0.00	0.37	0.02	1.00	0.06	0.04	0.07	-0.02	0.01
GSF_6	0.01	0.02	-0.00	-0.02	-0.01	0.05	0.02	-0.00	-0.14	0.03	0.06	1.00	-0.01	0.24	0.05	-0.00
GSF_4	0.01	0.02	-0.01	-0.02	-0.01	0.02	0.01	0.00	-0.06	-0.02	0.04	-0.01	1.00	-0.11	-0.01	-0.00
GSF_1	-0.02	0.24	0.01	-0.16	-0.43	-0.03	0.00	0.00	-0.13	-0.47	0.07	0.24	-0.11	1.00	-0.56	0.43
he_pi+	0.23	-0.05	-0.35	-0.03	0.00	-0.07	0.06	0.01	0.04	0.82	-0.02	0.05	-0.01	-0.56	1.00	0.00
GSF_3	0.00	0.01	-0.00	-0.00	-0.00	0.01	0.00	0.00	-0.02	-0.00	0.01	-0.00	-0.00	0.43	0.00	1.00

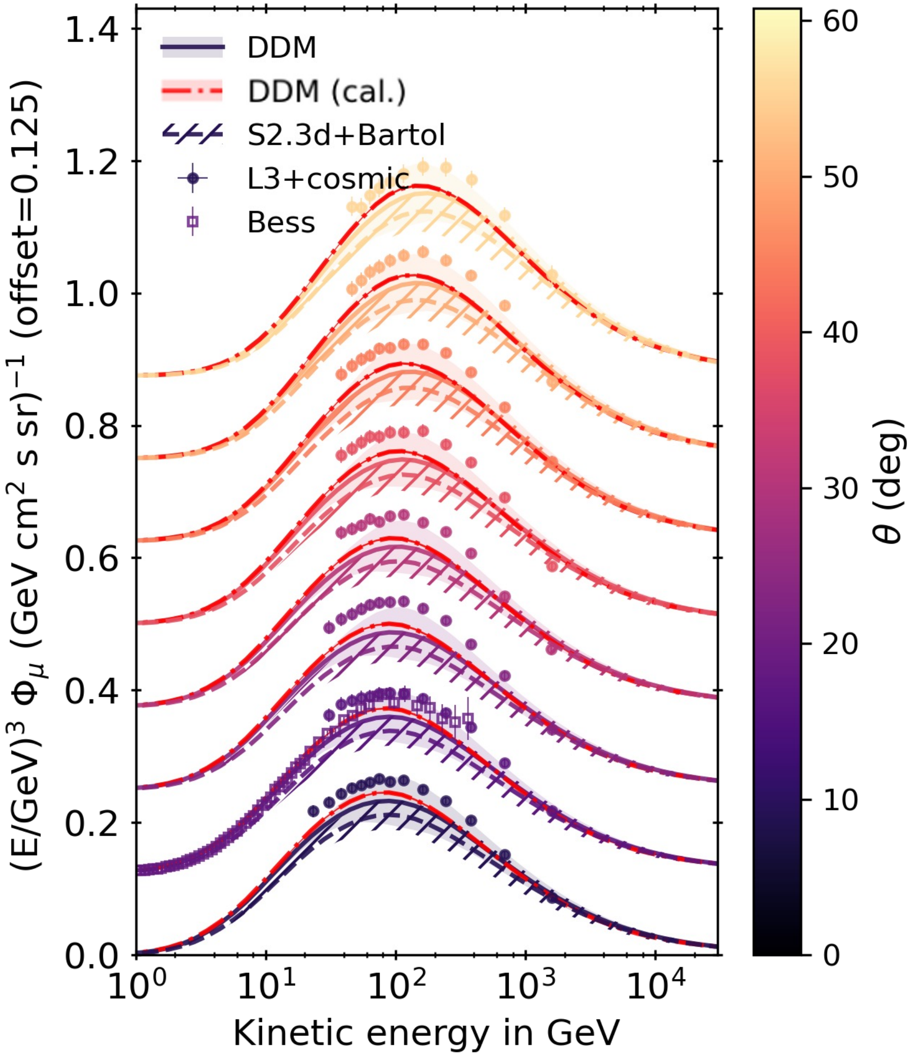
Chi2: ~409
410 dof (approximate)

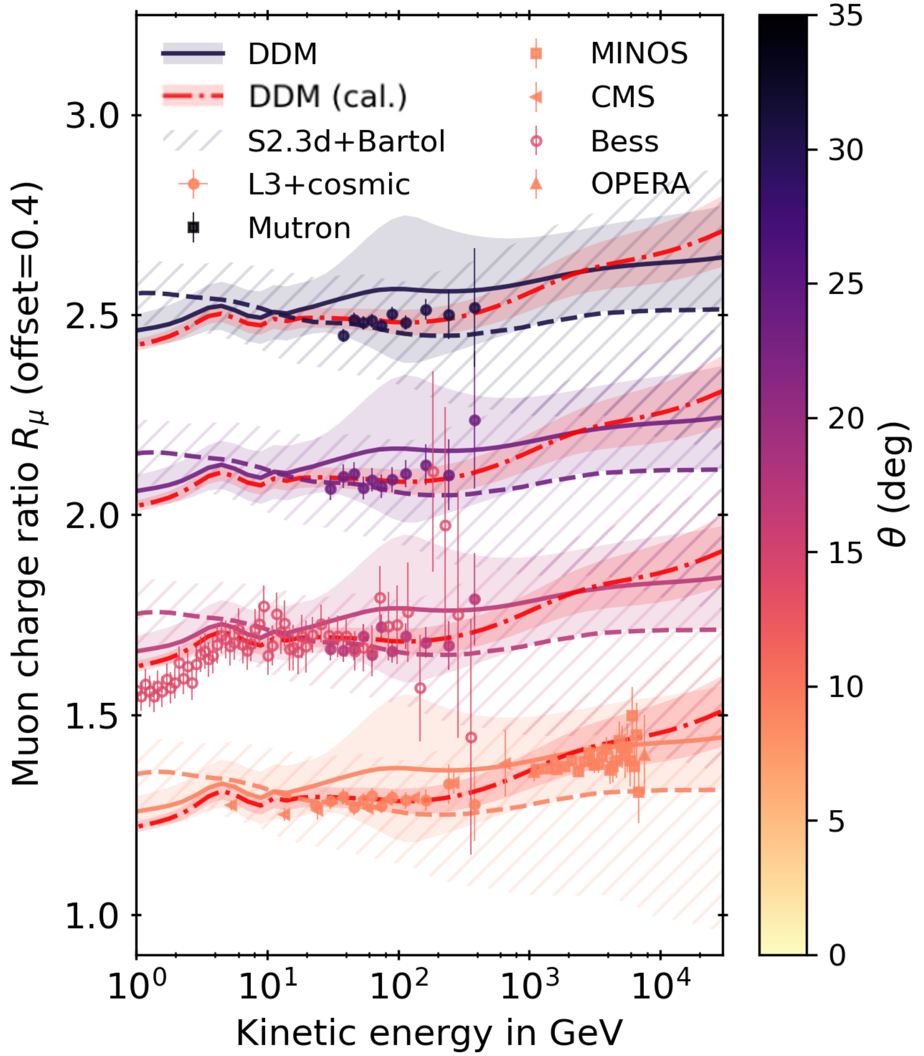
Impact of calibration: muons

hatched area: uncertainty from

Barr et al. PRD74, 094009 (2006) & AF, Dembinski, Engel, Riehn, Gaisser, Stanev ICRC 2017

SIBYLL2.3: AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, PRD 100 2019

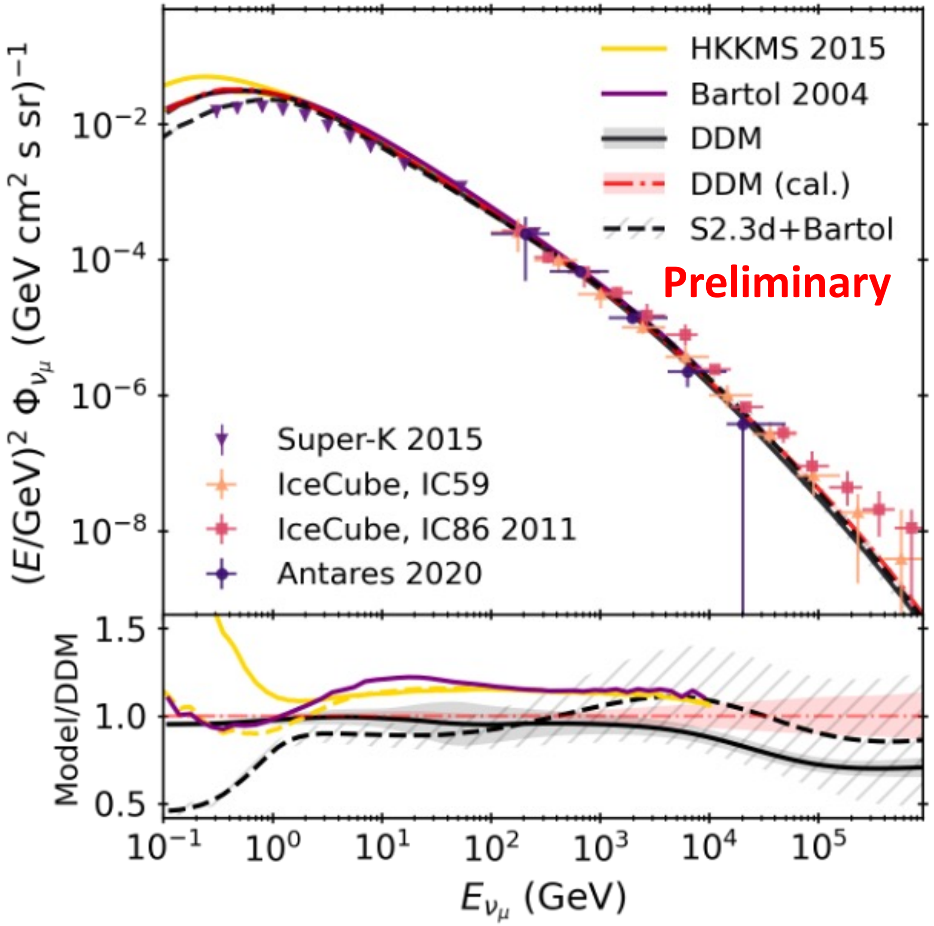




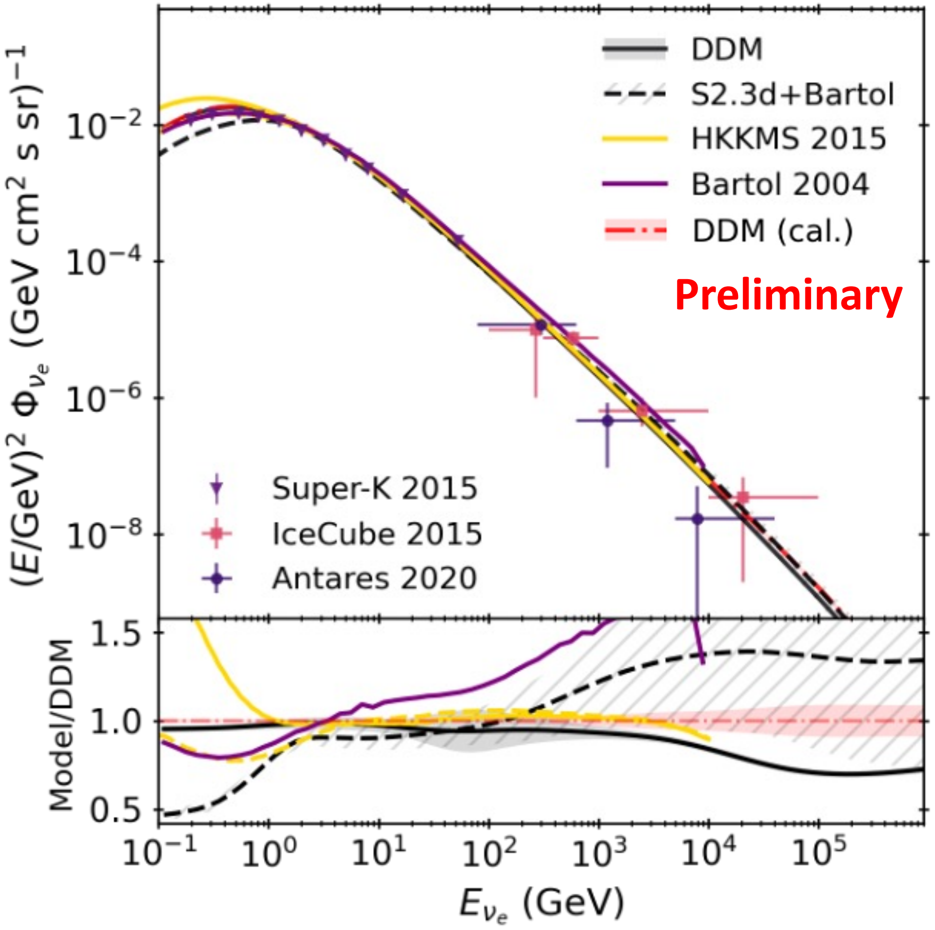
Shown data uncorrected for systematics

Neutrino fluxes

Muon neutrinos

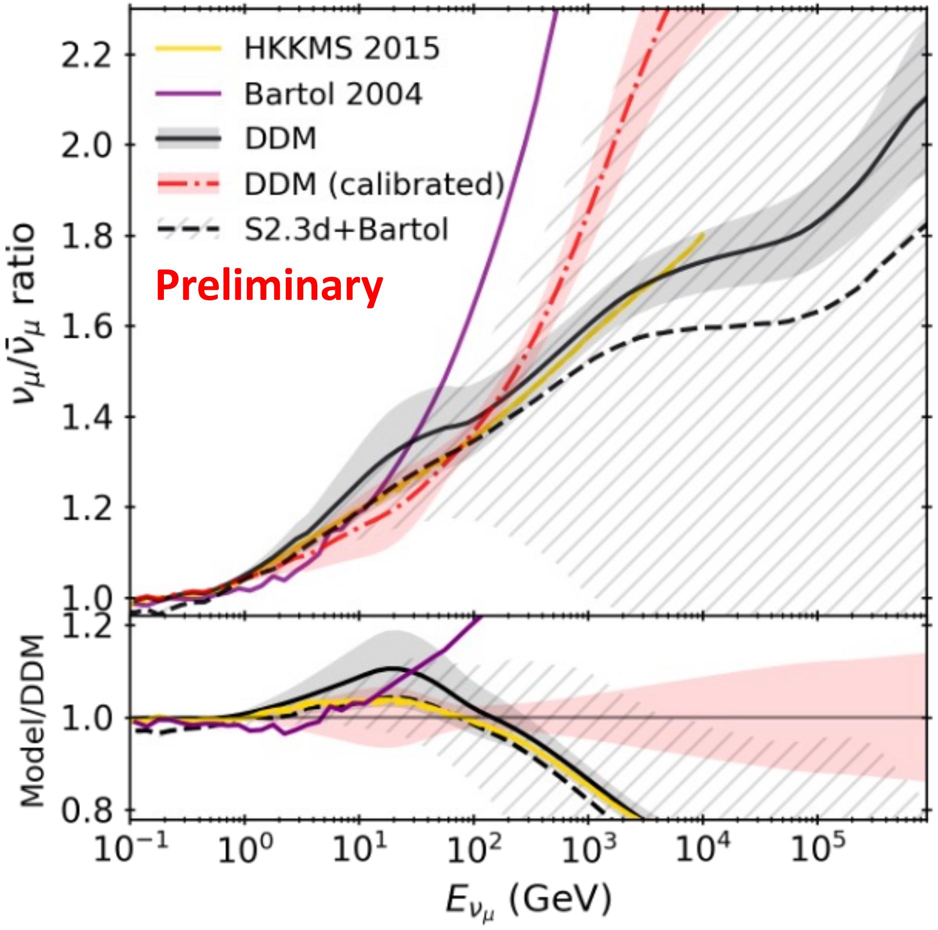


Electron neutrinos

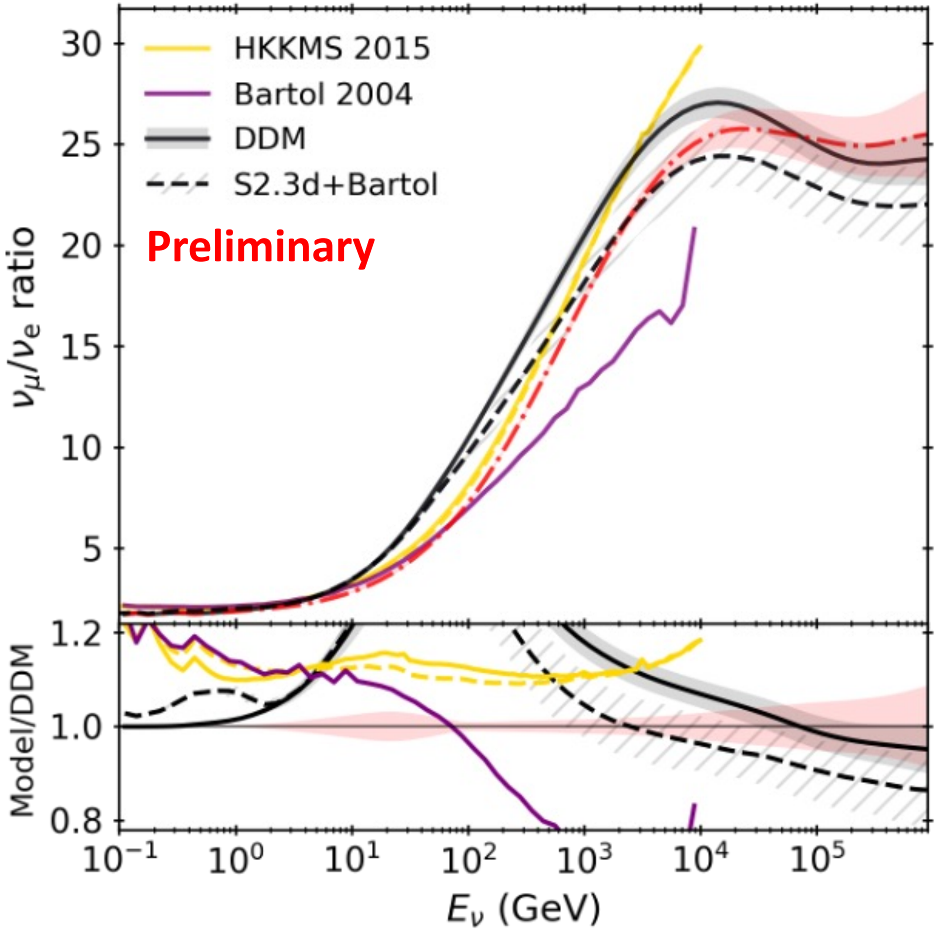


Neutrino ratios

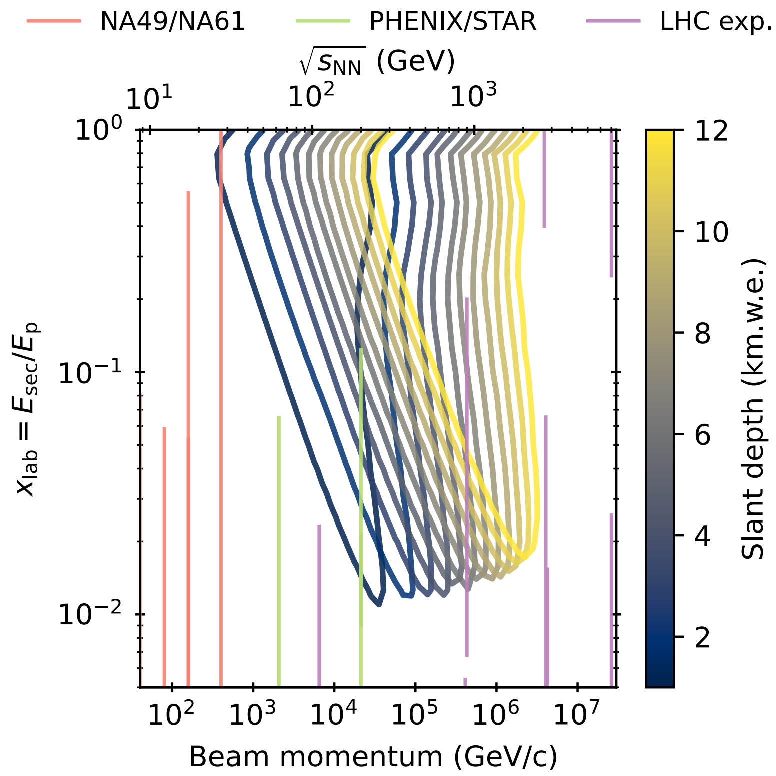
Muon neutrino/antineutrino ratio



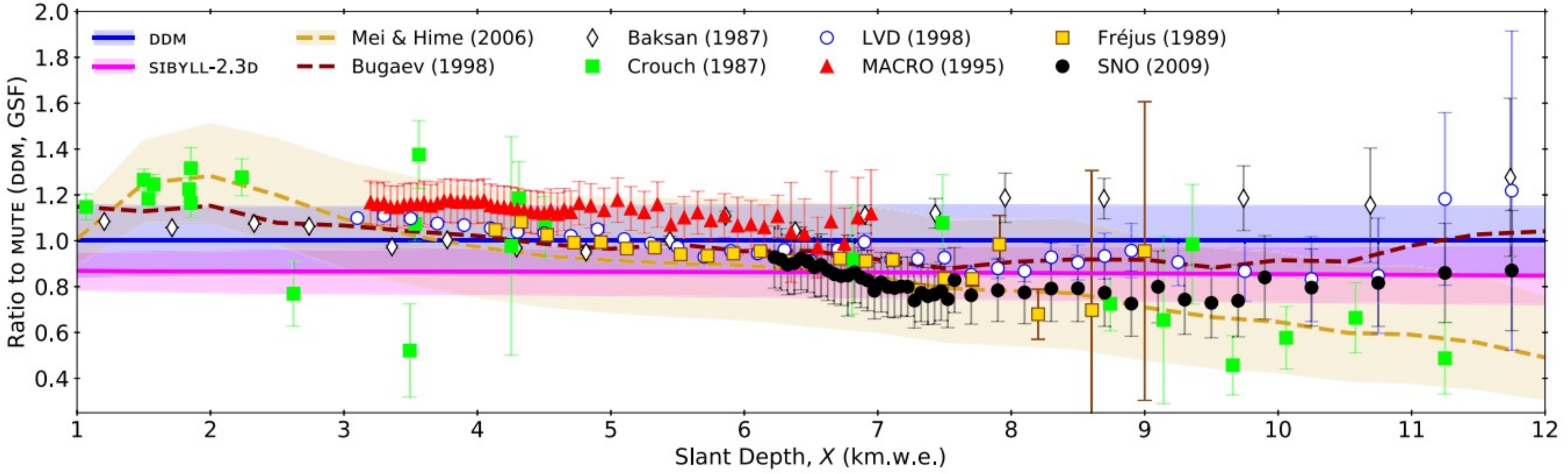
Flavor ratio



High energy constraints from deep underground μ



AF, W. Woodley, M.-C. Piro, *ApJ* **928** 27 (2022)



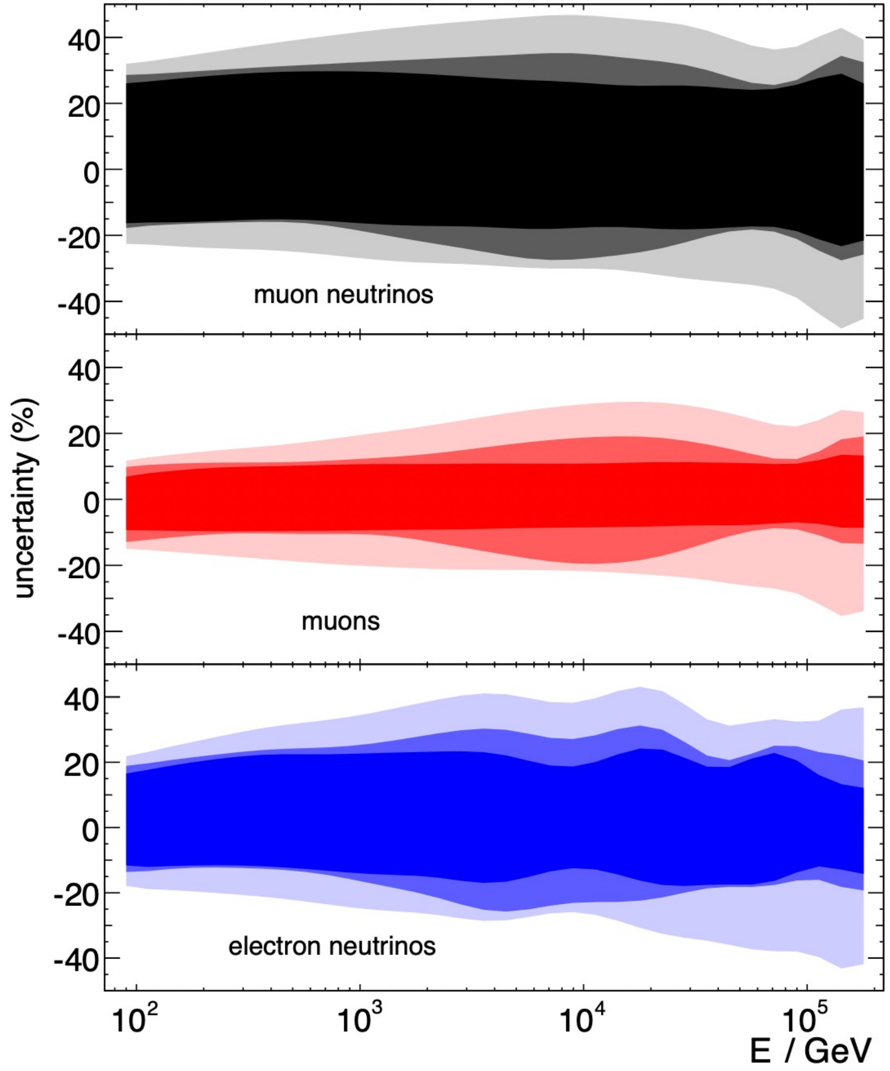
See poster by W. Woodley for progress

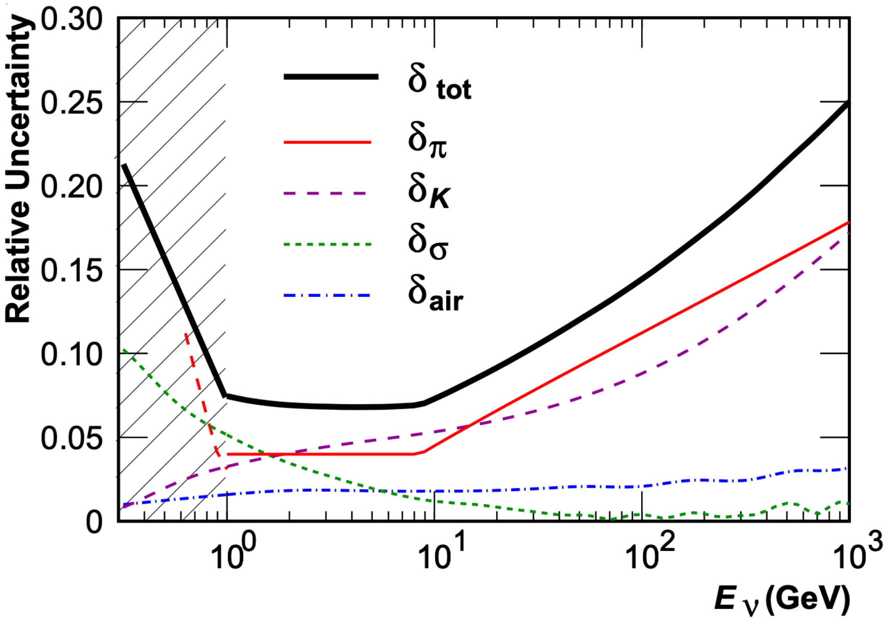
Summary

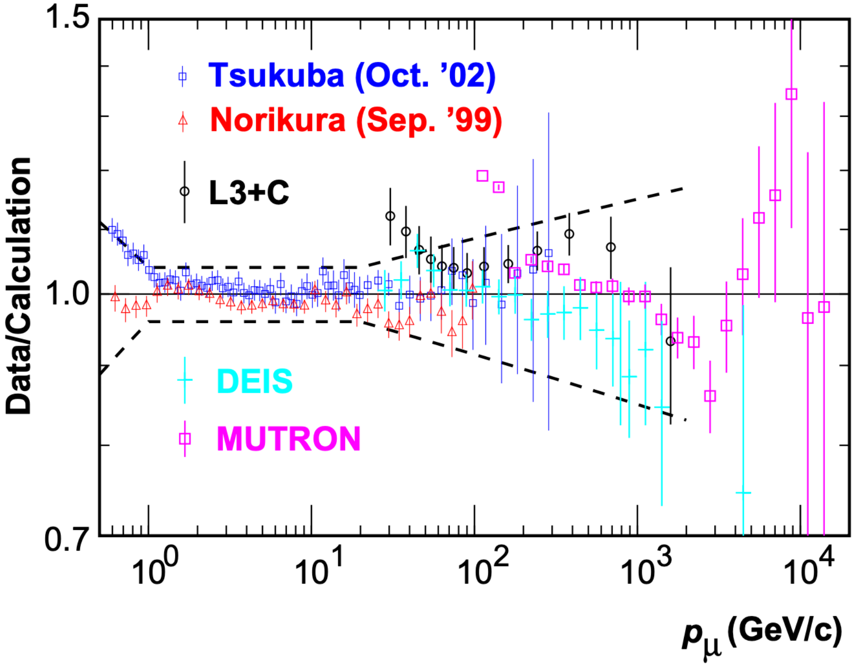
- Step forward made in high-energy flux predictions using data-driven models with uncertainties.
- Validation/calibration via muon surface fluxes challenging. But present results are robust + consistent, since (almost all) muon datasets mutually compatible after correcting for systematic errors.
- D-meson production and CR fluxes will be the dominant uncertainties at higher energies.
- First constraints on prompt neutrino fluxes more likely from neutrino observatories and FASER/FPF.
- Not evident how to use data from colliders in a DDM-like model due to phase space coverage. Collider (LHC) data useful to develop the “physical” models/event generators.
- For high-energy neutrino detectors proton-oxygen runs at 900 GeV center of mass could be very interesting. Higher energies miss the relevant (directly constraining) phase space.
- Global fit of model parameters using fixed-target data, direct + indirect CR data, surface and deep underground muon data will **reduce the neutrino flux uncertainty below 10% E < 500 TeV, to <5% below for E < 100 GeV**, down from >20—40%

Previous error estimates I

AF et al., PRD, 2012. Choose different models and estimate band by bracketing







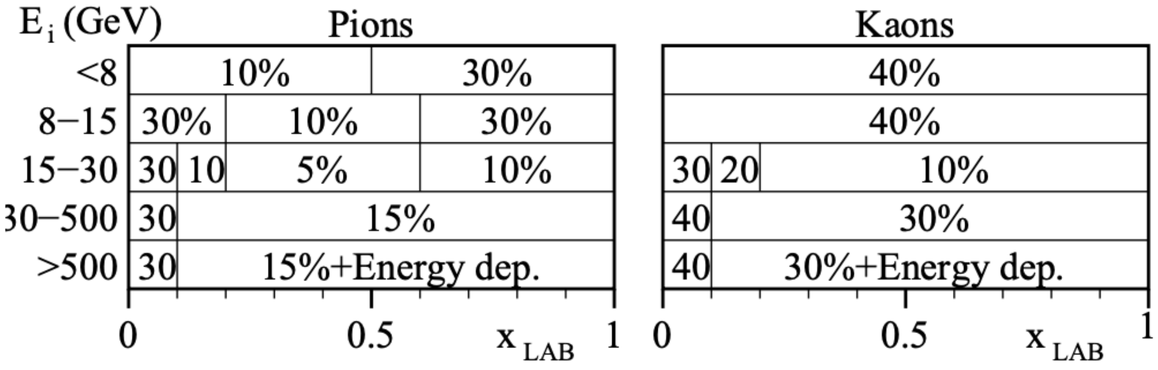
Honda et al. 2006 & Saniki et al. 2007:

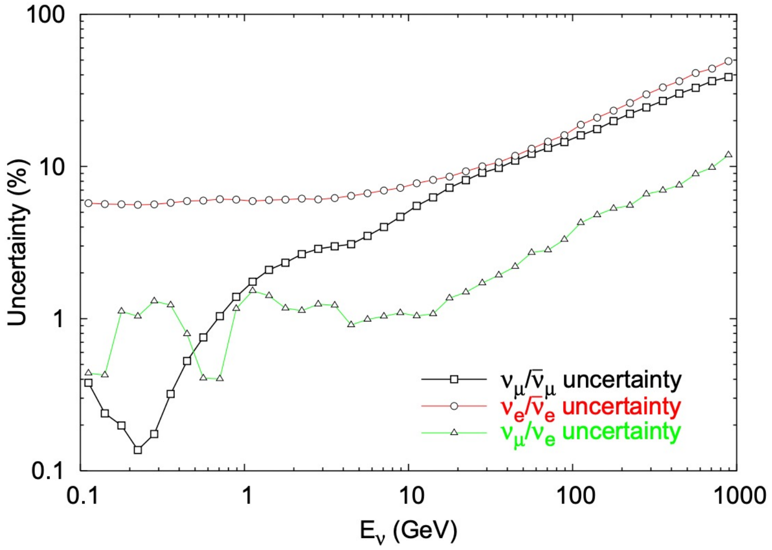
Uncertainty from eye-balling the description of muon data, and proportional rescaling to neutrino fluxes.

$$\frac{\Delta\phi_\mu}{\phi_\mu} \simeq \frac{\Delta\phi_{\nu_\mu}}{\phi_{\nu_\mu}} \simeq \frac{\Delta\phi_{\nu_e}}{\phi_{\nu_e}}$$

.. **This is a crude approximation** as one can see on the left plot…

The Bartol scheme and some of its problems





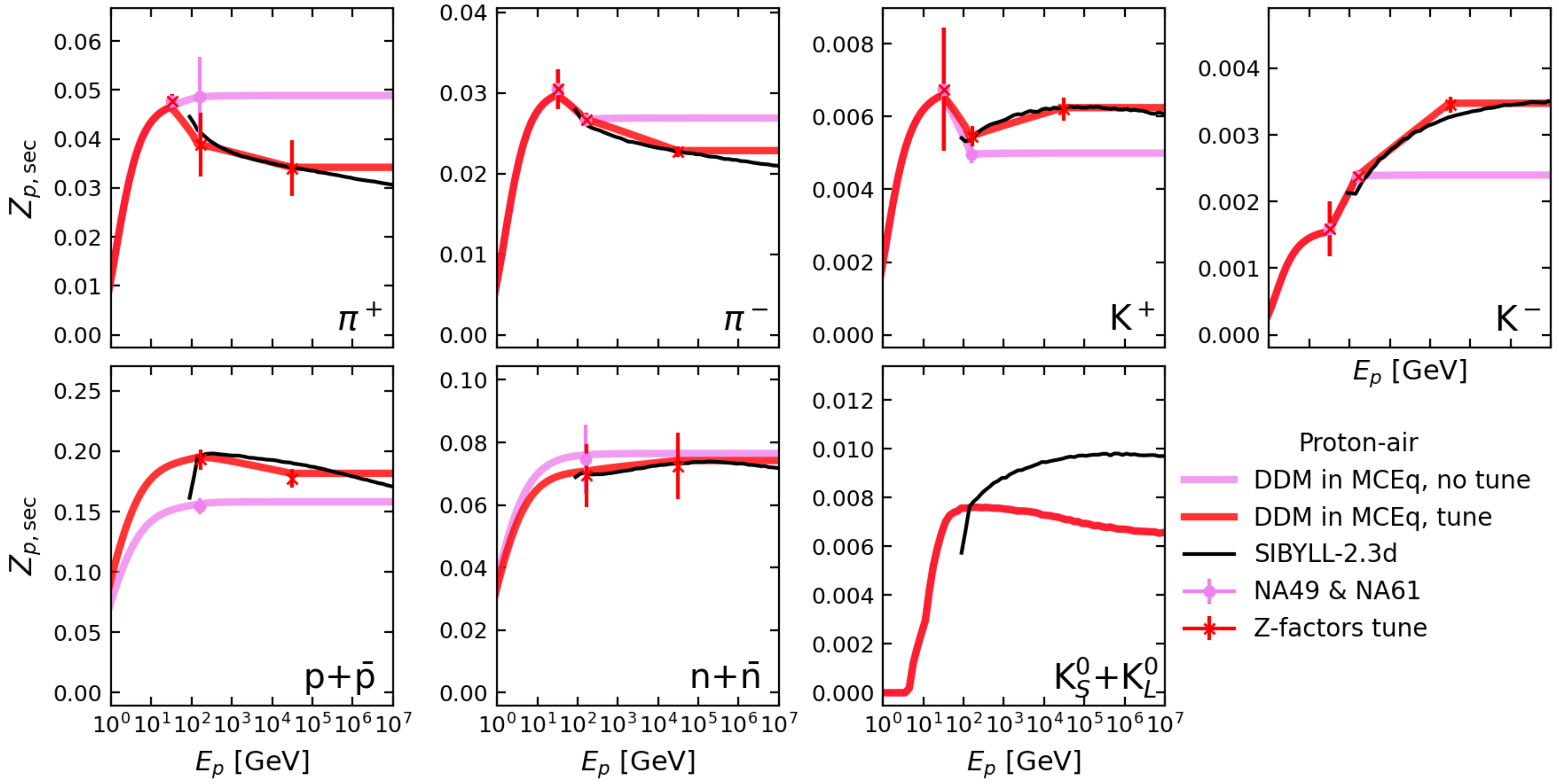
The scheme:

- For the interested, please read carefully page 4-5 of the Barr et al. PRD74, 2006
- Data from accelerators has been scouted carefully
- Experimental errors and an additional error if data disagrees between experiments were collected
- This error has been attached as weight to the Bartol 2004 flux MC calculation in 1D mode
- The calculation propagates the error from (E_proton, E_pion) phase space to the neutrino fluxes
- Similar approach with CR flux

Points to think about:

- The model used in the Bartol calculation never claimed to reproduce the data, and “probably no model ever will”
- Data not entirely checked for correctness, or sufficient completeness (true detector acceptance)
  - Extrapolation errors of the data from e.g. Aluminum → Nitrogen assigned by hand
  - No thick/thin target corrections
  - No feed-down corrections… not performed at ISR or anything before the 1990’s
- But blew up errors generously → **concervative estimate** → overestimation of errors

Energy inter- and extrapolation



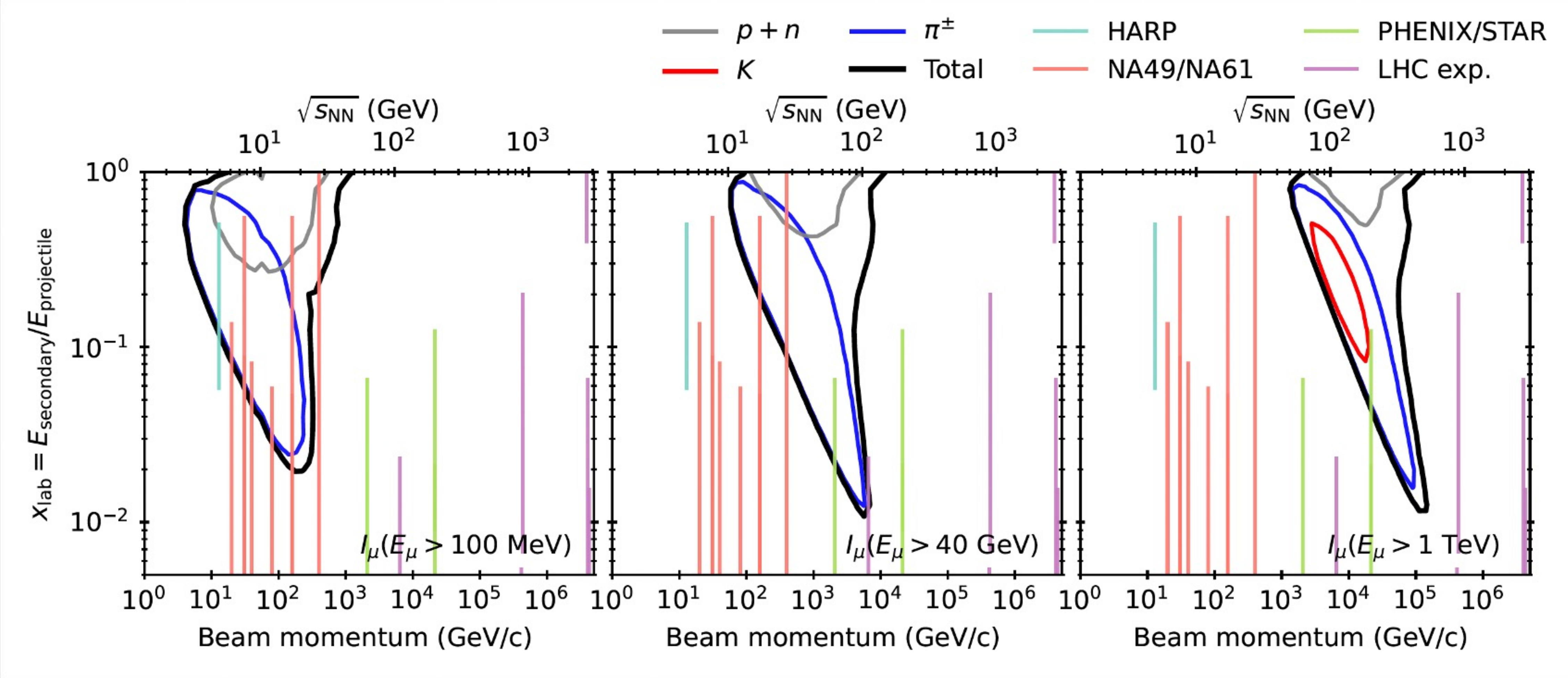
- 1 or 2 cross section "shapes" @ 31 & 158 GeV
- Interpolates linearly in log(E) between those
- Assumes Feynman scaling (shape of longitudinal spectrum constant)
- More points can be added to complicate energy dependence

Atm.-flux-relevant phase space
→ Spectrum-weighted moment:

$$Z_{\mathrm{N}h}(E_\mathrm{N}) = \int_0^1 \mathrm{d}x_\mathrm{Lab}\; x_\mathrm{Lab}^{\gamma(E_\mathrm{N})-1} \frac{\mathrm{d}N_{\mathrm{N}\to h}}{\mathrm{d}x_\mathrm{Lab}}(E_\mathrm{N})$$

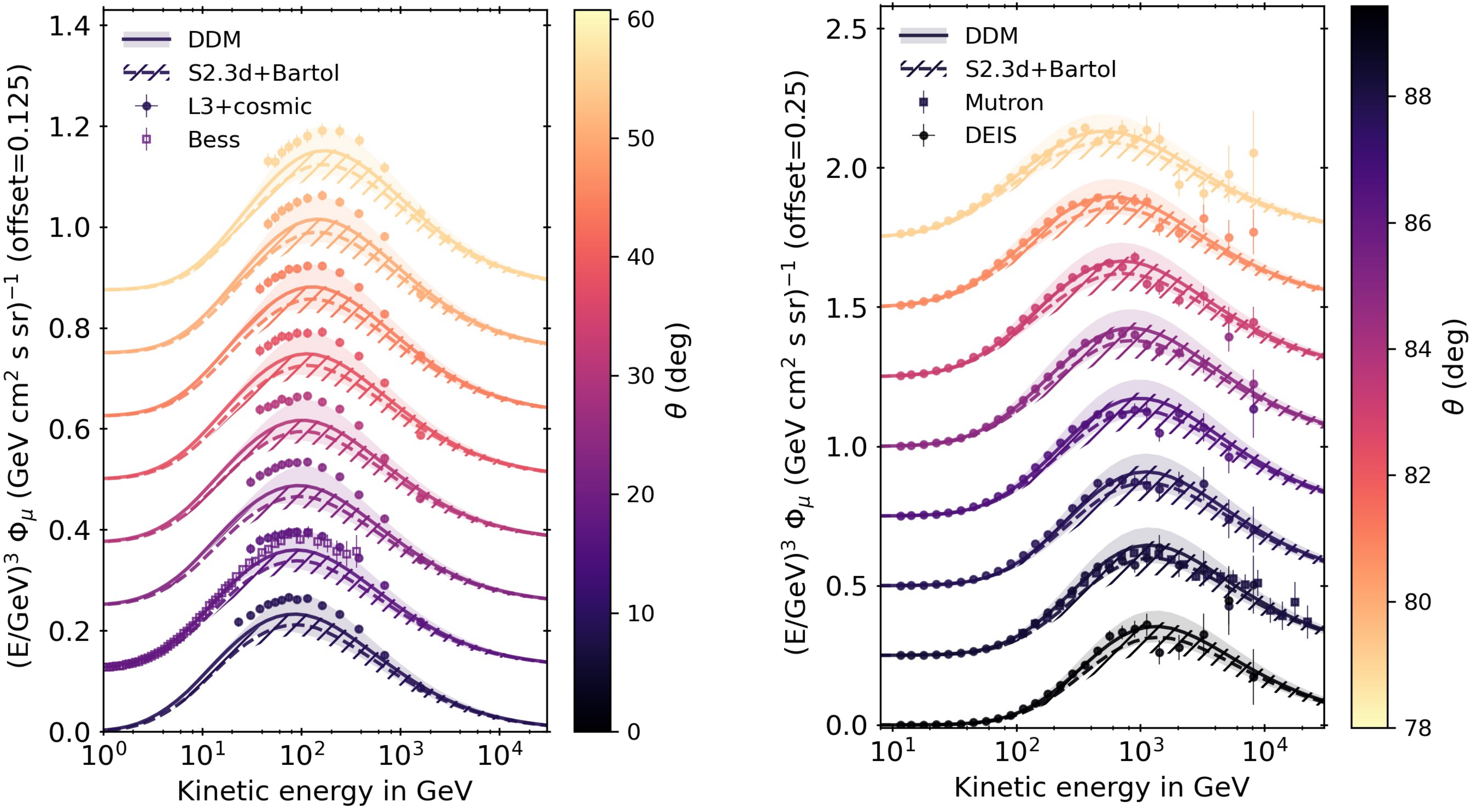
Phase space seen in surface **muon** measurements

AF & M. Huber, arXiv:2205.14766



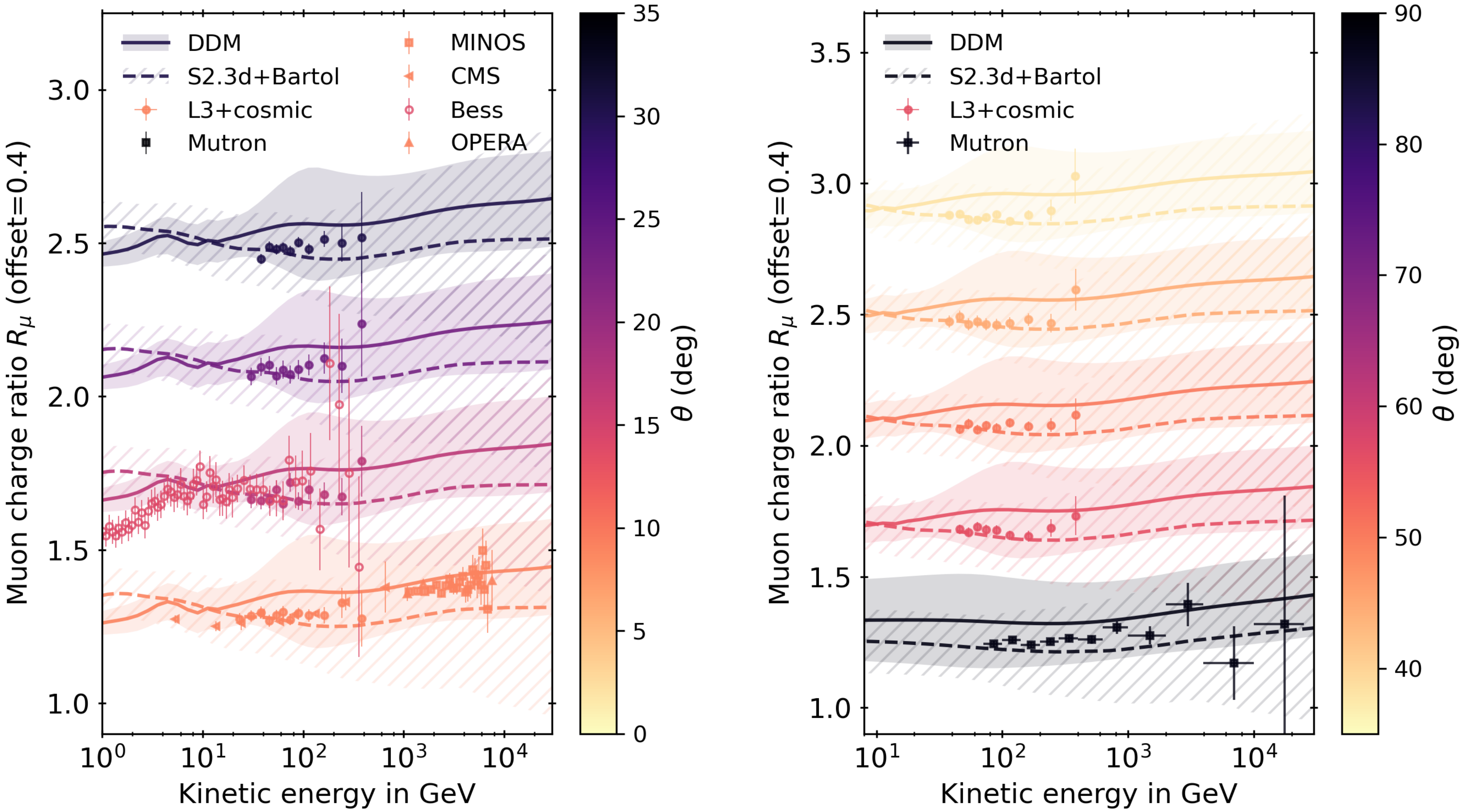
Contours = 90% of integral flux above indicated threshold

Description of muon data with DDM + GSF



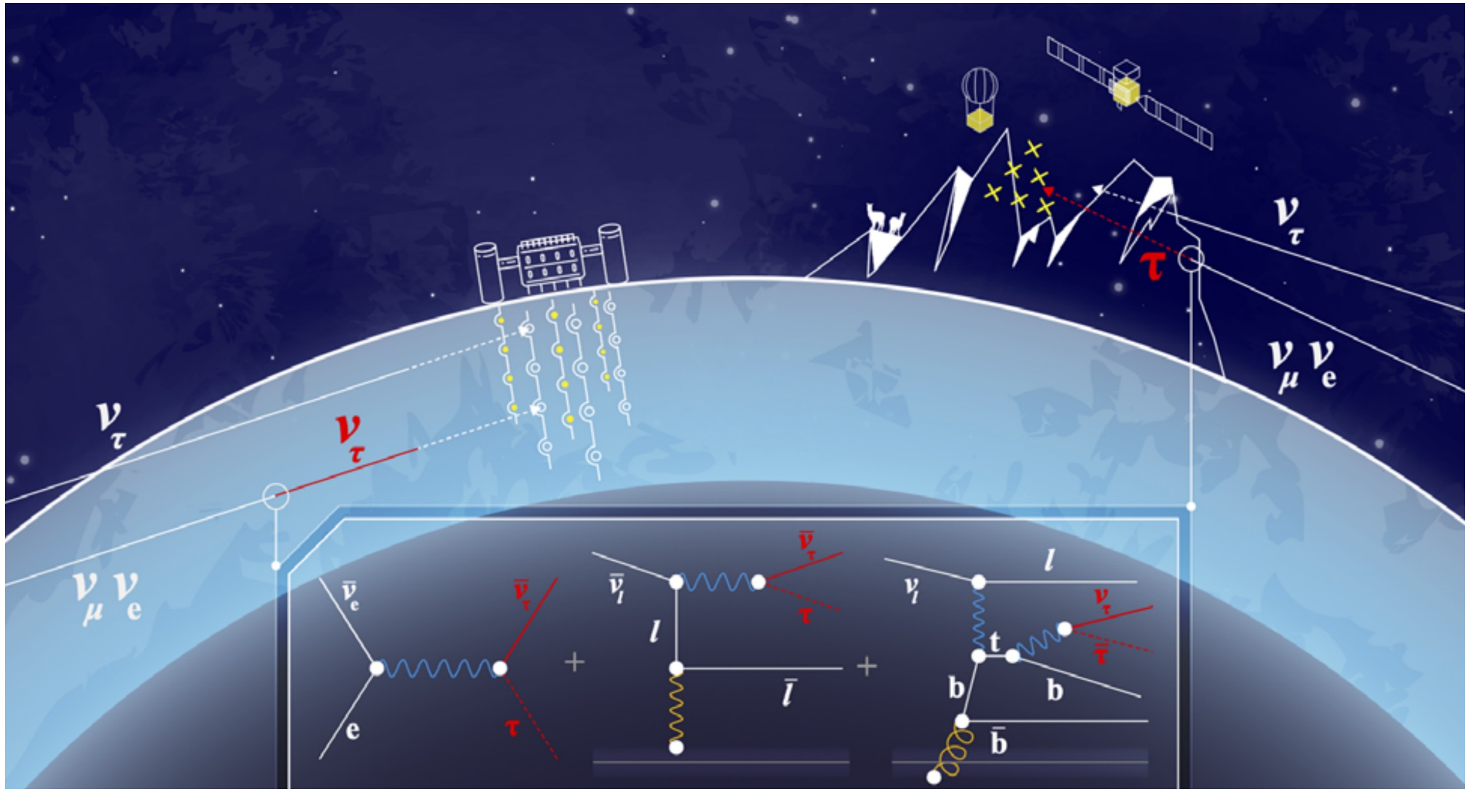
AF & M. Huber, arXiv:2205.14766

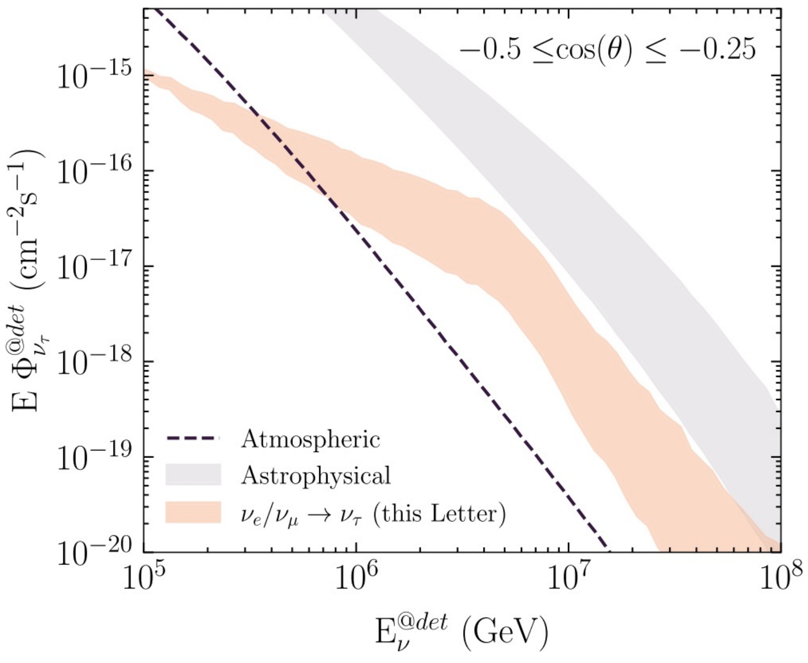
Results from DDM + GSF without muon calibration

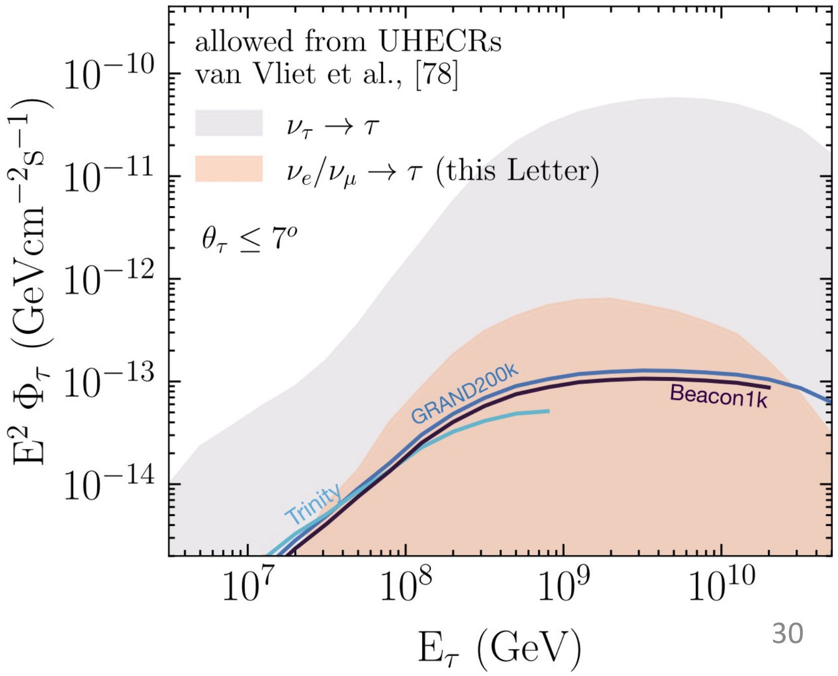


UHE tau neutrinos not entirely background free

A. Garcia Soto, P. Zhelnin, I. Safa, and C. A. Argüelles, PRL**128**, 171101 (2022)

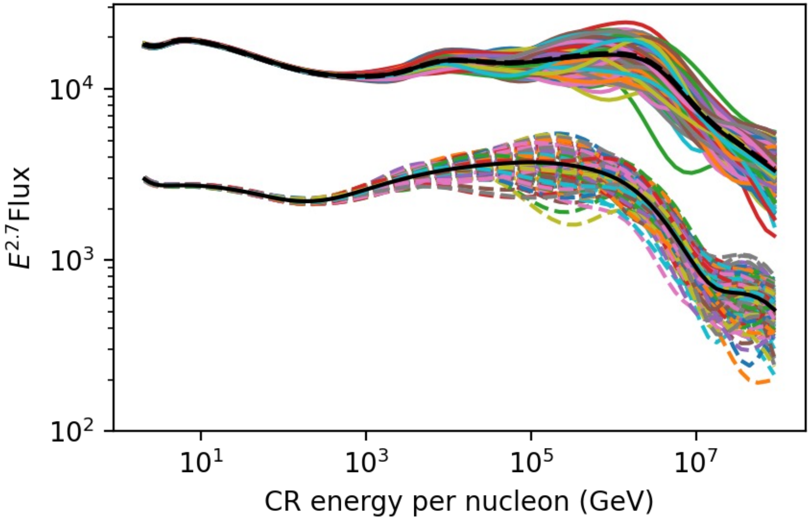


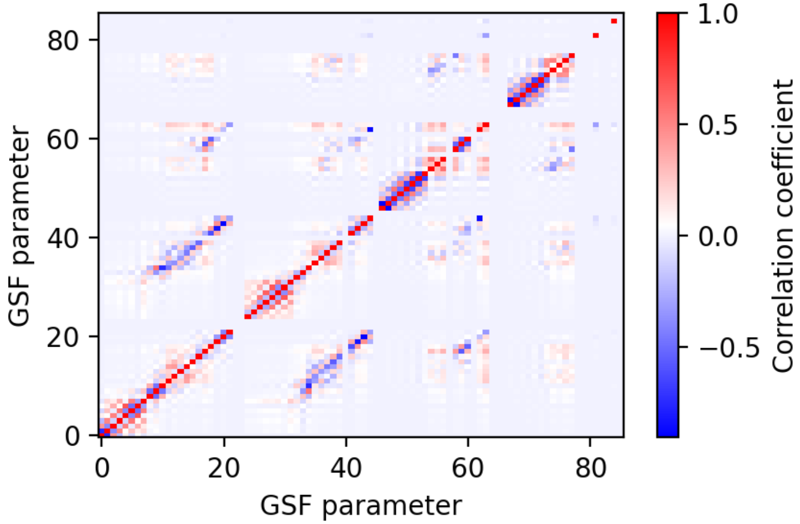


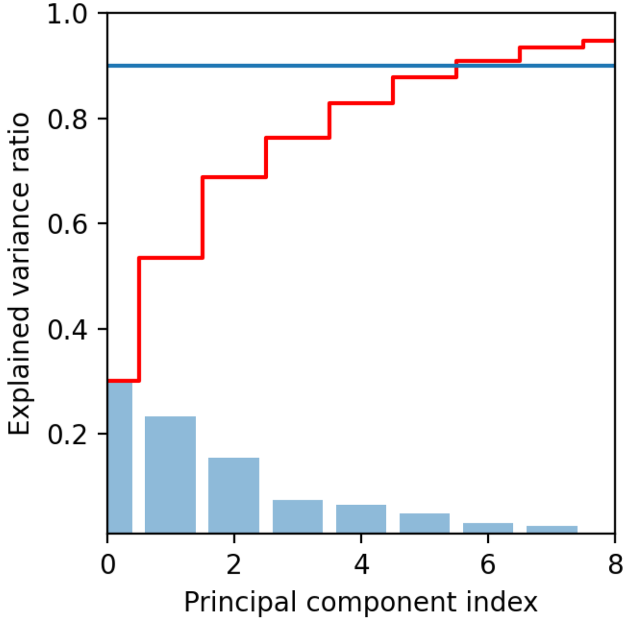


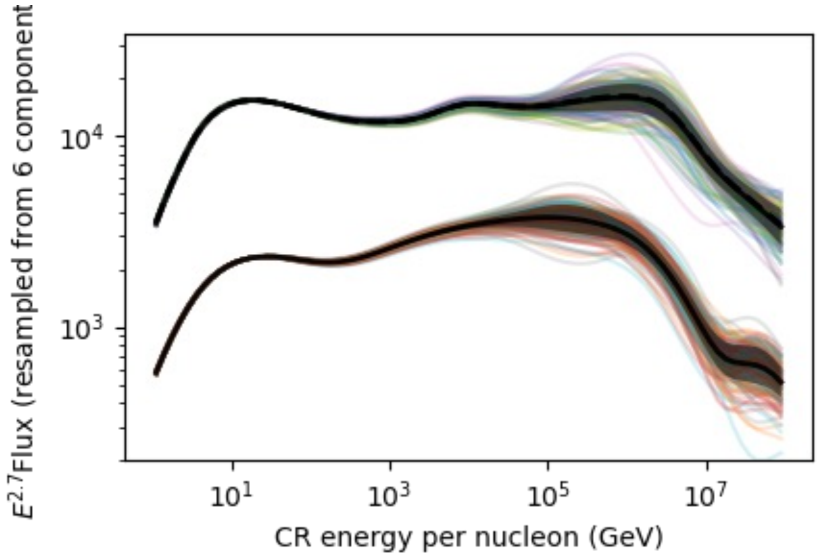
Atmospheric ν_τ production via “Glashow-resonance-like” processes from astrophysical ν_μ and ν_e

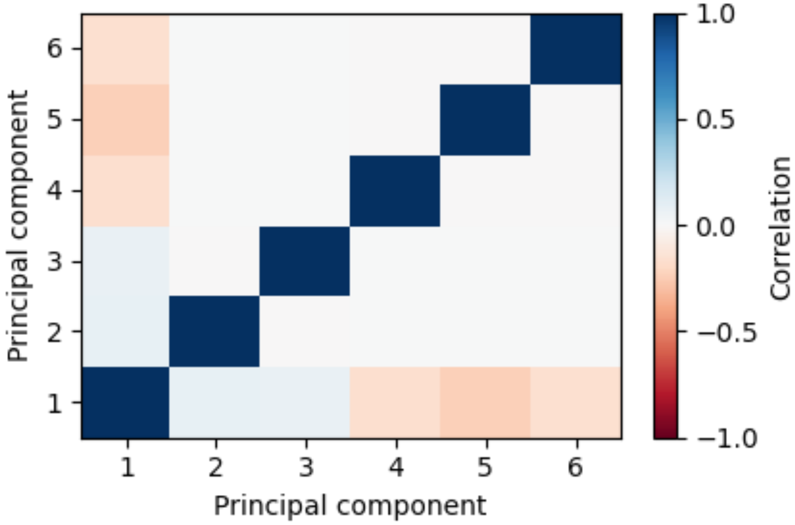
Dimensionality reduction to ~6 parameters



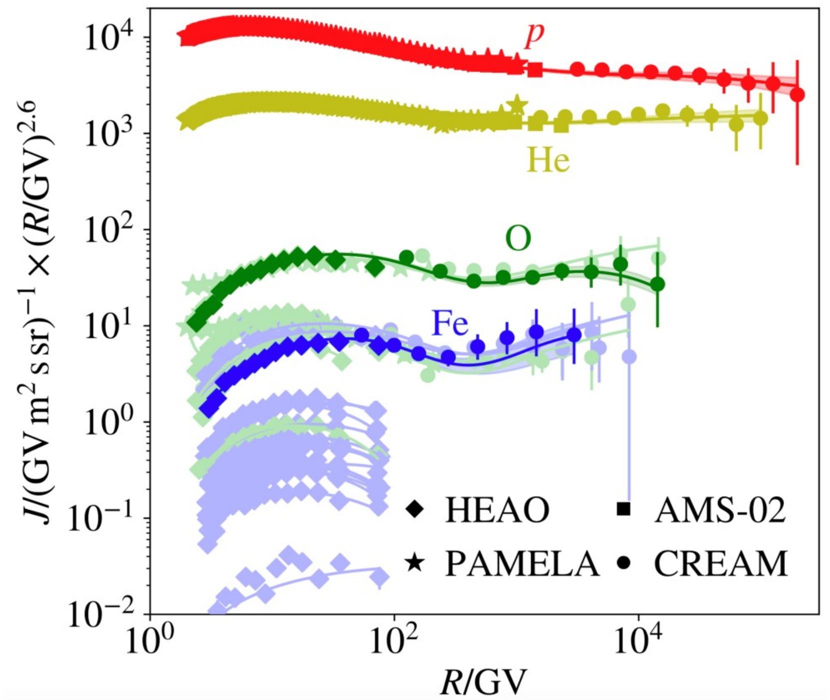


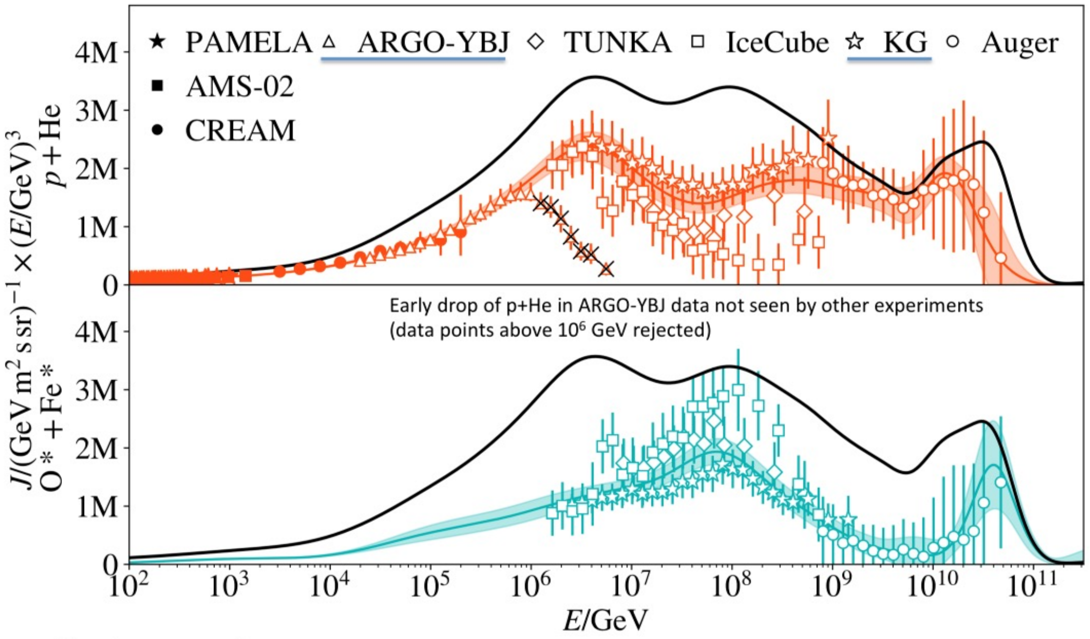






Parameterization of cosmic ray data – Global Spline Fit





Good agreement:
Auger, KASCADE-Grande, TUNKA

IceCube sees stronger mass gain above 10^7 GeV

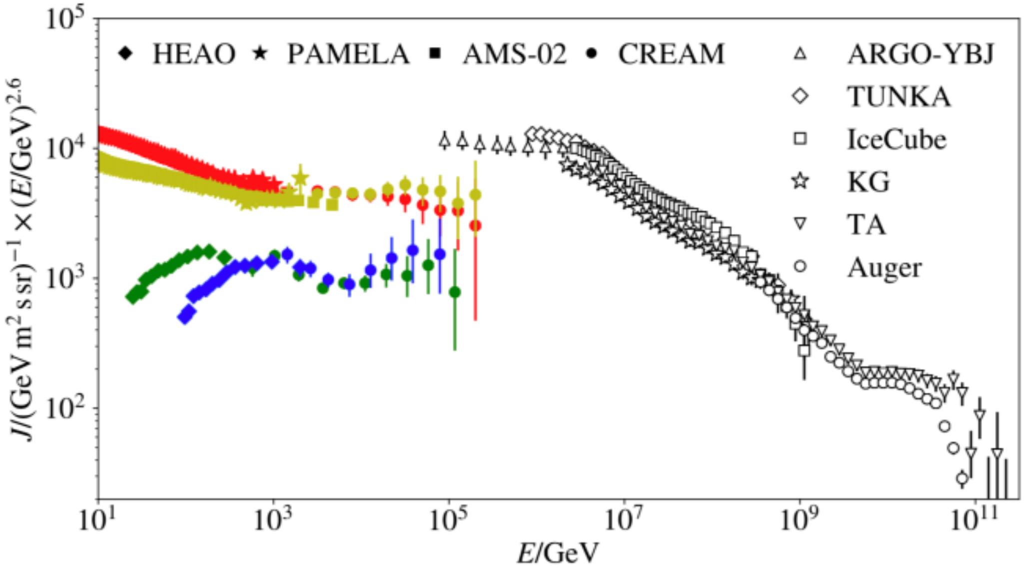
- Uses B-splines to fit 4 mass groups from GeV to 100 EeV
- Interpolates direct sattelite/balloon element data at low energies
- Fits mass groups to indirect experimental data
- Takes into account systematics carefully → only experiments with systematics

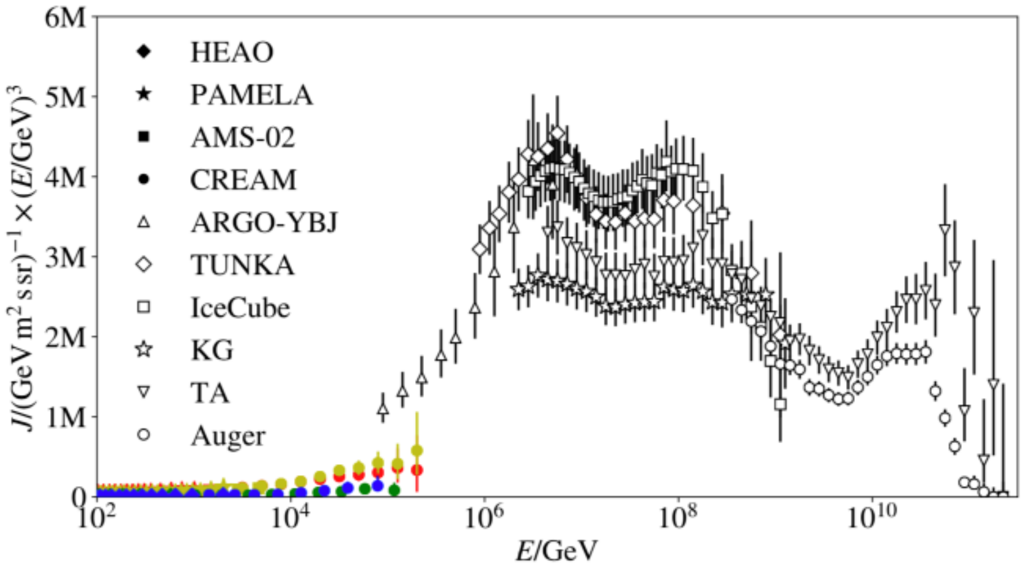
Dembinski, AF, Gaisser, ICRC 2017 & 2019

Combined fit to mass group flux and energy scale

H. Dembinski, AF, T. Gaisser
PoS(ICRC2017)533

Original data





- The determination of **energy scale in air-shower experiments is uncertain**
- This is caused by inconsistencies of **hadronic interaction models**
- Fit finds energy scales **within systematic uncertainties** of the experiment

$$\tilde{J}(\tilde{E}) = J(E)\frac{\mathrm{d}E}{\mathrm{d}\tilde{E}} = J\left(\frac{\tilde{E}}{1+z_E}\right)\frac{1}{1+z_E}$$

Flux distortion caused by energy-scale offset z_E

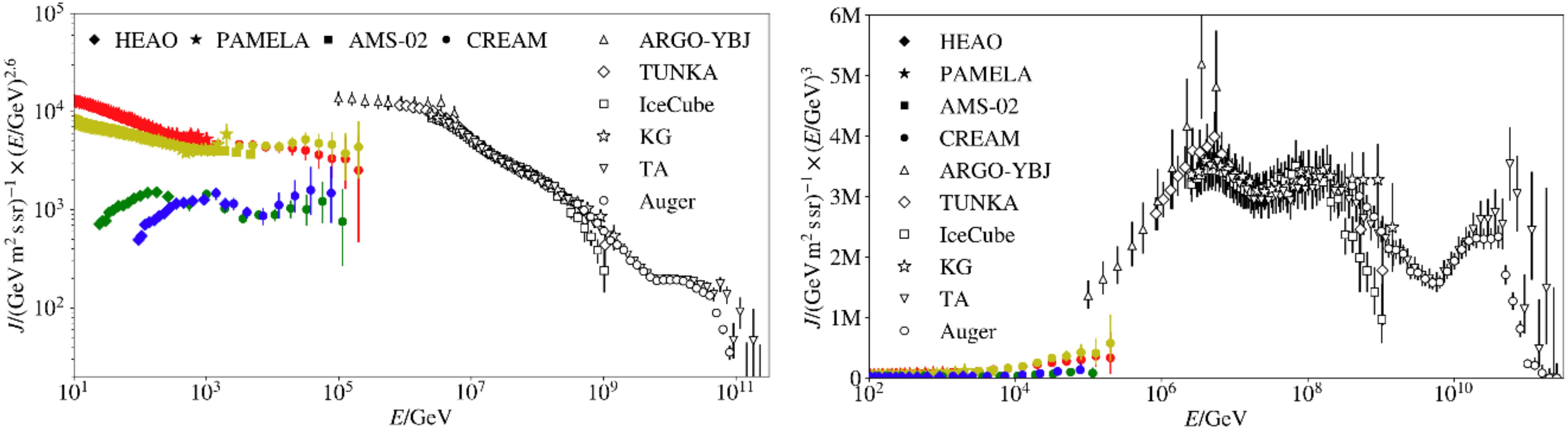
$$S = \sum_i z_i^2 + \sum_j \left(\frac{z_{Ej}}{(\sigma[E]/E)_j}\right)^2$$

Flux residuals — Energy-scale offset residuals

Combined fit to mass group flux and energy scale

H. Dembinski, AF, T. Gaisser
PoS(ICRC2017)533

Adjusted data



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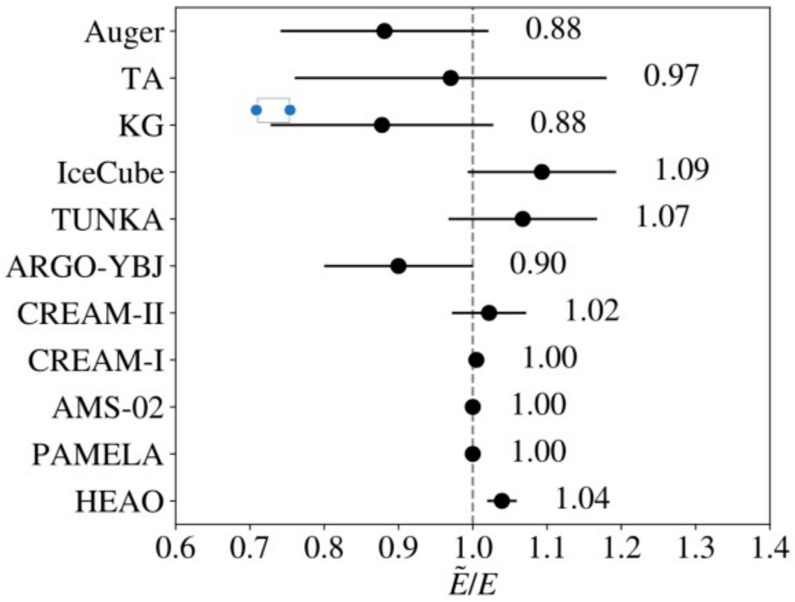
Flux residuals   Energy-scale offset residuals

Combined fit to mass group flux and energy scale



Fitted energy-scale offsets compatible with reported systematic uncertainties

GSF energy scale fixed by direct measurements

- The determination of **energy scale in air-shower experiments is uncertain**
- This is caused by inconsistencies of **hadronic interaction models**
- Fit finds energy scales **within systematic uncertainties** of the experiment

$$\tilde{J}(\tilde{E}) = J(E)\frac{\mathrm{d}E}{\mathrm{d}\tilde{E}} = J\left(\frac{\tilde{E}}{1+z_E}\right)\frac{1}{1+z_E}$$

Flux distortion caused by energy-scale offset z_E

$$S = \sum_i z_i^2 + \sum_j \left(\frac{z_{Ej}}{(\sigma[E]/E)_j}\right)^2$$

Flux residuals Energy-scale offset residuals

MCEq code inputs

Model building:

Hadronic interaction model sibyll 2.3c and inclusive lepton fluxes #8

Anatoli Fedynitch (DESY, Zeuthen), Felix Riehn (LIP, Lisbon and KIT, Karlsruhe, IKP), Ralph Engel (KIT, Karlsruhe, IKP), Thomas K. Gaisser (Delaware U., Bartol Inst.), Todor Stanev (Delaware U., Bartol Inst.) (Jun 11, 2018)

Published in: *Phys.Rev.D* 100 (2019) 10, 103018 • e-Print: 1806.04140 [hep-ph]

pdf DOI cite 67 citations

Hadronic interaction model Sibyll 2.3d and extensive air showers #9

Felix Riehn (Santiago de Compostela U., IGFAE and LIP, Lisbon and Delaware U. and KIT, Karlsruhe, IKP), Ralph Engel (KIT, Karlsruhe, IKP), Anatoli Fedynitch (Tokyo U., ICRR and DESY, Zeuthen and KIT, Karlsruhe, IKP), Thomas K. Gaisser (Delaware U., Bartol Inst.), Todor Stanev (Delaware U., Bartol Inst.) (Dec 6, 2019)

Published in: *Phys.Rev.D* 102 (2020) 6, 063002 • e-Print: 1912.03300 [hep-ph]

pdf DOI cite 67 citations

Data-driven model of the cosmic-ray flux and mass composition from 10 GeV to 10^{11} GeV #7

Hans Peter Dembinski (Heidelberg, Max Planck Inst.), Ralph Engel (KIT, Karlsruhe), Anatoli Fedynitch (DESY, Zeuthen), Thomas Gaisser (Delaware U., Bartol Inst.), Felix Riehn (LIP, Lisbon) et al. (Jul 28, 2017)

Published in: *PoS* ICRC2017 (2018) 533 • Contribution to: ICRC 2017, 533 • e-Print: 1711.11432 [astro-ph.HE]

pdf links DOI cite 75 citations

Hadronic uncertainties of inclusive atmospheric lepton fluxes from fixed-target experiments #3

Anatoli Fedynitch (Tokyo U., ICRR), Matthias Huber (Munich, Tech. U.) (2021)

Published in: *PoS* ICRC2021 (2021) 1227 • Contribution to: ICRC 2021, 1227

pdf DOI cite 1 citation

Equations solved, open-source code published:

Calculation of conventional and prompt lepton fluxes at very high energy #4

Anatoli Fedynitch (KIT, Karlsruhe and KIT, Karlsruhe, Dept. Phys. and CERN), Ralph Engel (KIT, Karlsruhe and KIT, Karlsruhe, Dept. Phys.), Thomas K. Gaisser (Delaware U., Bartol Inst.), Felix Riehn (KIT, Karlsruhe and KIT, Karlsruhe, Dept. Phys.), Todor Stanev (Delaware U., Bartol Inst.) (Mar 2, 2015)

Published in: *EPJ Web Conf.* 99 (2015) 08001 • Contribution to: ISVHECRI 2014 • e-Print: 1503.00544 [hep-ph]

pdf DOI cite 114 citations

Some ongoing MCEq projects

Calibration of atmospheric neutrino flux calculations using cosmic muon flux and charge ratio measurements #25

Juan-Pablo Yáñez (Alberta U.), Anatoli Fedynitch (Alberta U.), Tyler Montgomery (Alberta U.) (Sep 18, 2019)

Published in: *PoS* ICRC2019 (2020) 881 • Contribution to: ICRC 2019, 881 • e-Print: 1909.08365 [astro-ph.HE]

pdf DOI cite 7 citations

A Numerical Approach to Angular Distributions in Hadronic Cascades #6

Tetiana Kozynets (Bohr Inst.), Anatoli Fedynitch (Tokyo U., ICRR), D.Jason Koskinen (Bohr Inst.) (2021)

Published in: *PoS* ICRC2021 (2021) 1209 • Contribution to: ICRC 2021, 1209

pdf DOI cite 1 citation

On the Accuracy of Underground Muon Intensity Calculations #1

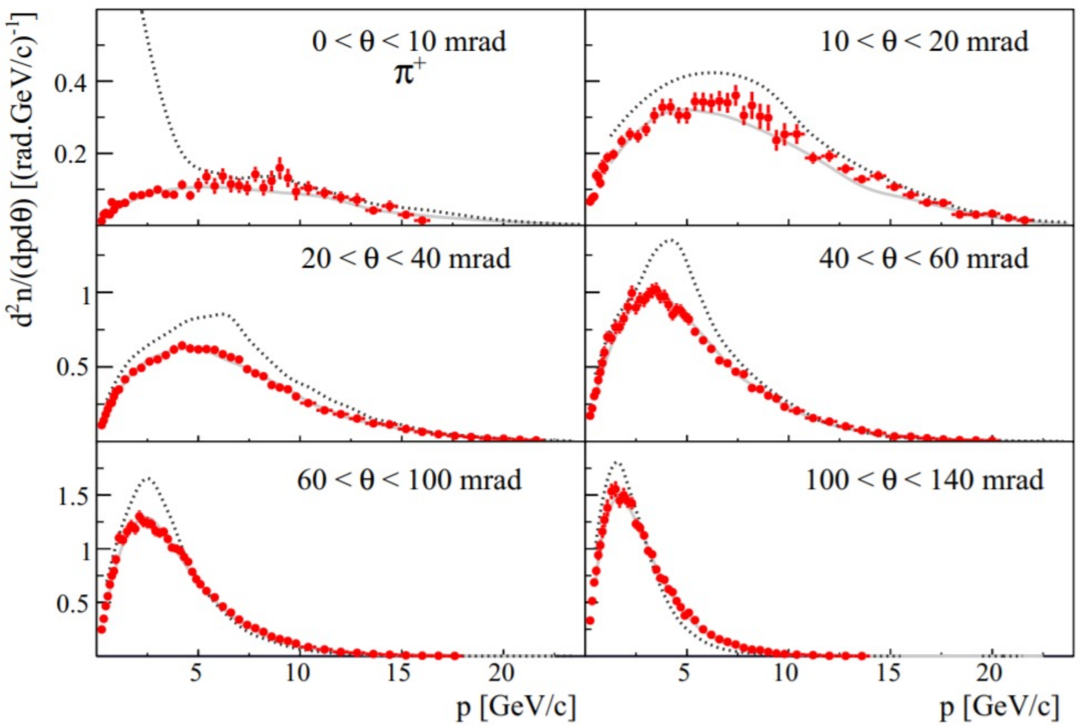
Anatoli Fedynitch (Tokyo U., ICRR and Taiwan, Inst. Phys.), William Woodley (Alberta U.), Marie-Cecile Piro (Alberta U.) (Sep 23, 2021)

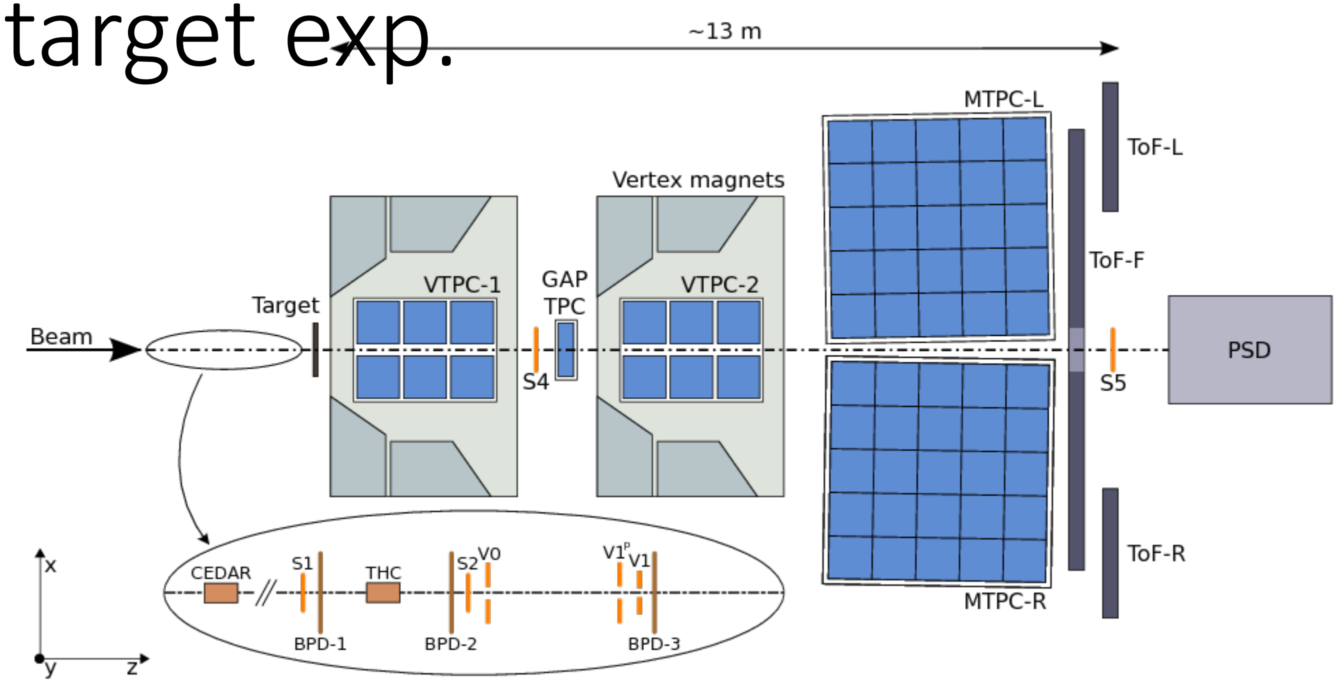
Published in: *Astrophys.J.* 928 (2022) 1, 27 • e-Print: 2109.11559 [astro-ph.HE]

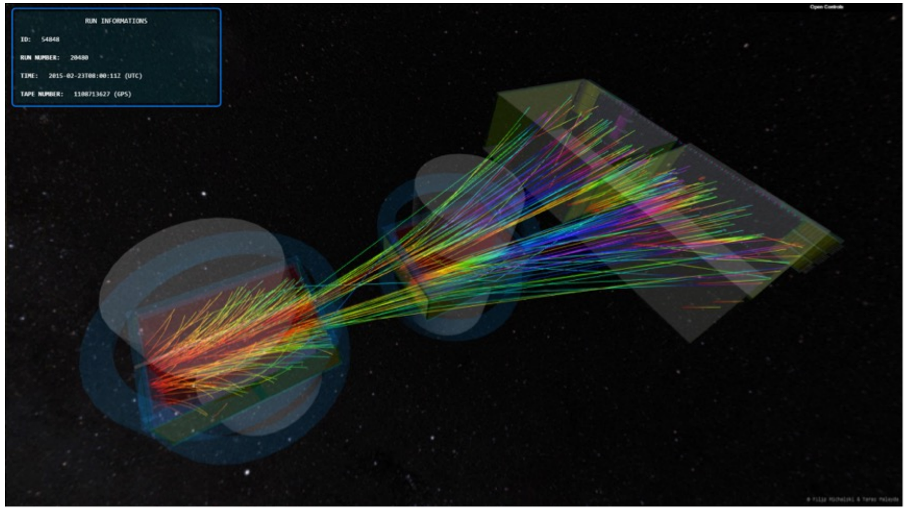
pdf DOI cite 0 citations

NA49/NA61/SHINE fixed-target exp.

Proton on carbon, 31 GeV, NA61, EPJC 76, 2016







Courtesy of CERN (home.cern)

How do the accelerators enter?

For atmospheric leptons

$$p_z \sim \text{TeV} - \text{PeV}$$

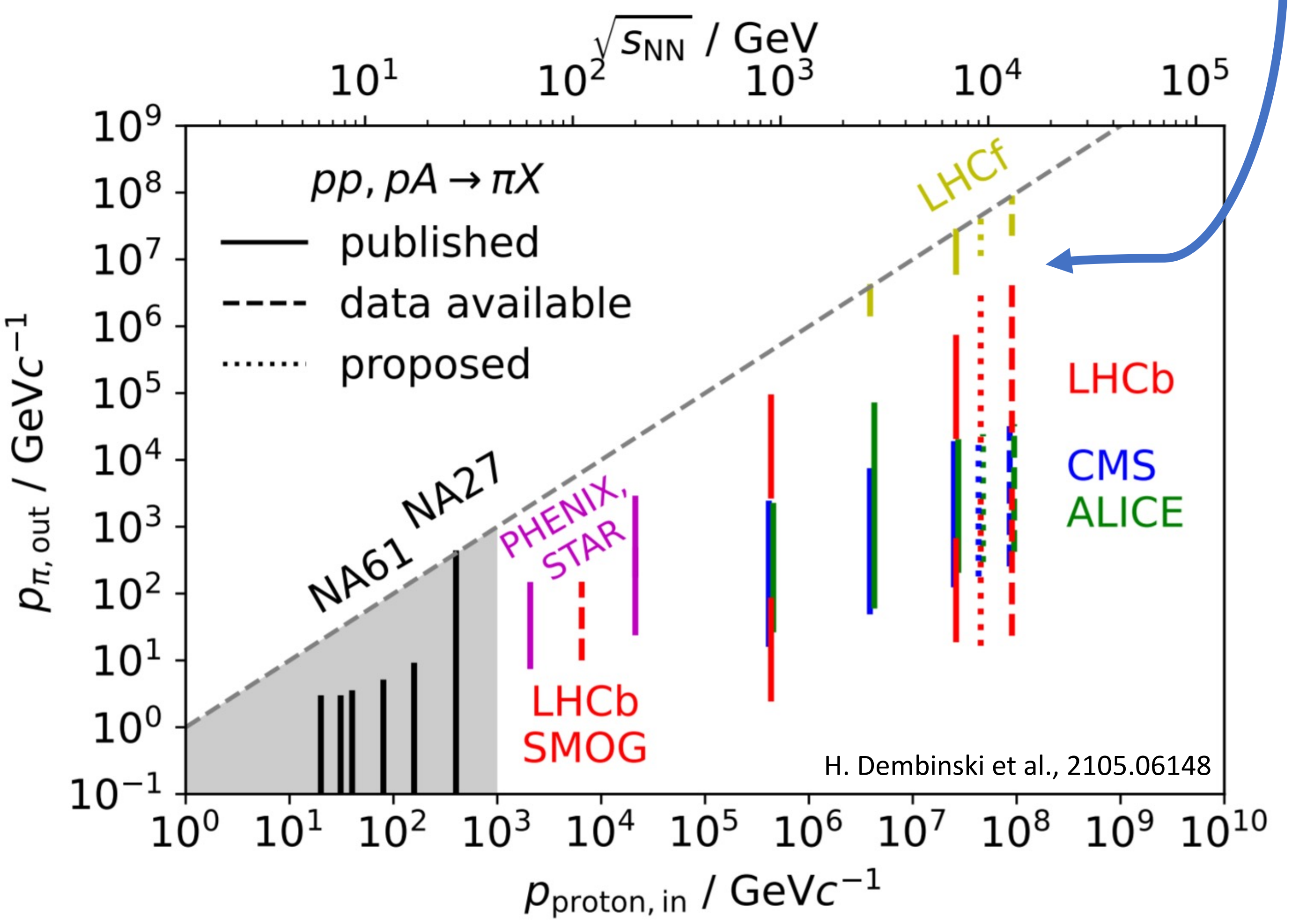
$$p_T \sim \text{few GeV}$$

$$\theta \sim \mu\text{rad}$$

$$x_{\text{lab}} = \frac{E_{\text{secondary}}}{E_{\text{primary}}} \approx \frac{p_{z,\text{secondary}}}{E_{\text{primary}}}$$

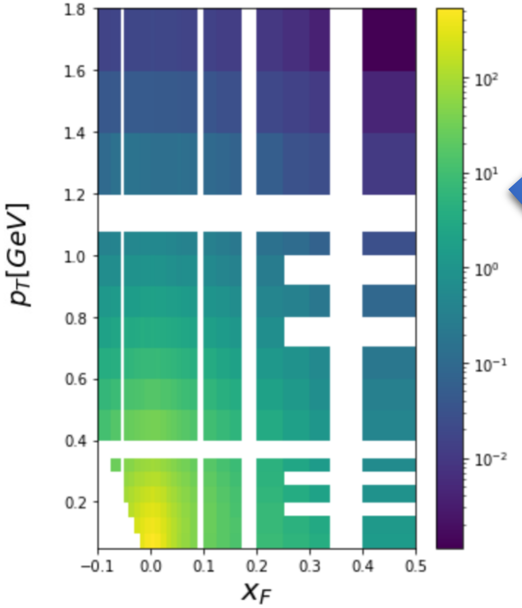
$$x_{\text{lab}} > 0.1, \quad \eta \to \infty$$

Beam pipe!



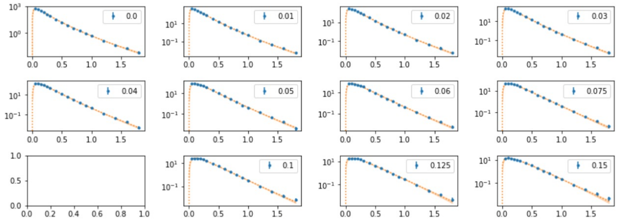
Building the DDM

NA49 proton-carbon @ 158 GeV

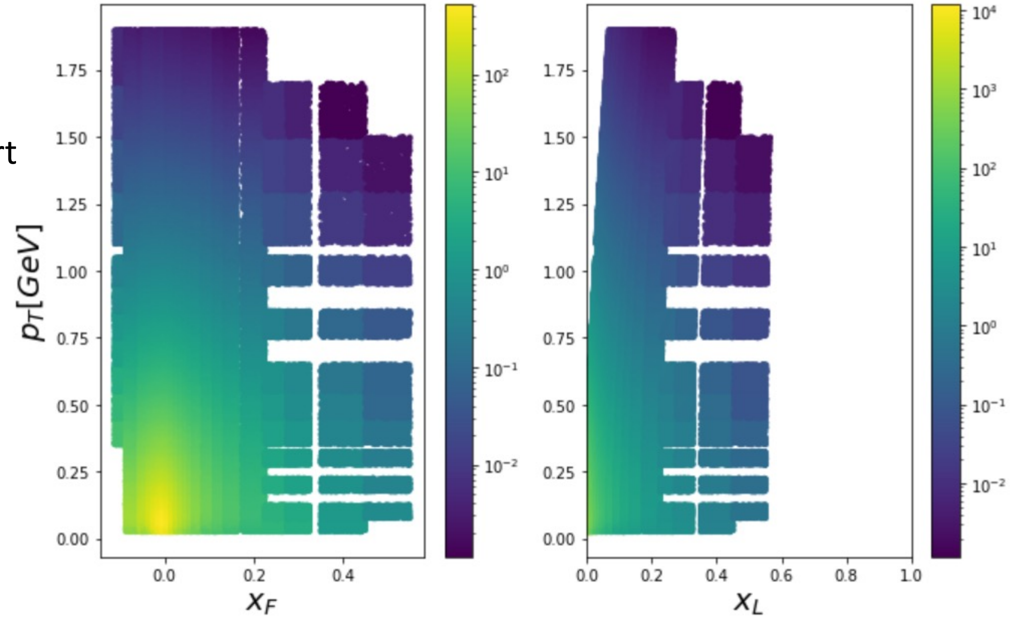


Sample from
x_F = pz/sqrt(s) and convert
into $x_L = E_{secondary}/E_{proj}$

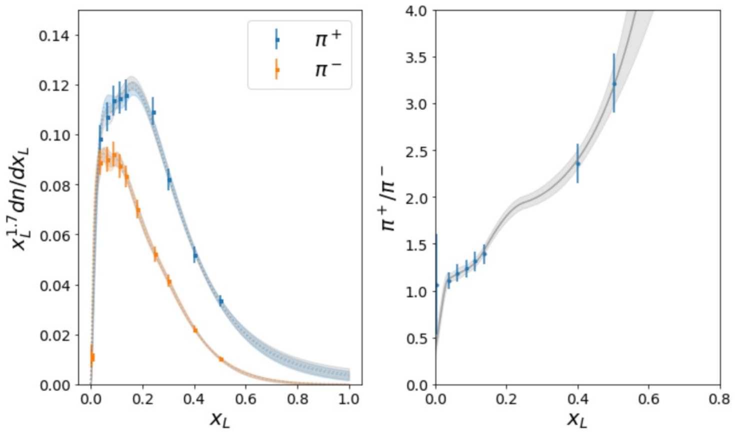
$$x_{Lab} = \frac{E_c}{E_a} = \frac{\gamma\sqrt{m_c^2 + \frac{1}{4}x_F^2 E_{c.m.}^2 + p_{c,T}^{*2}} + \frac{1}{2}\gamma\beta x_F^2 E_{c.m.}}{E_a}$$



Fit pT in each x_F bin using $$\frac{\mathrm{d}n}{\mathrm{d}p_\perp} = a_0 p_\perp^{a_1}\ e^{a_2 p_\perp^{a_3}}$$



Fit dn/dx_L with splines, get covariance matrix

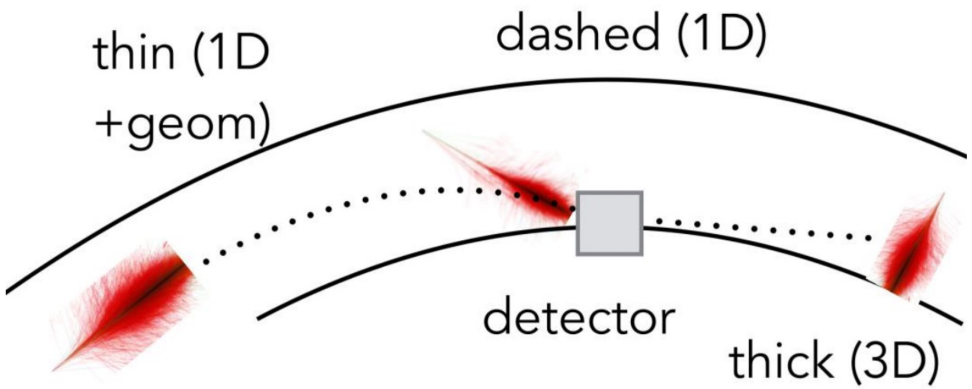


Included data

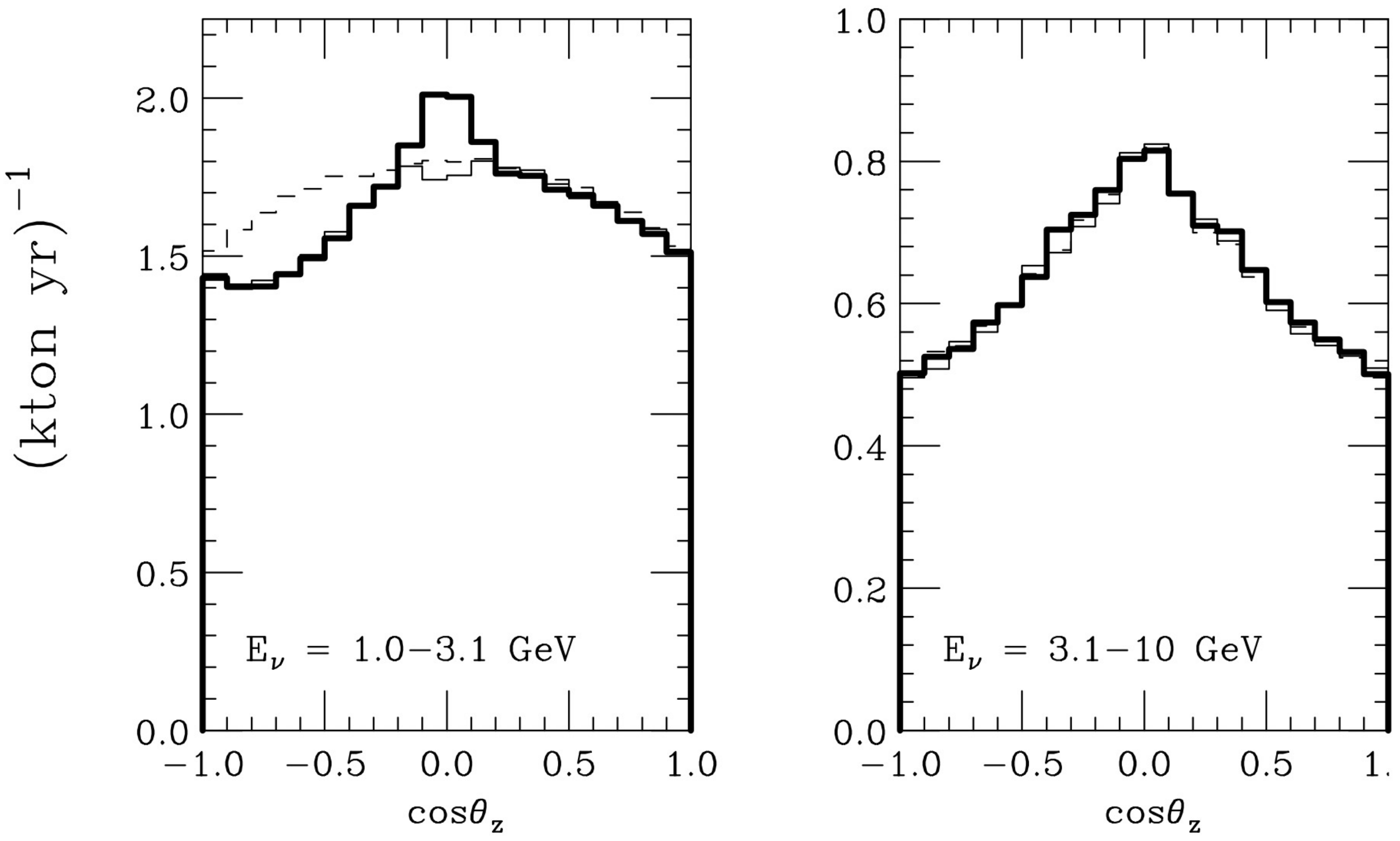
Experiment	beam	E_{beam}/GeV	Secondaries	Variables
NA49	pC	158	$\pi^\pm, \overset{(-)}{\mathrm{p}}, n$	$x_F, p_\perp$
NA49	pp	158	$\mathrm{K}^\pm$	$x_F, p_\perp$
NA61/SHINE	pC	31	$\pi^\pm, K^\pm, K_S^0, \Lambda$	p, θ
NA61/SHINE	π^-C	158, 350	$\pi^\pm, K^\pm, \overset{(-)}{\mathrm{p}}$	$p, p_\perp$

(In the next iteration we would like to include new results from NA61 and old results from NA59 that require Be->C extrapolation.

Bartol and HKKM calculations focus on low energy and 3D



There's an appreciable difference between the $\cos\theta_z$ distributions in 1D vs 3D calculations at <3 GeV.



*Lipari 2000, ν_μ-CC rate at the North Magnetic Pole

Groups involved in modeling atmospheric neutrinos