

# Theoretical models of neutrino masses

Virtual Seoul

May 30 - June 4

$\nu$  2022

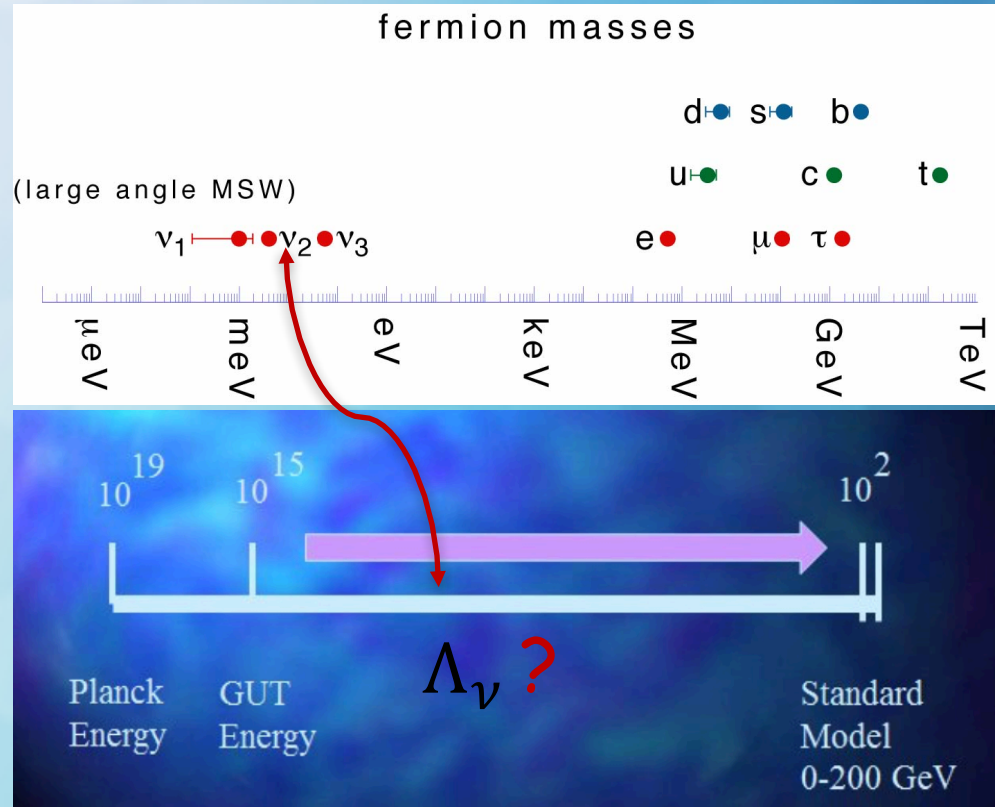


1<sup>st</sup> June 2022

Ferruccio Feruglio   INFN Padova

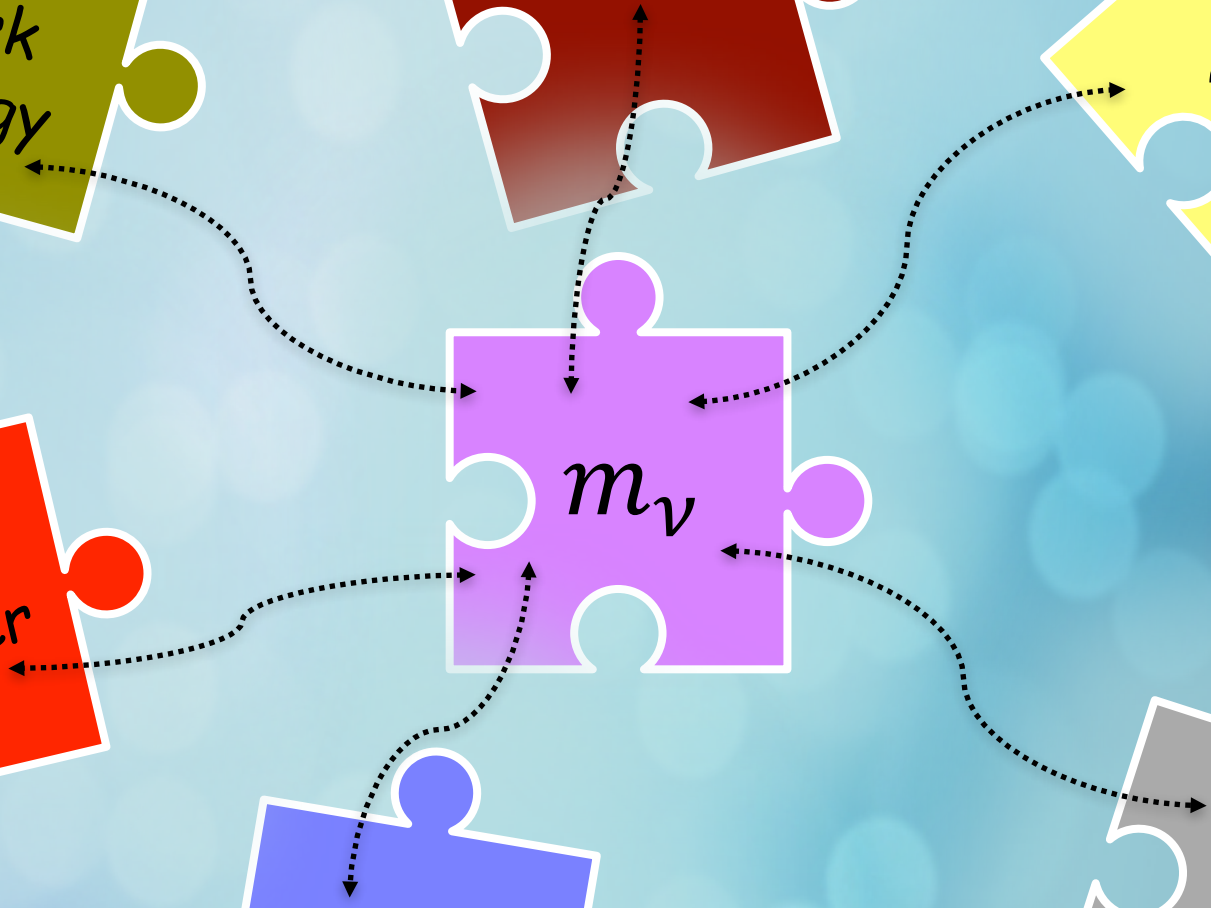
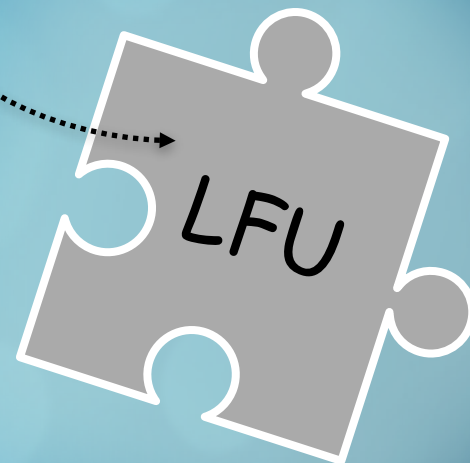
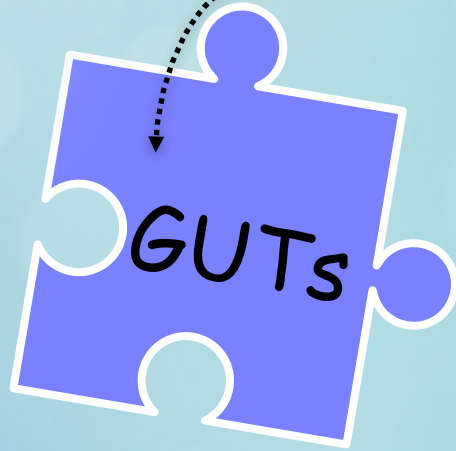
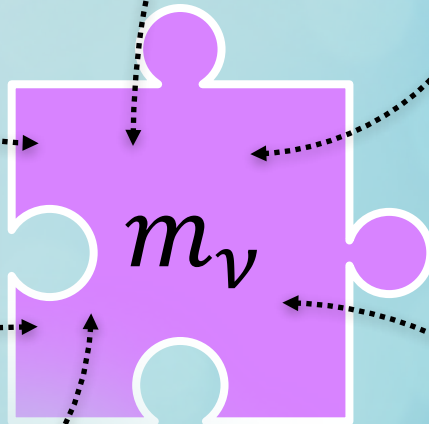
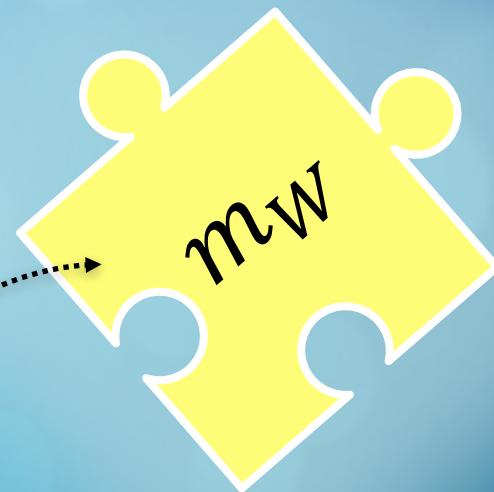
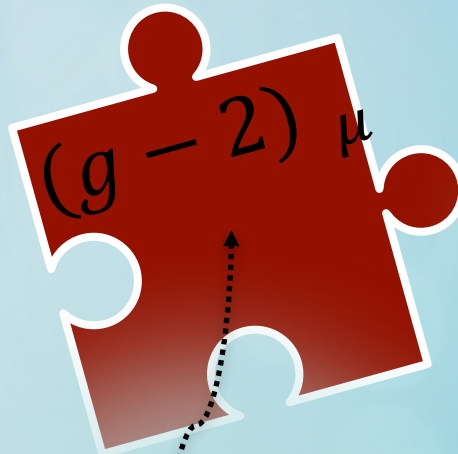
# two (among many) open questions

mechanism of neutrino mass generation



peculiar mixing pattern and flavour symmetries

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \text{PMNS} \\ \text{matrix} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



which mass scale?

$$\mathcal{L} = \mathcal{L}_{SM} + v^c y_\nu (H \ell) + \ell \mathcal{C}_4 \ell \Delta$$

d=4



which mass scale?

$$\mathcal{L} = \mathcal{L}_{SM} + v^c y_\nu (H\ell) + \ell \mathcal{C}_4 \ell \Delta$$

d=4

$$+ \frac{1}{\Lambda_5} (H\ell) \mathcal{C}_5 (H\ell)$$

d=5

$$+ \frac{1}{\Lambda_6^2} \overline{(H\ell)} \mathcal{C}_6 \gamma^\mu \partial_\mu (H\ell) + \dots$$

d=6

which mass scale?

$$\begin{aligned}\mathcal{L} = & \mathcal{L}_{SM} + v^c \mathcal{Y}_\nu(H\ell) + \ell \mathcal{C}_4 \ell \Delta \\ & + \frac{1}{\Lambda_5} (H\ell) \mathcal{C}_5 (H\ell) \\ & + \frac{1}{\Lambda_6^2} \overline{(H\ell)} \mathcal{C}_6 \gamma^\mu \partial_\mu (H\ell) + \dots\end{aligned}$$

d=4

d=5

d=6

most plausible option: d=5

$$m_\nu \approx v \frac{v}{\Lambda_5}$$

$$m_\nu \approx \sqrt{\Delta m_{atm}^2}$$



$$\Lambda_5 \approx 10^{15} \text{ GeV}$$

smallness of  $m_\nu$  due to  $\Lambda_5$  very large

unique window on physics at very large scales,  
inaccessible in present man-made experiments



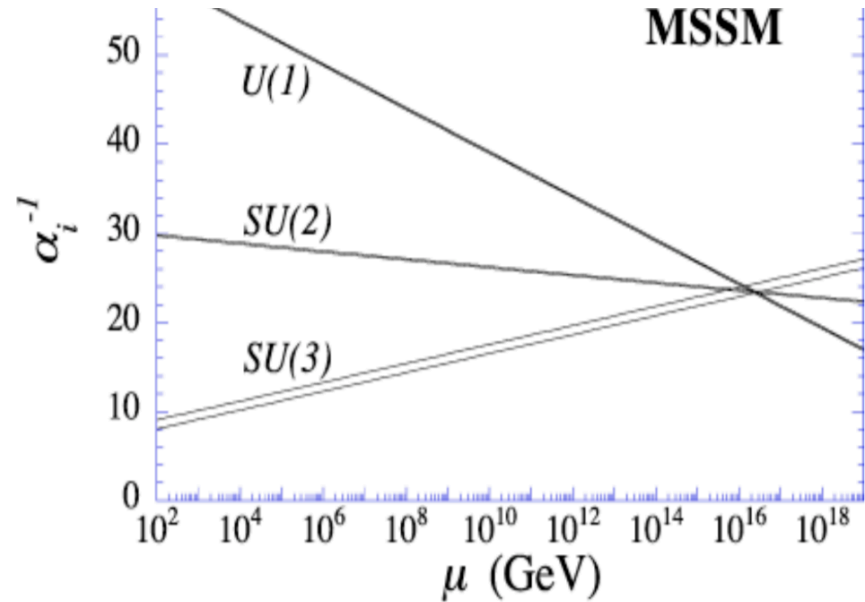
independent indication of a new physics threshold close to  $GUT$  scale

$SO(10)$  GUT:  $16 = (1 \text{ SM family} + \nu^c)$

heavy  $\nu^c$  exchange give rise to  
(a specific version of) d=5

see-saw

$$\frac{C_5}{\Lambda_5} \approx y_\nu^T M^{-1} y_\nu$$



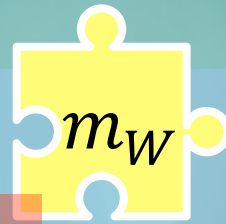
d=5 breaks (B-L) by two units

B-L violated, in general, when unifying particle interactions (in  $GUT$ s and by quantum gravity)

(B-L) violation welcome in baryogenesis

baryogenesis through out-of-equilibrium, CP and (B-L)-violating decays of heavy  $\nu^c$

# $m_\nu$ related to $m_W$ anomaly?



$$(m_W)_{CDF} = 80,433.5(9.4) \text{ MeV}$$

$$\Delta = (1, 3, +1)$$

$$(m_W)_{SM} = 80,354.5(5.7) \text{ MeV}$$

$$\mathcal{L} = \mathcal{L}_{SM} + \ell \mathcal{C}_4 \ell \Delta$$

$$\frac{\delta m_W^2}{m_W^2} = -\frac{2 \cos^2 \theta}{\cos 2\theta} \frac{\langle \Delta \rangle^2}{v^2} + \text{loops}$$

[Heeck 2204.10274]

wrong  
sign



$$\langle \Delta \rangle < 1 \text{ GeV}$$

$$100 \text{ GeV} \leq m_\Delta \leq 300 \text{ GeV}$$

$$\mathcal{C}_4 \approx \frac{m_\nu}{\langle \Delta \rangle} \ll 1$$

to suppress  $\mu \rightarrow e\gamma$   
and  $\Delta^{++} \rightarrow \ell^+ \ell^+$

$$1 \text{ MeV} < \langle \Delta \rangle < 1 \text{ GeV}$$

$$10^{-11} < \mathcal{C}_4 < 10^{-8}$$

no clue on smallness of  $m_\nu$

tests  $0.1 \text{ pb} < \sigma(pp \rightarrow \Delta^{++} \Delta^{--}) < 1 \text{ pb}$

# $m_\nu$ related to $m_W$ anomaly?

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda_5} (H\ell) \mathcal{C}_5 (H\ell) + \frac{1}{\Lambda_6^2} \overline{(H\ell)} \mathcal{C}_6 \gamma^\mu \partial_\mu (H\ell) + \dots$$

relevant e.g. in inverse seesaw

$$\frac{\mathcal{C}_5}{\Lambda_5} \approx y_\nu^T M^{-1} \mu M^{-1} y_\nu$$

$$\frac{\mathcal{C}_6}{\Lambda_6^2} \approx y_\nu^+ M^{-1} M^{-1} y_\nu$$

lepton mixing matrix no more unitary

$$\mathcal{U}_{fi} \rightarrow (\delta_{ff'} - \eta_{ff'}) \mathcal{U}_{f'i}$$

$$\eta_{ff'} = \frac{\mathcal{C}_6}{4} \frac{v^2}{\Lambda_6^2} \quad \eta_{ff} > 0$$

$$\frac{\delta m_W^2}{m_W^2} = + \frac{\sin^2 \theta}{\cos 2\theta} \frac{\delta G_F}{G_F} = + \frac{\sin^2 \theta}{\cos 2\theta} (\underbrace{\eta_{ee} + \eta_{\mu\mu}}_{\approx 2 \cdot 10^{-3}})$$

right sign

$$\approx 2 \cdot 10^{-3}$$

$$M \approx y_\nu 3.9 \text{ TeV}$$

mixing active/sterile  $\sim 0.1$

$$\mu \approx \mathcal{O}(10 \text{ eV})$$



# LFU in $\pi, K, \tau$ decays

[Blennow, Coloma, Fernandez-Martinez, Gonzalez-Lopez, 2204.04559]

$$\eta_{ee} \approx \eta_{\mu\mu}$$

$$1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 = (1.5 \pm 0.6) \times 10^{-3} \quad \text{PDG2020}$$

$$V_{ud} = (V_{ud})_{EXP} (1 - \eta_{\mu\mu})$$



forbidden in  
minimal framework

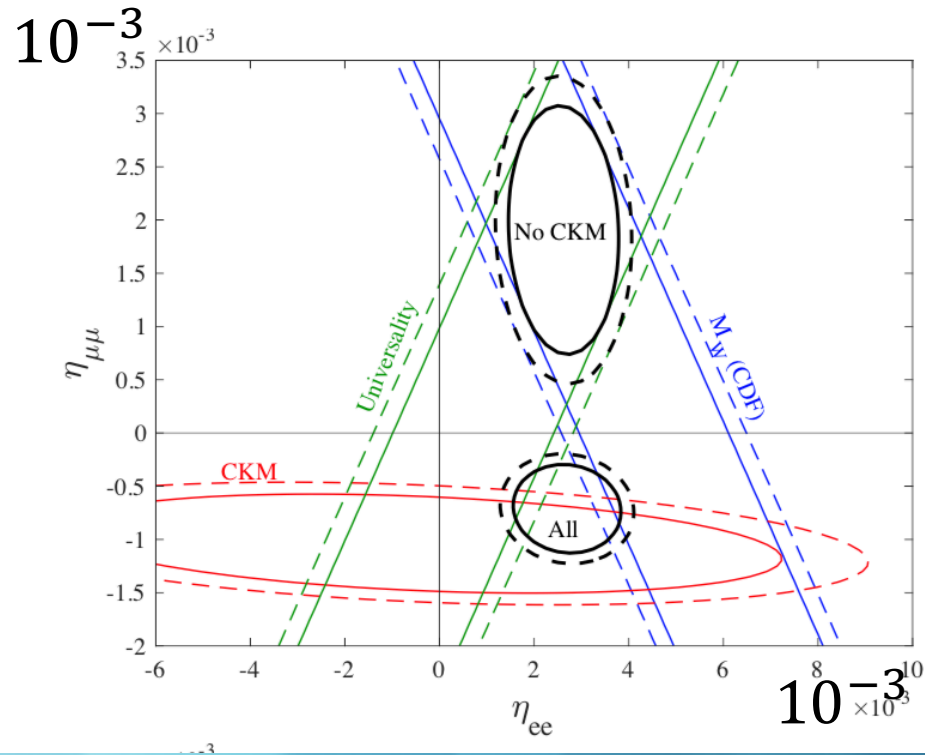
$$\eta_{\mu\mu} < 0$$

$$BR(\mu \rightarrow e\gamma) \approx \frac{3\alpha}{2\pi} |\eta_{e\mu}|^2 < 4.2 \times 10^{-13}$$

$$|\eta_{e\mu}| \leq \mathcal{O}(10^{-5})$$

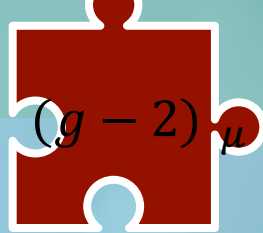


$$\eta_{ee} \approx \eta_{\mu\mu} \approx \mathcal{O}(10^{-3})$$



[Fernandez-Martinez, Hernandez-Garcia,  
Lopez-Pavon 1605.08774]

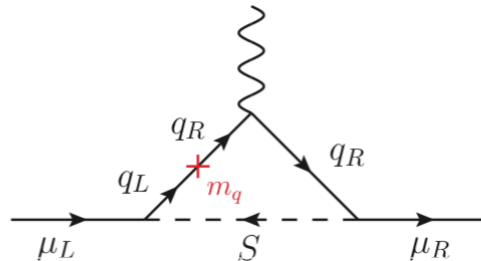
# $m_\nu$ related to $(g-2)_\mu$ anomaly?



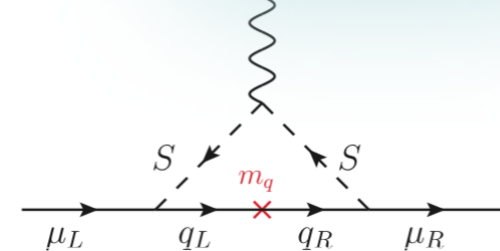
$$\Delta a_\mu = +2.51(59) 10^{-9}$$

can be produced by  
scalar LQ exchange

chiral enhancement:  
 $\approx m_{top}/m_\mu$



[fig. from 1910.03877]



$S_1 = (\bar{3}, 1, +1/3)$  [not automatically p-decay free]

$R_2 = (3, 2, +7/6)$

[Djouadi, Kohler, Spira, Tutas, 1990  
Chakraverty, Choudhury, Datta 0102180  
Cheung 0102238  
Biggio, Bordone 1411.6799]

can help explaining part of LFUV in B decays

scalar LQ exchange



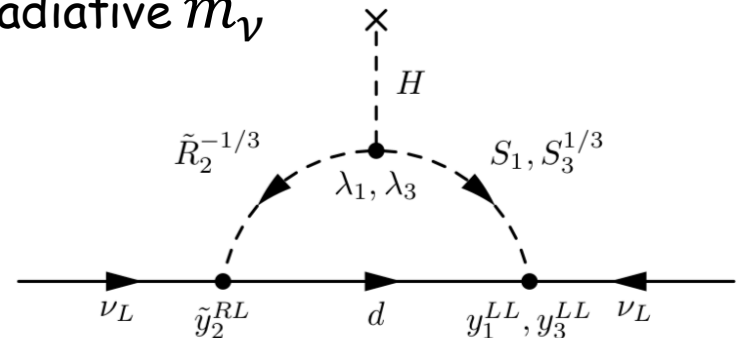
as in the  
Zee model

$$\eta^+ \rightarrow S_1 = \left(\bar{3}, 1, \frac{1}{3}\right)$$

$$H_2 \rightarrow \tilde{R}_2 = \left(\bar{3}, 2, \frac{1}{6}\right)$$

[fig. from 1701.08322]

radiative  $m_\nu$



[Klein, Lindner, Ohmer 1901.03225]

$$\Delta\mathcal{L} = S_1 q y_1 \ell + \bar{S}_1 u^c y_1' e^c + \tilde{R}_2 d^c y_2 \ell$$

[Di Zhang 2105.08670]

$$m_\nu \approx \frac{3 \sin 2\theta}{32\pi^2} \log\left(\frac{M_2^2}{M_1^2}\right) (y_2^T \hat{m}_d y_1 + y_1^T \hat{m}_d y_2)$$

$$m_\nu \approx 0.05 \text{ eV} \quad \Rightarrow \quad (y_2)_{3i} (y_1)_{3j} \approx 10^{-9}$$

$$\Delta a_\mu = +2.51(59) \cdot 10^{-9} \quad \Rightarrow \quad (y_1)_{32} (y_1')_{32} \approx -5 \times 10^{-3} \left(\frac{M_1}{1\text{TeV}}\right)^2$$

collider bounds

pair production  $S_1 \bar{S}_1$   
from  $gg$  and  $q\bar{q}$  at LHV  
 $S_1 \rightarrow t\mu$

$$M_1 > 1420 \text{ GeV}$$

[Angelescu, Becirevic, Faroughy,  
Sumensari 1808.08179  
CMS 1809.05558]

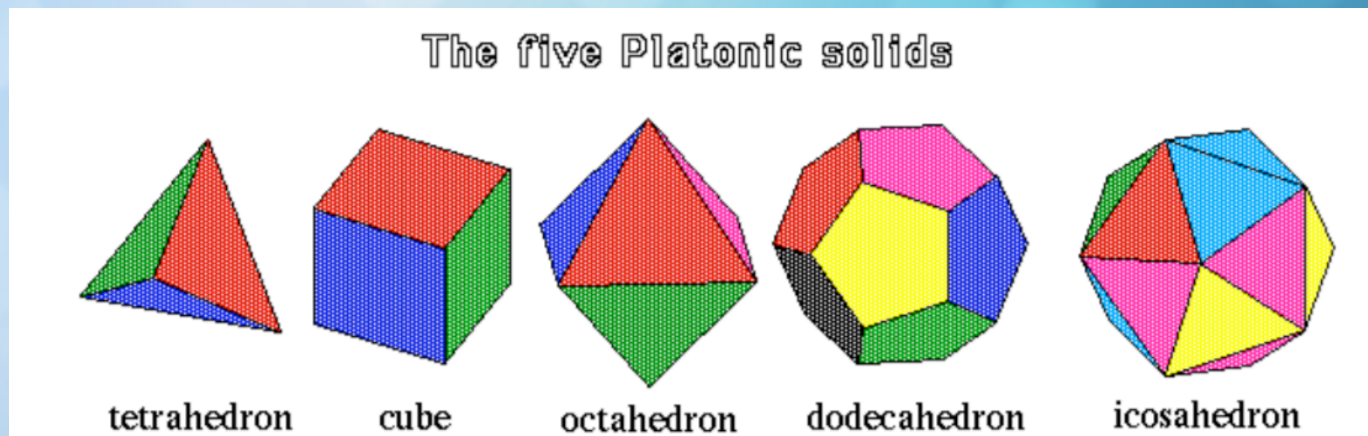
LFV  $\mu \rightarrow e\gamma$

$$(y_1)_{32} (y_1')_{31}, (y_1)_{31} (y_1')_{32} < 5.4 \times 10^{-8} \left(\frac{M_1}{1\text{TeV}}\right)^2$$

$$(y_1)_{22} (y_1')_{21}, (y_1)_{21} (y_1')_{22} < 1.3 \times 10^{-6} \left(\frac{M_1}{1\text{TeV}}\right)^2$$

[Mandal, Pich 1908.11155]

# lepton mixing and flavour symmetries



# "traditional" Flavour Symmetries

$$g \in G_{fl}$$

$$\begin{aligned}\Psi &\xrightarrow{g} \varrho(g) \Psi \\ \Psi^c &\xrightarrow{g} \varrho_c(g) \Psi^c\end{aligned}$$

$$\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \dots$$



$$\Psi^c \mathcal{Y} \Psi$$

$$\varrho_c^*(g) \mathcal{Y} \varrho(g)^+ = \mathcal{Y}$$

can be combined with CP

$$\Psi \xrightarrow{CP} X_{CP} \Psi^*$$

example: Isospin SU(2) in strong interactions

$$m_p = m_n$$

flavour symmetries of this type are necessarily broken

- many parameters from symmetry breaking sector
- vacuum alignment problem  $\langle \varphi \rangle$



# new type of symmetries

## modular transformations

$\tau$  a complex scalar field,  
 $\text{Im}(\tau) > 0$

$$\tau \xrightarrow{\gamma} \frac{a\tau + b}{c\tau + d}$$

$a, b, c, d$  integers  
 $ad - bc = 1$

$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$  infinite discrete group

$SL(2, \mathbb{Z})$  always broken: non-linearly realized

$$\Psi \xrightarrow{\gamma} (c\tau + d)^{-k} \varrho(\gamma) \Psi$$

$\varrho(\gamma)$  representation of the group  $SL(2, \mathbb{Z}_N)$

| $N$                   | 2      | 3      | 4      | 5      | 6   |
|-----------------------|--------|--------|--------|--------|-----|
| $SL(2, \mathbb{Z}_N)$ | $S_3'$ | $A_4'$ | $S_4'$ | $A_5'$ | ... |

$\mathcal{Y}(\tau)$  depends on  $\tau$  to compensate the  $(c\tau + d)^{-(k+k_c)}$  factors

very restrictive if the model is SUSY



# Example

$$SL(2, \mathbb{Z}_3) \approx A_4'$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \rightarrow (c\tau + d)^{-1} \rho(\gamma) \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$k_\nu = +1$$

$\sim 3$  of  $SL(2, \mathbb{Z}_3)$

$$w(\tau, \nu) = \nu Y(\tau) \nu + h.c.$$

$$(c\tau + d)^{-2} \rho(\gamma)^T Y(\gamma\tau) \rho(\gamma) = Y(\tau)$$

has a unique solution:  
modular form

[F.F. 1706.08749]

$$m(\tau) = m_0 \begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix}$$

Yukawas completely  
determined in terms of  $\tau$   
up to an overall constant

no corrections from higher order operators in the exact SUSY limit

by scanning  $\tau$  VEVs the best agreement is obtained for

$$\tau = 0.0111 + 0.9946i$$

|                   | $\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}$ | $\sin^2 \vartheta_{12}$ | $\sin^2 \vartheta_{13}$ | $\sin^2 \vartheta_{23}$ | $\frac{\delta_{CP}}{\pi}$ | $\frac{\alpha_{21}}{\pi}$ | $\frac{\alpha_{31}}{\pi}$ |
|-------------------|---------------------------------------------|-------------------------|-------------------------|-------------------------|---------------------------|---------------------------|---------------------------|
| <i>Exp</i>        | 0.0292                                      | 0.297                   | 0.0215                  | 0.5                     |                           | —                         | —                         |
| $1\sigma$         | 0.0008                                      | 0.017                   | 0.0007                  | 0.1                     |                           | —                         | —                         |
| <i>prediction</i> | 0.0292                                      | 0.295                   | 0.0447                  | 0.651                   | 1.55                      | 0.22                      | 1.80                      |

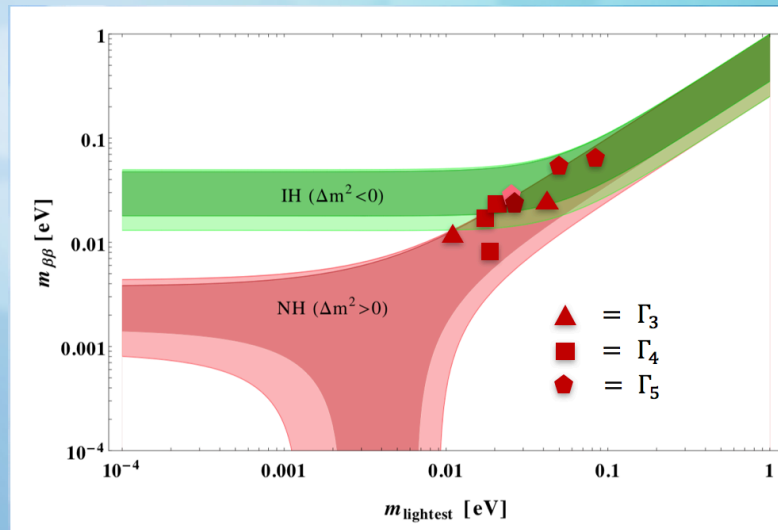
2-parameter fit to  
5 physical quantities

many  
 $\sigma$  away

8 dimensionless physical  
quantities independent on  
any coupling constant!

brought to agreement by a small contribution from the charged lepton

many models  
with predictive  
power



solutions with NO prefer  
 $m_1 > 10 \text{ meV}$  and  $|m_{ee}|$  on  
the high side of allowed  
range

Bottom-up modular invariance approaches to the lepton flavour problem have been exploited first using the groups

$\Gamma_3 \simeq A_4$  (F. Feruglio, 1706.08479; J.C. Criado, F. Feruglio, 1807.01125);

$\Gamma_2 \simeq S_3$  (T. Kobayashi et al., 1803.10391);

$\Gamma_4 \simeq S_4$  (J.T. Penedo, S.T. Petcov, 1806.11040, minimal, no flavons).

After these first studies, the interest in the approach grew significantly and models based on different groups have been constructed and extensively studied:

$\Gamma_4 \simeq S_4$

(Novichkov:2018ovf, Kobayashi:2019mna, Okada:2019lzv, Kobayashi:2019xvz, Gui-JunDing:2019wap, Wang:2019ovr, Wang:2020dbp, Gehrlein:2020jnr);

$\Gamma_5 \simeq A_5$

(P.P. Novichkov et al., 1812.02158; Ding:2019xna, Gehrlein:2020jnr);

$\Gamma_3 \simeq A_4$

(Kobayashi:2018scp, Novichkov:2018yse, Nomura:2019jxj, Nomura:2019yft, Ding:2019zxx, Okada:2019mjf, Nomura:2019lnr, Asaka:2019vev, Gui-JunDing:2019wap, Zhang:2019ngf, Nomura:2019xsb, Kobayashi:2019gtp, Wang:2019xbo, Abbas:2020vuy, Okada:2020dmb, Ding:2020yen, Behera:2020sfe, Nomura:2020opk, Nomura:2020cog, Behera:2020lpd, Asaka:2020tmo, Nagao:2020snm, Hatauruk:2020xtk);

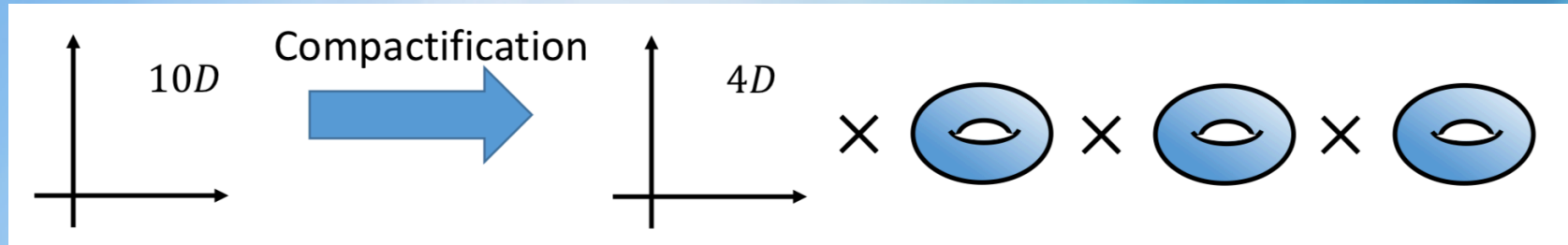
$\Gamma_2 \simeq S_3$  (Okada:2019xqk, Mishra:2020gxg);

$\Gamma_7 \simeq PSL(2, \mathbb{Z}_7)$  (G.-J. Ding et al., 2004.12662).

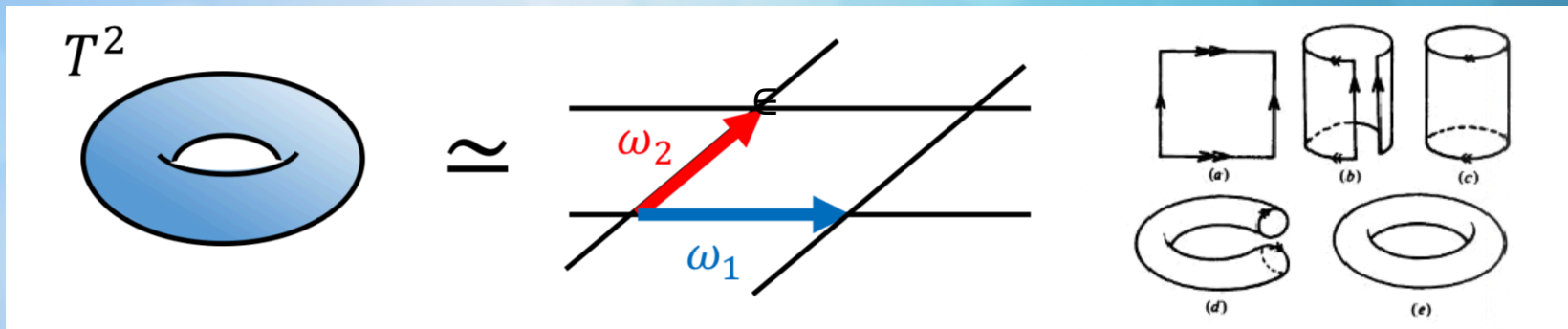
Similarly, attempts have been made to construct viable models of **quark flavour and of quark-lepton unification (including based on GUTs):**

# geometrical interpretation

string theory in  $d=10$  need 6 compact dimensions



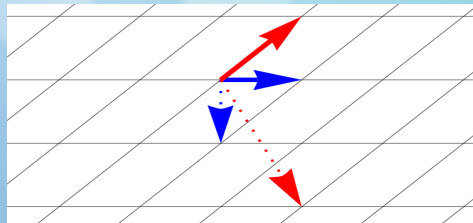
simplest compactification: 3 copies of a torus  $T^2$



tori parametrized by

$$\mathcal{M} = \left\{ \tau = \frac{\omega_2}{\omega_1} \mid \text{Im}(\tau) > 0 \right\}$$

lattice left invariant by modular transformations:



$$\tau \rightarrow \gamma\tau \equiv \frac{a\tau + b}{c\tau + d} \in SL(2, \mathbb{Z})$$

$a, b, c, d$  integers  
 $ad - bc = 1$

# conclusion

Neutrino mass generation:  
no evidence but several mechanisms interesting to explore

Learn more at Neutrino 2022 poster session:

Enrique Arrieta-Diaz, Xin Wang, Soumita Pramanick,  
Xu Li, Driya Mishra, Papia Panda, Camilo Cortes Pana,  
Mitesh Behera, Yuta Hyodo, Manuel Ettemberg, Nicolas Benoit

Flavour symmetries: a non conventional realization?

- relation to string theories:

[Nilles, Ramos-Sanchez, Trautner, Vaudrevange, Chen, Knapp-Perez,  
Ramos-Hamud, Ramos-Sanchez, Ratz and Shukla, Kobayasji, Otsuka,...]

- Modular invariance and CP violation
- Modular invariance and fermion mass hierarchy
- Modulus stabilization

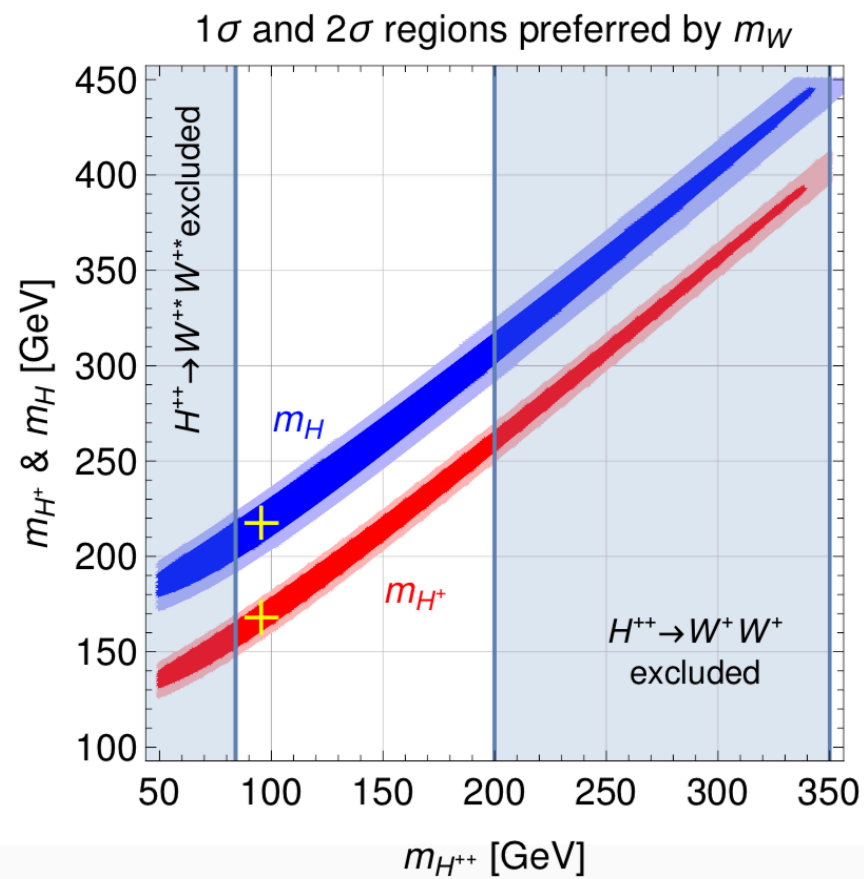
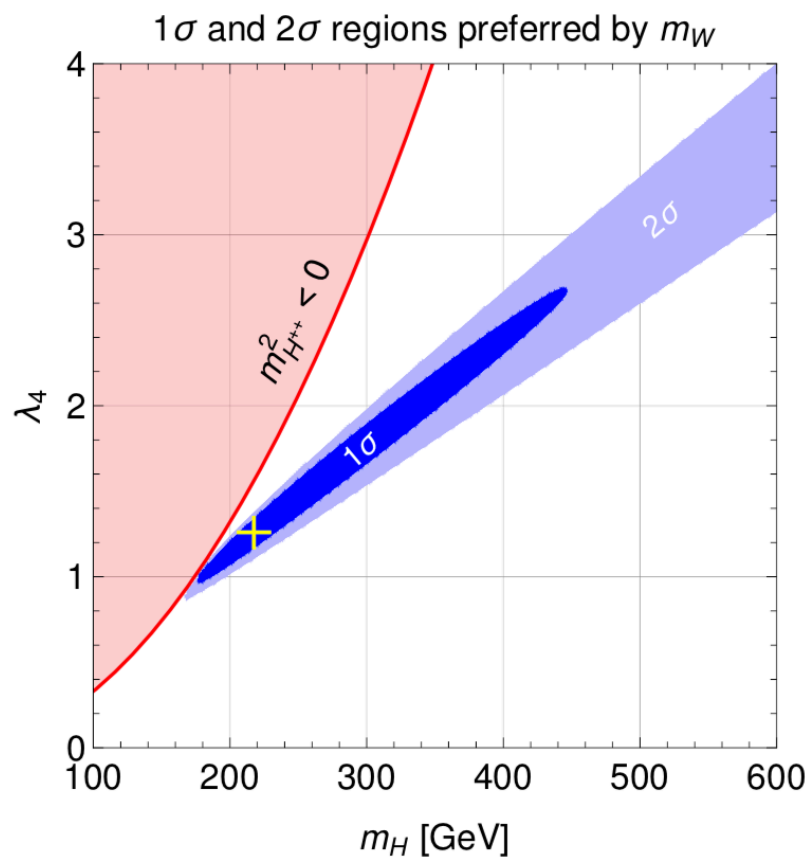
[Gherardi, Romanino, King, Novichkov, Penedo, Petcov, Titov, Ding, Liu, Tanimoto,...]

**THANK  
YOU!**



back-up slides

$$\mathcal{L} = \mathcal{L}_{SM} + \ell \mathcal{C}_4 \ell \Delta$$



[CMS 1802.02965v2]

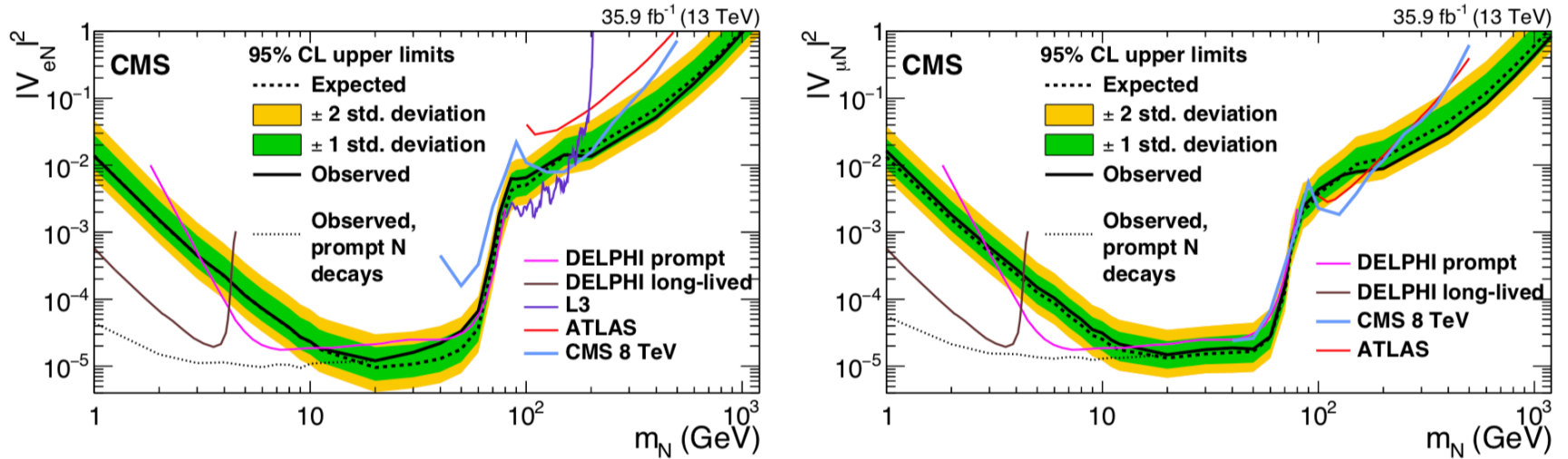


Figure 2: Exclusion region at 95% CL in the  $|V_{eN}|^2$  vs.  $m_N$  (left) and  $|V_{\mu N}|^2$  vs.  $m_N$  (right) planes. The dashed black curve is the expected upper limit, with one and two standard-deviation bands shown in dark green and light yellow, respectively. The solid black curve is the observed upper limit, while the dotted black curve is the observed limit in the approximation of prompt  $N$  decays. Also shown are the best upper limits at 95% CL from other collider searches in L3 [41], DELPHI [38], ATLAS [28], and CMS [27].

# any relation to the muon (g-2) ?

among all possible 1-particle extensions of the SM a special property enjoyed by scalar LQ that couples to quarks of BOTH chiralities

$$S_1 = (\bar{3}, 1, +1/3)$$

[not automatically p-decay free]

$$R_2 = (3, 2, +7/6)$$

contributions to dipole transitions can be chirally enhanced

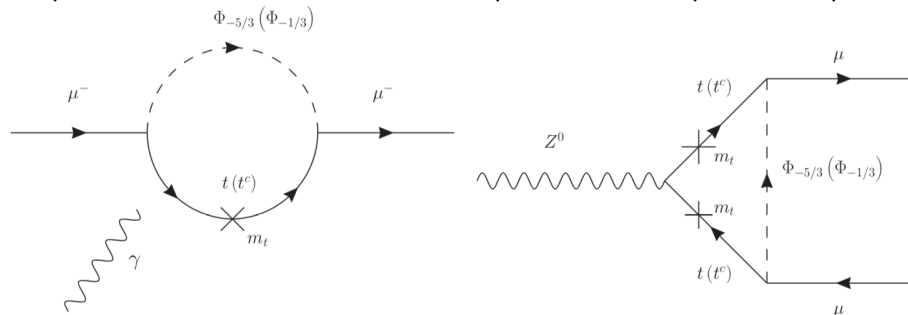
| $\delta a_\ell$                                                             | $\Gamma(\ell \rightarrow \ell' \gamma)$                                                                   |
|-----------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| $\frac{1}{16\pi^2} \frac{m_\ell m_{top}}{M_{LQ}^2} g_{t\ell}^L g_{t\ell}^R$ | $\frac{\alpha_{em}}{256\pi^4} \frac{m_\ell^3 m_{top}^2}{M_{LQ}^4}  g_{t\ell}^{L(R)} g_{t\ell'}^{R(L)} ^2$ |

[Djouadi, Kohler, Spira, Tutas, 1990  
Chakraverty, Choudhury, Datta 0102180  
Cheung 0102238  
Biggio, Bordone 1411.6799]

$\delta a_\mu$  of correct size for  $M_{LQ} \approx 1$  TeV in a weak coupling regime

1-to-1 correlation to (chirally enhanced) deviations in Z-coupling to leptons

Bauer, Neubert 1511.01900; Leskow, D' Ambrosio, Crivellin, Muller 1612.06858<sup>2</sup>



$$\delta BR(Z \rightarrow \ell^+ \ell^-) \approx \frac{1}{16\pi^2} \frac{m_{top}^2}{M_{LQ}^2} |g_{t\ell}^L|^2$$

many models addressing B-anomalies include  $S_1$  or  $R_2$  in their spectrum

[NC anomaly requires special care:  
no contribution to  $b \rightarrow s \ell^+ \ell^-$  from tree-level  $S_1$  exchange;  
 $C_9 = +C_{10}$  from  $R_2$  exchange]

Bauer, Neubert 1511.01900  
Chen, Nomura, Okada 1607.04857  
Caio, Gargalionis, Schmidt, Volkas 1704.05849  
Becirevic, Sumensari, 1704.05835  
Chauhan, Kindra, Narang, 1706.04598  
Crivellin, Muller, Ota, 1703.09226  
...

$S_1$   
 $R_2 + S_3$   
 $S_1$   
 $R_2$   
 $R_2$   
 $S_1 + S_3$

if LQ couples mainly to top and 2<sup>nd</sup> lepton generation

$$\delta a_\mu \approx +3 \times 10^{-9}$$



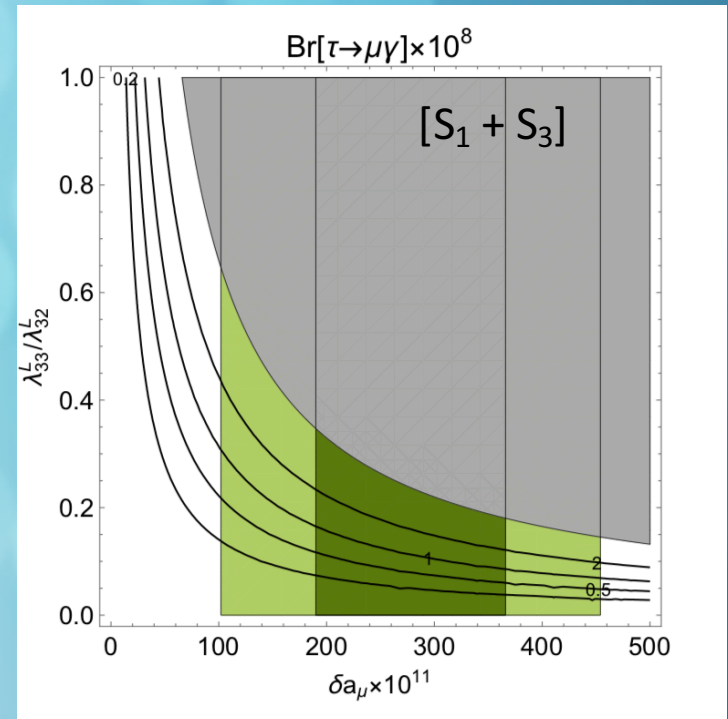
$$\delta BR(Z \rightarrow \mu^+ \mu^-) \approx 10^{-4}$$

If LQ couples also to top and 3<sup>rd</sup> lepton generation

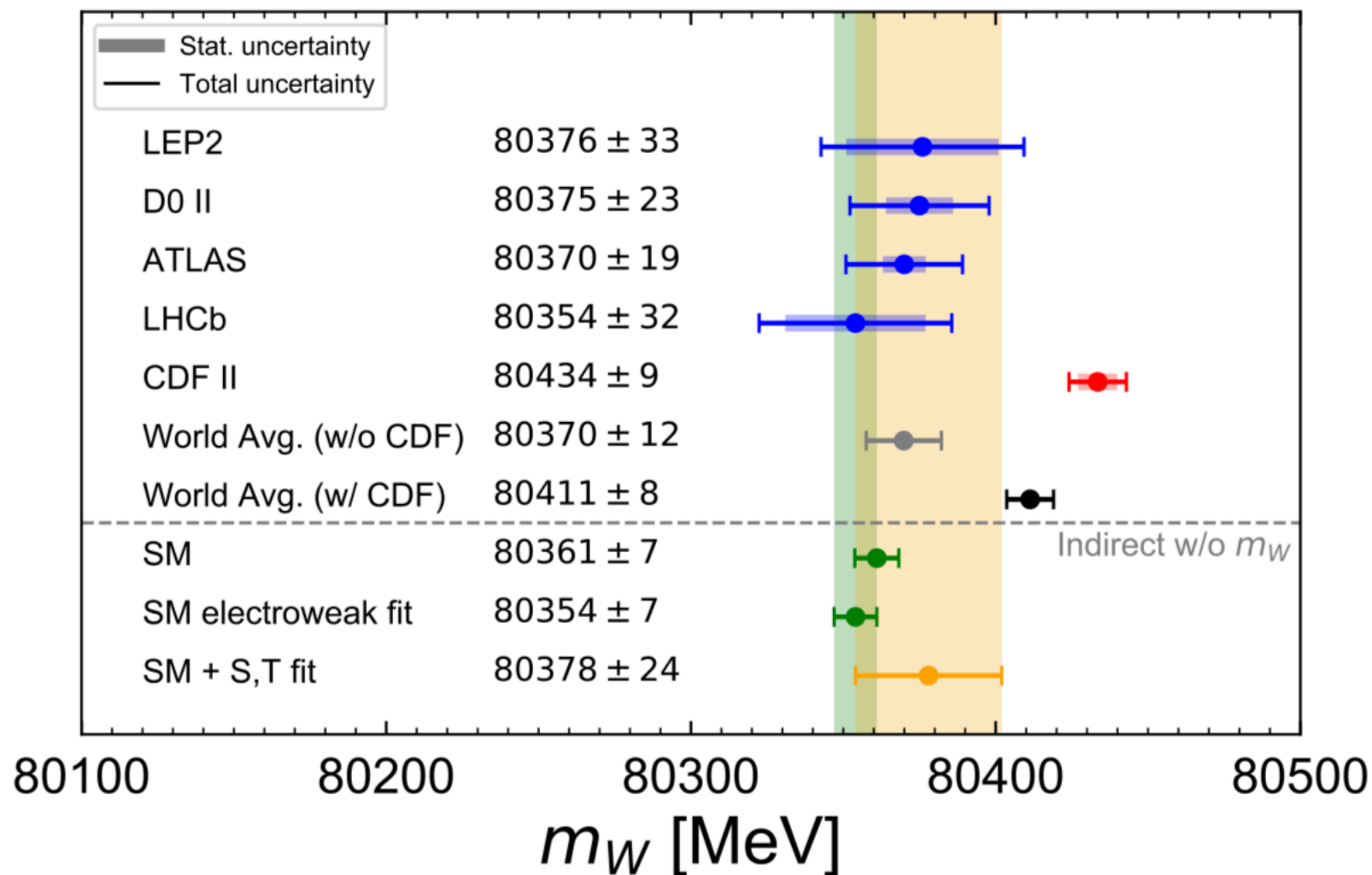


$$\frac{BR(\tau \rightarrow \mu \gamma)}{(\delta a_\mu)^2} \geq \frac{9 \times 10^{-7}}{(+3 \times 10^{-9})^2} \left( \frac{g_{33}^L}{g_{32}^L} \right)^2$$

$$BR(Z \rightarrow \tau^\pm \mu^\mp) \approx \frac{10^{-8} (g_{33}^L g_{32}^L)^2}{M_{LQ}^4 (TeV)}$$



[Crivellin, Muller, Ota, 1703.09226]



[Emanuele Bagnaschi, John Ellis, Maeve Madigan, Ken Mimasu, Veronica Sanze, and Tevong You, 2204.05260v2]



# tests of modulus couplings

G-J. Ding, FF,  
2003.13448

non standard neutrino  
interactions

$$\mathcal{L} = i \sum_{f=e,e^c,\nu} \bar{f} \bar{\sigma}^\mu \partial_\mu f + \frac{1}{2} \partial_\mu \varphi_\alpha \partial^\mu \varphi_\alpha - \frac{1}{2} M_\alpha^2 \varphi_\alpha^2$$

$$- (m_e + \mathcal{Z}_\alpha^e \varphi_\alpha) e^c e - \frac{1}{2} \nu (m_\nu + \mathcal{Z}_\alpha^\nu \varphi_\alpha) \nu + h.c. + \dots$$

$$\tau = \langle \tau \rangle + \frac{\varphi_u + i \varphi_v}{\sqrt{2}}$$

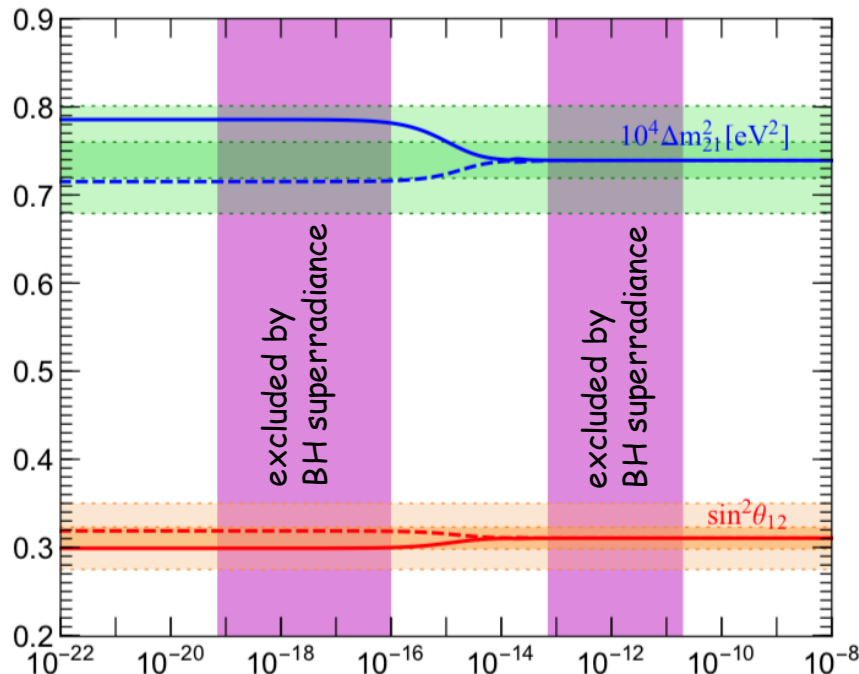


in medium with non-zero  
electron number density

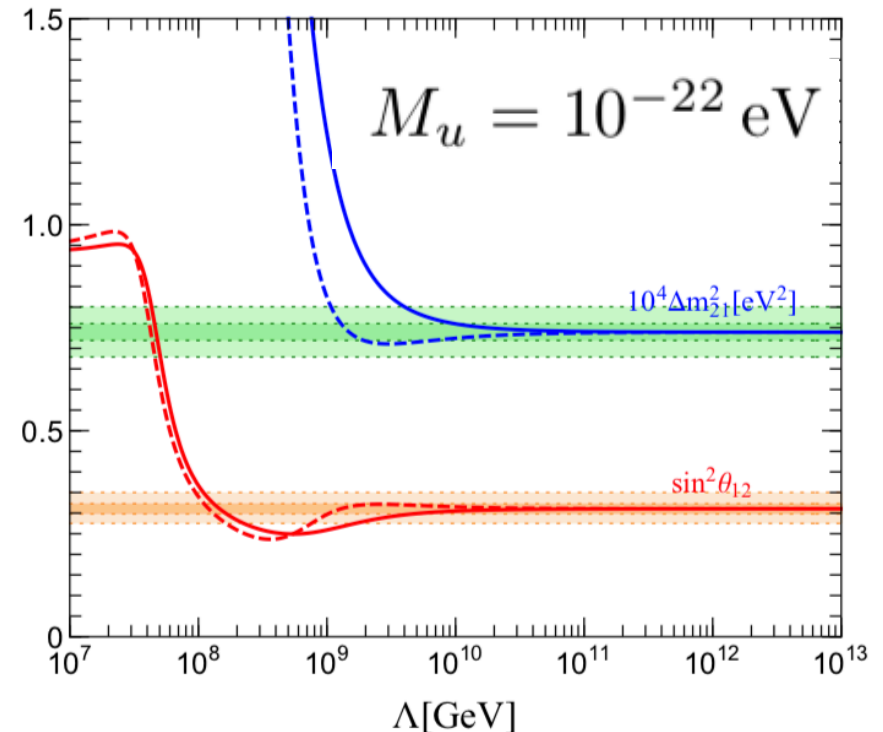
small, unless the modulus is very light

$$\delta m_\nu(0) = -n_e^0 \frac{\text{Re}(\mathcal{Z}^e) \mathcal{Z}^\nu}{M^2(R)},$$

in the sun:



$$\Lambda = 5 \times 10^9 \text{ GeV} \quad \begin{matrix} M_u [\text{eV}] \\ [\text{modulus VEV}] \end{matrix}$$



|                      | “flavor+electroweak”<br>$m > \text{EW}$ ( $2\sigma$ limit) | “Averaged-out oscillations”<br>$\Delta m^2 \gtrsim 0.1 \text{ eV}^2$ (90% CL) |
|----------------------|------------------------------------------------------------|-------------------------------------------------------------------------------|
| $\alpha_{ee}$        | $1.3 \cdot 10^{-3}$ [36]                                   | $8.4 \cdot 10^{-3}$ [55]                                                      |
| $\alpha_{\mu\mu}$    | $2.2 \cdot 10^{-4}$ [36]                                   | $5.0 \cdot 10^{-3}$ [15]                                                      |
| $\alpha_{\tau\tau}$  | $2.8 \cdot 10^{-3}$ [36]                                   | $6.5 \cdot 10^{-2}$ [56]                                                      |
| $ \alpha_{\mu e} $   | $6.8 \cdot 10^{-4}$ ( $2.4 \cdot 10^{-5}$ ) [36]           | $9.2 \cdot 10^{-3}$                                                           |
| $ \alpha_{\tau e} $  | $2.7 \cdot 10^{-3}$ [36]                                   | $1.4 \cdot 10^{-2}$                                                           |
| $ \alpha_{\tau\mu} $ | $1.2 \cdot 10^{-3}$ [36]                                   | $1.1 \cdot 10^{-2}$                                                           |

Table 1: **Current upper bounds on the  $\alpha$  parameters.** The bounds in the middle column (shown at  $2\sigma$ , 1 d.o.f.) apply for  $\Delta m^2 \gtrsim 100 \text{ eV}^2$  and are taken to a global fit to flavor and electroweak precision observables from Ref. [36]. The bounds in the right column (shown at 90% CL, for 1 d.o.f.) apply for  $\Delta m^2 \gtrsim 0.1 \text{ eV}^2$  and come from neutrino oscillation searches, when the sterile neutrinos would be in the averaged-out regime for such a mass scale. The second number quoted in parenthesis for the  $\alpha_{\mu e}$  element includes the  $\mu \rightarrow e\gamma$  observable, which can in principle be evaded [28]. The bounds without a reference are indirectly obtained from constraints on the diagonal parameters via  $\alpha_{\alpha\beta} \leq 2\sqrt{\alpha_{\alpha\alpha}\alpha_{\beta\beta}}$ , which follows from the unitarity of the full  $\mathcal{U}$  mixing matrix.

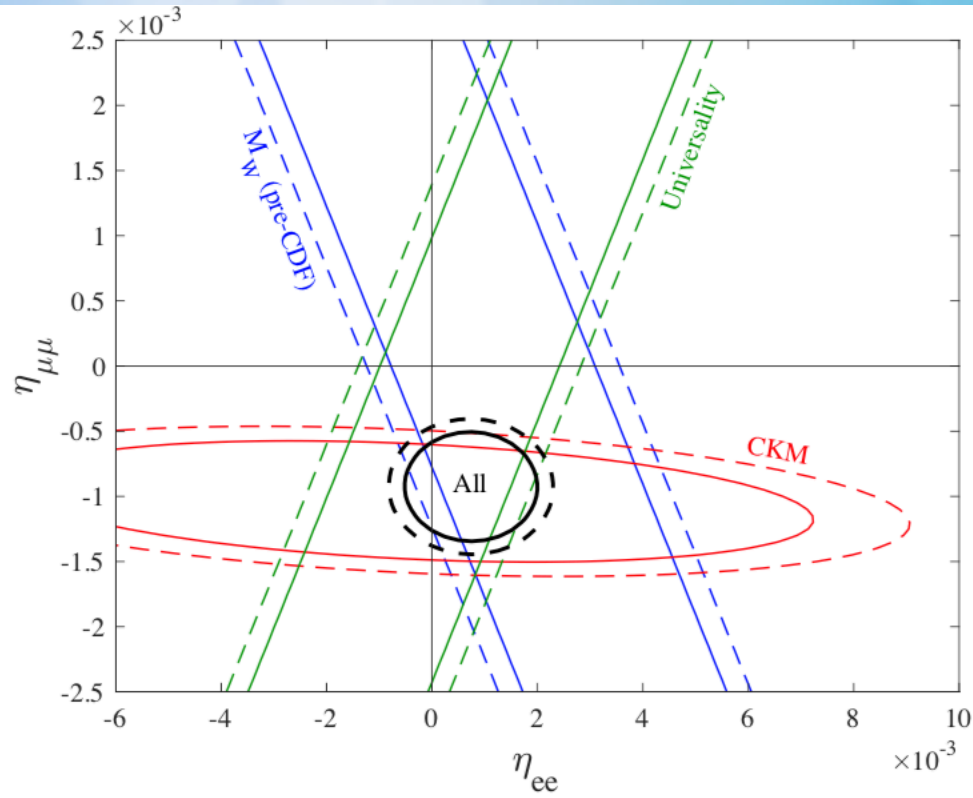


Figure 1. Results of our fit, projected onto the  $\eta_{ee}$ - $\eta_{\mu\mu}$ -plane. The solid (dashed) curves correspond to the allowed regions at 95% CL (99% CL), for 2 d.o.f.. The constraints are labelled as ‘ $M_W$ ’ for the measurement of the  $W$  mass, ‘Universality’ for the lepton flavour universality bounds, and ‘CKM’ for the bounds coming from beta and kaon decays. In the upper (lower) panel we take the constraints on  $M_W$  from CDF II [7] (the global average before CDF II [42]). The black contours correspond to combinations of different data sets, see text for details.