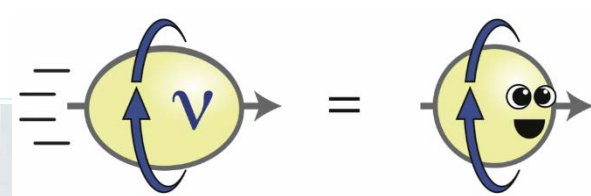


SEOUL, May 30 (Mon) – June 4 (Sat)



Overview of current experimental and theoretical status of $0\nu\beta\beta$

Fedor Šimkovic

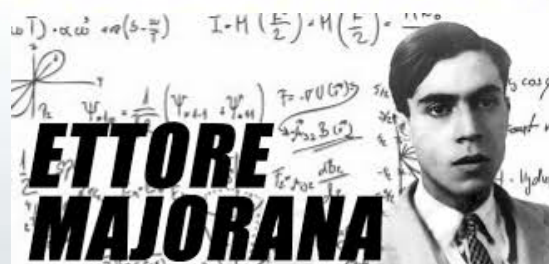
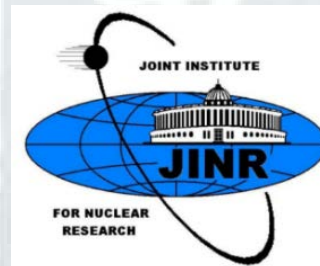
$\nu\bar{\nu}$ ARE NEUTRINOS
THEIR OWN?
ANTIPARTICLES?



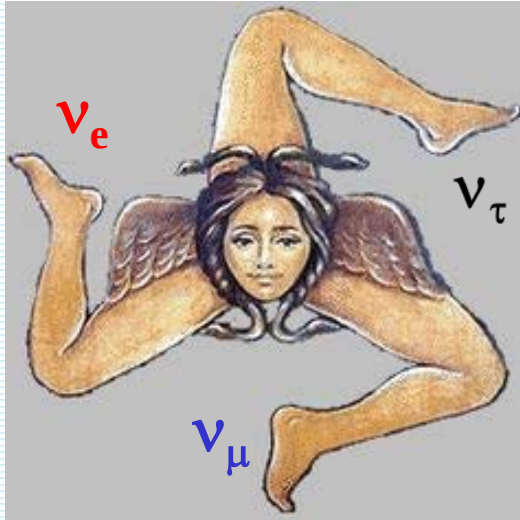
$\nu\bar{\nu}$ ARE NEUTRINOS
THEIR OWN?
ANTIPARTICLES?



NEUTRINO 2022



OUTLINE



- I. *Introduction* (brief history, Majorana ν 's,)
- II. *The $0\nu\beta\beta$ -decay experiments, status and perspectives*
(Gerda/Majorana/Legend, EXO, KamLAND-Zen, NEXT, CUORE, CUPID, etc.)
- III. *ν -mass $0\nu\beta\beta$ -decay mechanisms*
(QCSS scenario, sterile ν , LR symmetric model, LNV at LHC)
- IV. *The $2\nu\beta\beta$ -decay and new physics*
(sterile heavy ν , right-handed ν)
- V. *The $0\nu\beta\beta$ -decay NMEs, current status*
(nuclear structure approaches, uncertainties, contact term)
- VI. *Supporting nuclear experiments and effective g_A*
(β -decay, $2\nu\beta\beta$ -decay, muon capture, heavy-ion DCX)
- V. *Outlook*

Acknowledgements:

$0\nu\beta\beta$ -decay exp.: A. Giuliani, G. Gratta, Ch. Marques, I. Shimizu, L. Szimard, D. Waters

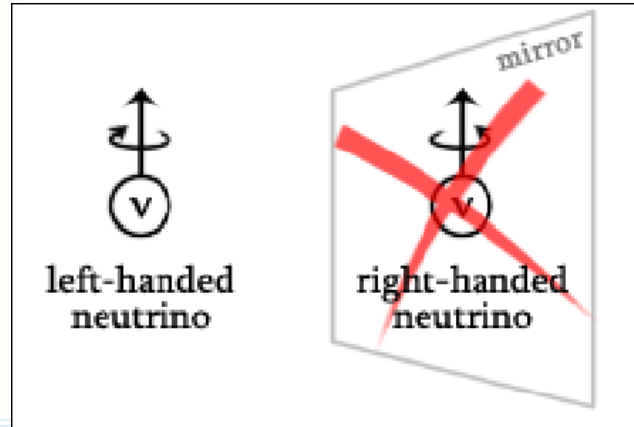
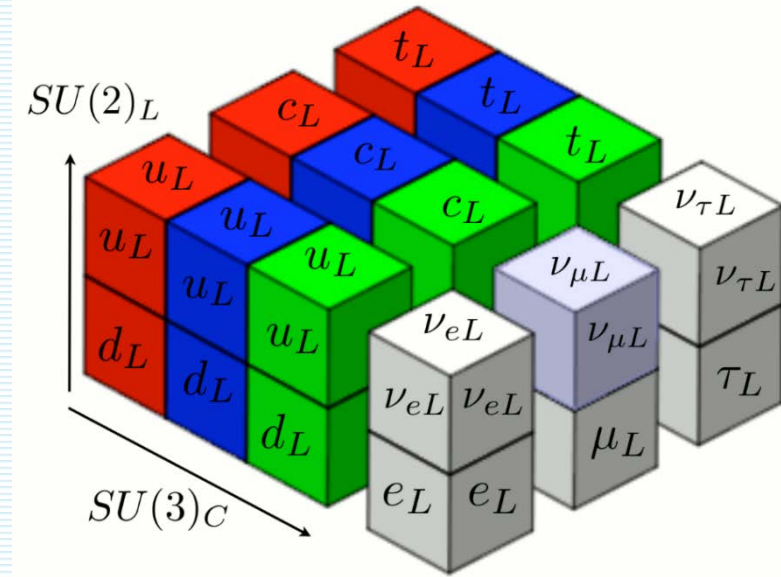
$0\nu\beta\beta$ -decay theory: S. Bilenky, V. Cirigliano, S. Kovalenko, M. Krivoruchenko, S. Petcov, A. Smetana

$0\nu\beta\beta$ -decay NMEs: C. Agodi, F. Cappuzzello, J. Engel, E. Lisi, P. Vogel,
and other colleagues and friends.

Standard Model
(an astonishing successful theory,
based on few principles)

ν is a **special particle** in SM:

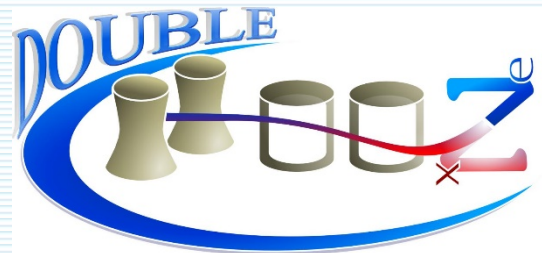
- It is the only fermion that **does not carry electric charge** (like γ , g , H^0)
- There are only **left-handed ν 's** (ν_{eL} , $\nu_{\mu L}$, $\nu_{\tau L}$)
- **ν -mass** can not be generated with any renormalizable coupling with the Higgs fields through SSB



ν 's oscillations experiments
 $\Rightarrow$ **tiny neutrino masses (!)**
 $\Rightarrow$ **Beyond SM physics (!)**



6/3/2022



, etc





Majorana fermions

Ettore Majorana

Teoria simmetrica dell'elettrone e del positrone
(A symmetric theory of electrons and positrons).
Il Nuovo Cimento, 14: 171–184, 1937.) 171

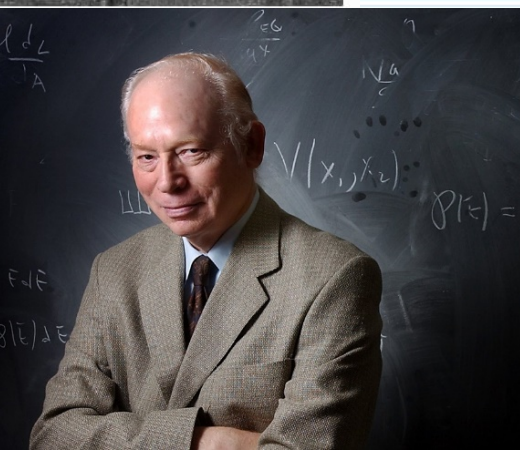
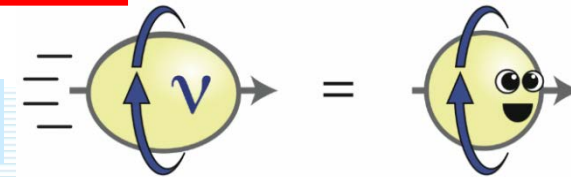
ν is its own
antiparticle



Bruno Pontecorvo
*Inverse beta processes and
nonconservation of lepton charge*
Zhur. Eksptl'. i Teoret. Fiz.
34, 247 (1958)

It follows from the above assumptions that in vacuum a neutrino can be transformed into an antineutrino and vice versa. This means that the neutrino and antineutrino are “mixed” particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 of different combined parity.⁵

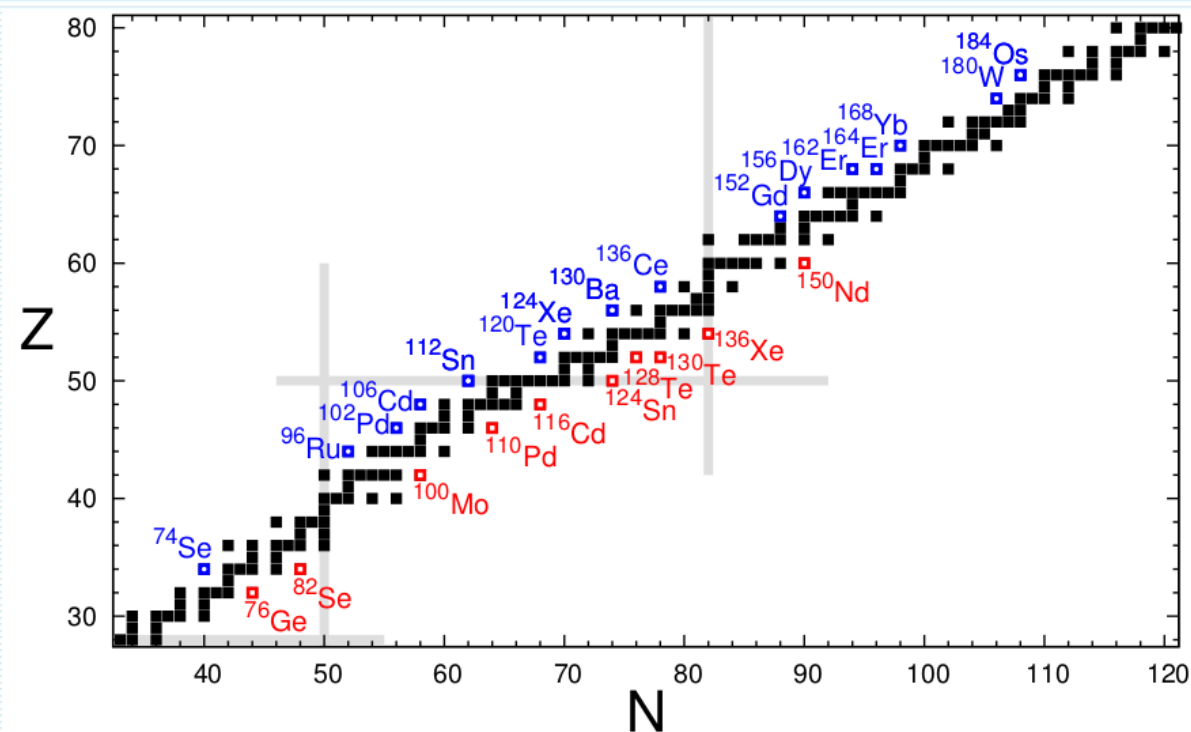
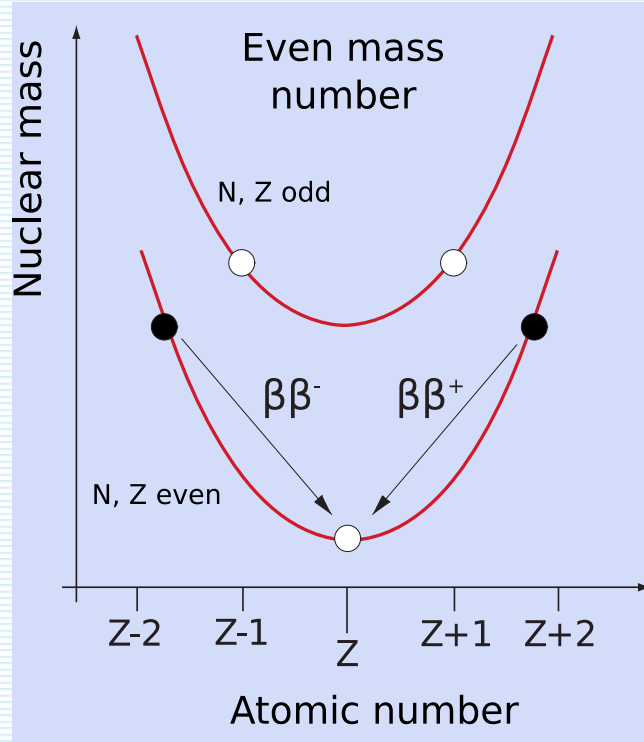
$\nu \leftrightarrow \text{anti-}\nu$ oscillation



Steve Weinberg
 ν -mass generation
via d=5 eff. oper.
related to unknown
high energy scale (GUT?)

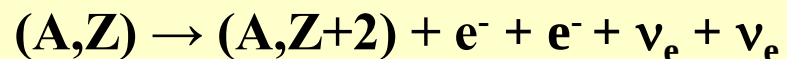
thought massless back in 1979. Weinberg does not take credit for predicting neutrino masses, but he thinks it's the right interpretation. What's more, he says, the non-renormalisable interaction that produces the neutrino masses is probably also accompanied with non-renormalisable interactions that produce proton decay and other things that haven't been observed, such as violation of baryon-number conservations. “We don't know anything about the details of those terms, but I'll swear they are there.”

Nuclear double- β decay (even-even nuclei, pairing int.)



Phys. Rev. 48, 512 (1935)

Two-neutrino double- β decay – LN conserved

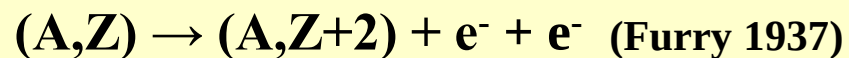


Goepert-Mayer – 1935. 1st observation in 1987

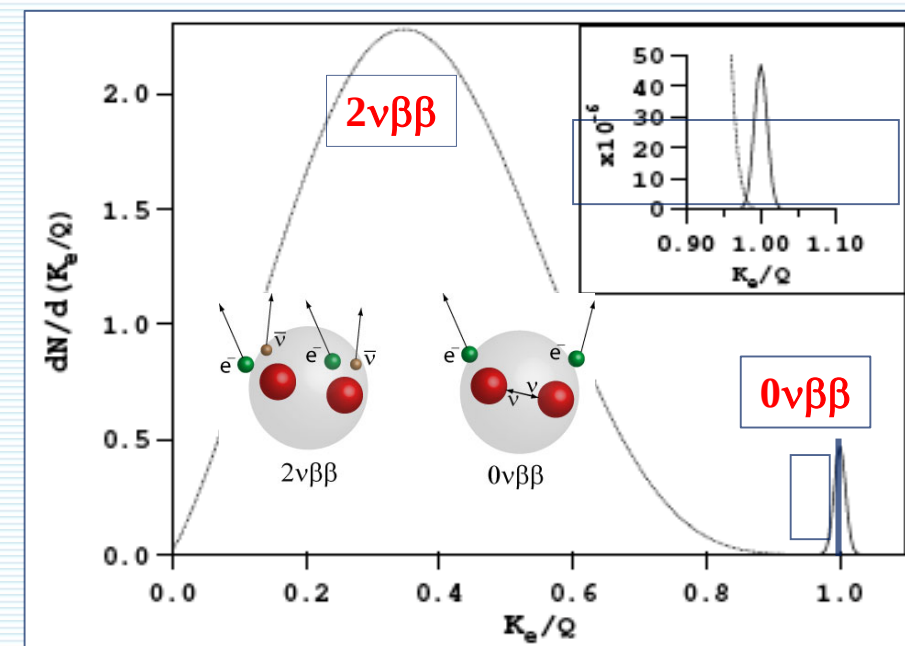
Nuovo Cim. 14, 322 (1937)

Phys. Rev. 56, 1184 (1939)

Neutrinoless double- β decay – LN violated



Not observed yet. Requires massive Majorana ν 's



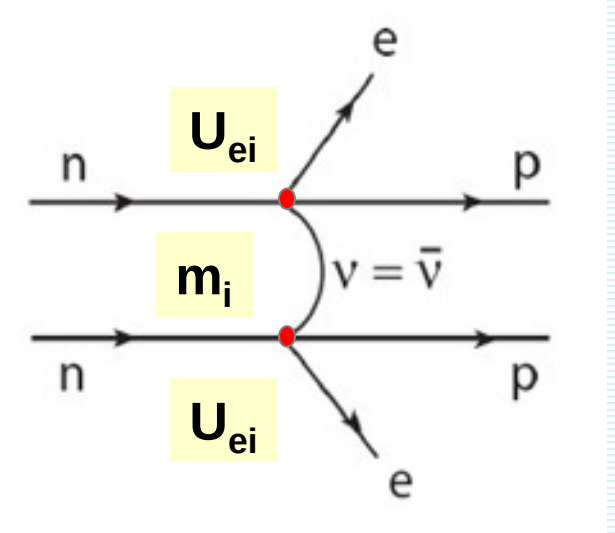
$$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$$

$0\nu\beta\beta$ -decay
(LNV at $\approx GUT$ scale, exchange of three light ν)

$$\left(T_{1/2}^{0\nu}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M_\nu^{0\nu}\right|^2 G^{0\nu}$$

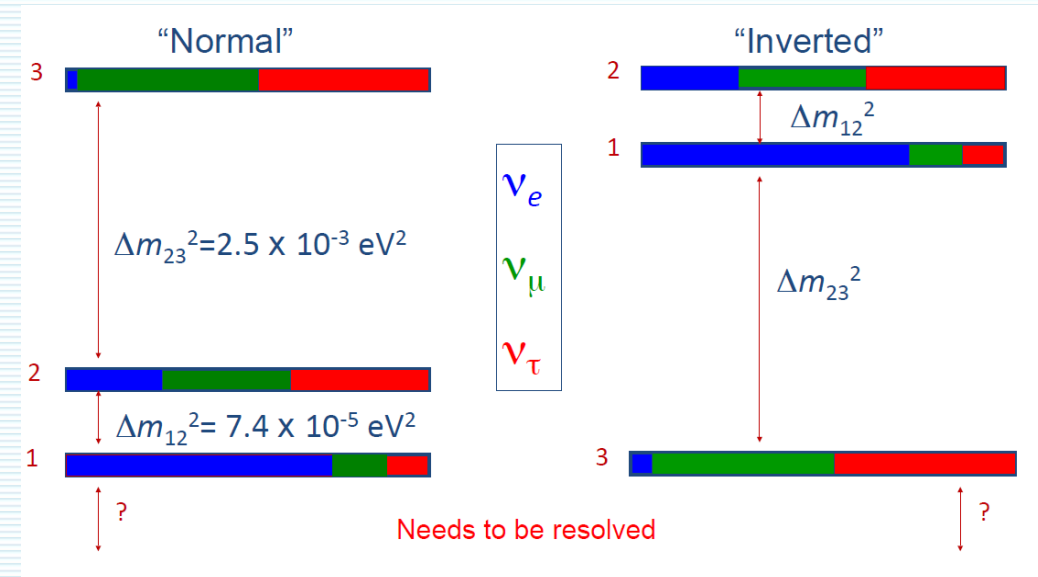
Phase space factor
well understood

*NME must be evaluated
using tools of nuclear theory*



Effective Majorana mass
can be evaluated. It depends on
 $m_1, m_2, m_3, \theta_{12}, \theta_{13}, \alpha_1, \alpha_2$
**(3 unknown parameters: $m_1/m_3, \alpha_1, \alpha_2$
 and ν -mass hierarchy)**

$$m_{\beta\beta} = \left| c_{13}^2 c_{12}^2 e^{i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3 \right|$$



$$U^{PMNS} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -e^{i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

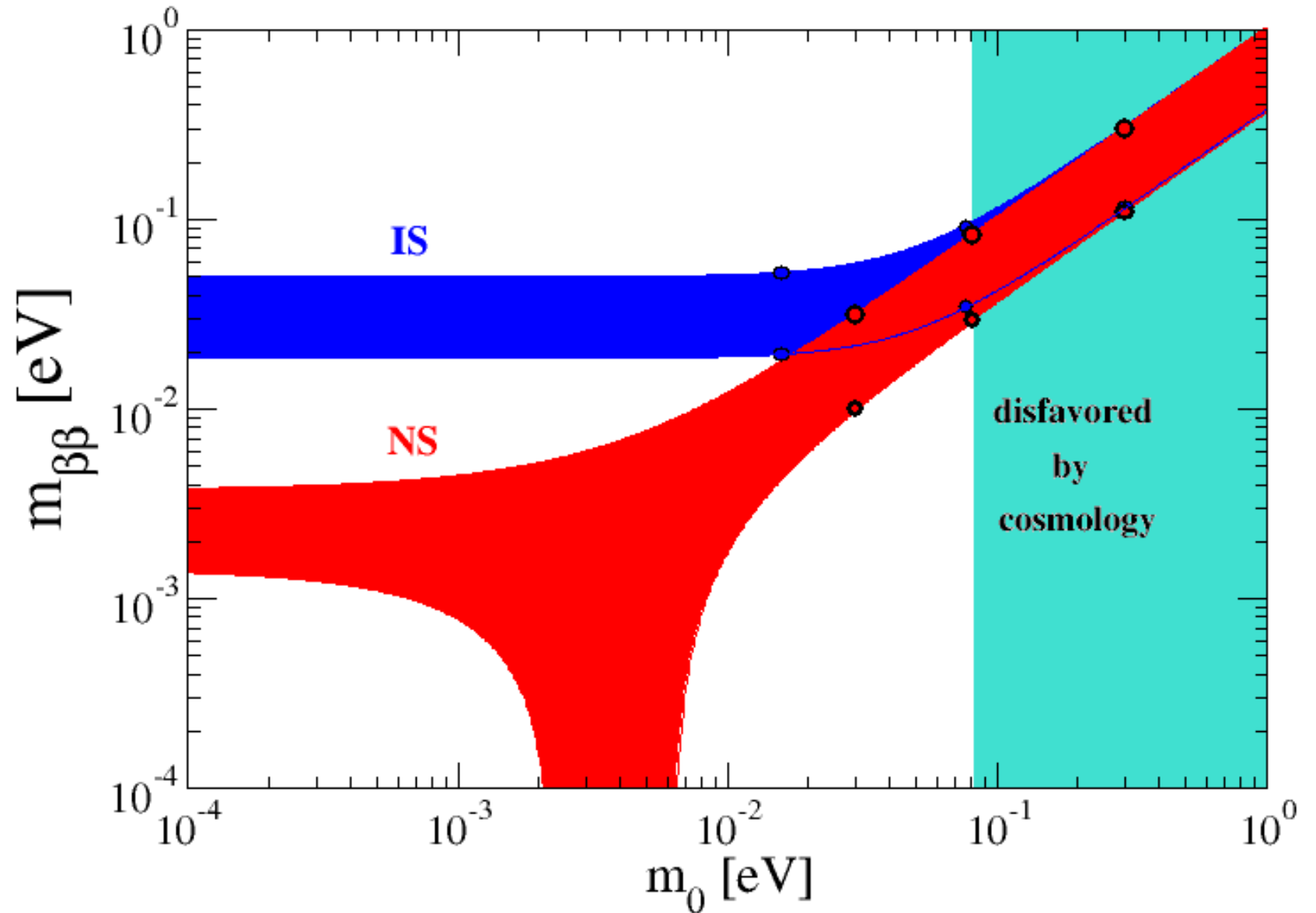
**Effective Majorana
 ν -mass $m_{\beta\beta}$
 (prediction
 due ν -oscillations)**

Constraint from cosmology

$$\begin{aligned}\Sigma &= m_1 + m_2 + m_3 \\ &< 0.90 \text{ eV} \\ &< 0.26 \text{ eV (Planck coll.)} \\ &< 0.12 \text{ eV}\end{aligned}$$

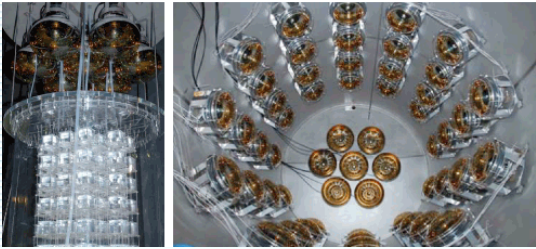
**Contrary, the constraint from
 $0\nu\beta\beta$ -decay (KLZ)**

$$\begin{aligned}m_{\beta\beta} &< 0.036\text{--}0.156 \text{ eV} \\ \text{implies} \\ \Sigma &< 0.12 \text{ eV}\end{aligned}$$

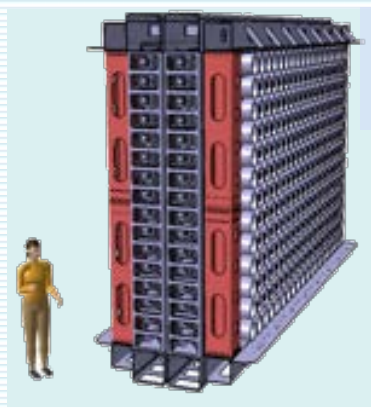


**$0\nu\beta\beta$ decay
isotopes
and
experiments**

[Current CANDLES detector]



CANDLE
CaF
scintillating
crystal



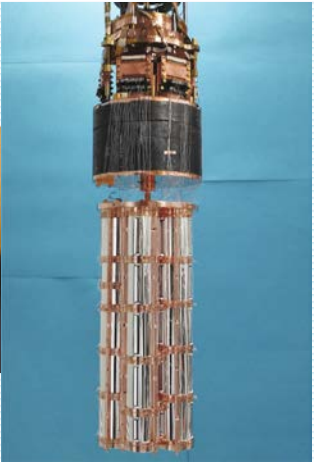
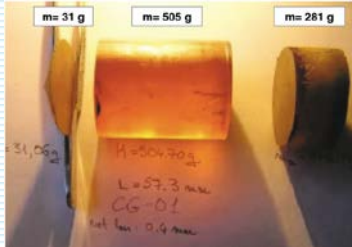
SuperNEMO
Se source foil

GERDA, MAJORANA
Ge crystal

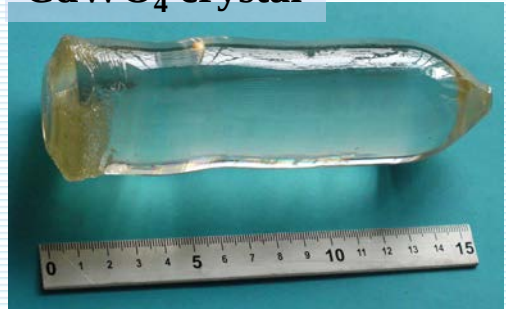


Candidates	$Q_{\beta\beta}$ (MeV)	N.A. (%)
$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$	4.268	0.187
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	2.039	7.8
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	2.998	8.8
$^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$	3.356	2.8
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	3.034	9.7
$^{110}\text{Pd} \rightarrow ^{110}\text{Cd}$	2.017	11.7
$^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$	2.813	7.5
$^{124}\text{Sn} \rightarrow ^{124}\text{Te}$	2.293	5.8
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	2.528	34.1
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	2.458	8.9
$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$	3.371	5.6

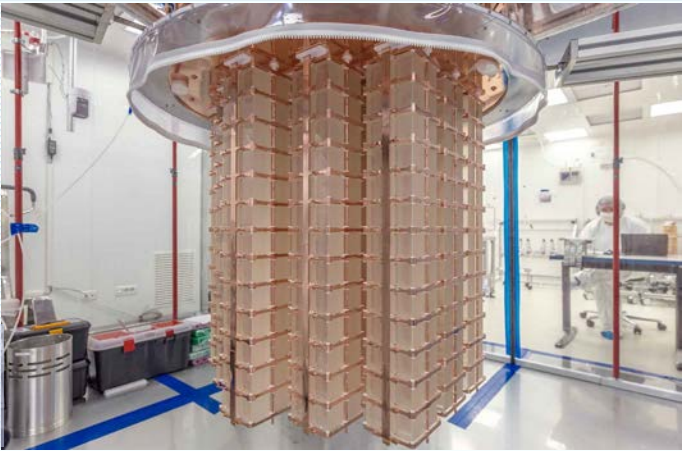
CUPID-0
ZnSe
scintillating
crystal



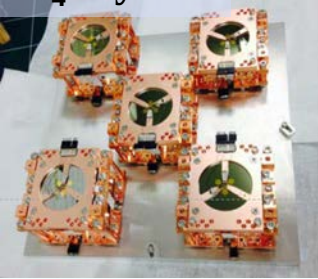
Aurora
 CdWO_4 crystal



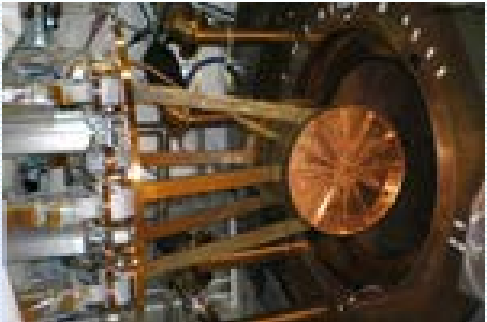
CUORE
 TeO_2 crystal



Amore
 CaMoO_4 crystal



EXO, KamLAND-Zen
Liquid Xe



Enrichments of isotopes

^{76}Ge : Semiconductor industry enriches Ge already – ^{76}Ge can in principle be extracted as by product.

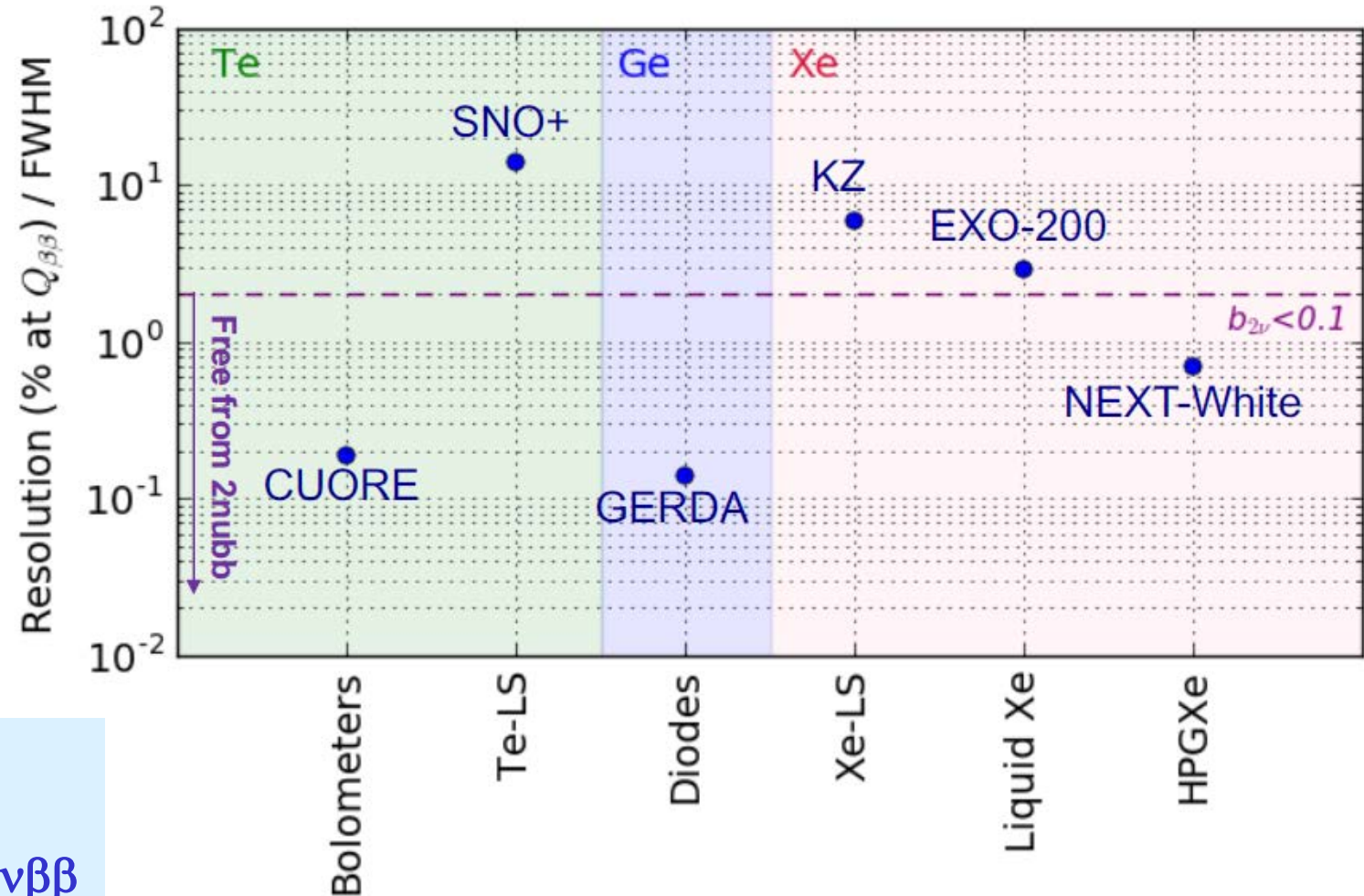
^{100}Mo : New capacity for enrichment of Mo in ^{100}Mo for nuclear medicine is needed. Parasitic?

^{130}Te : Comes naturally enriched. Natural Te is the most viable for an unenriched experiment.

^{136}Xe : Atmospheric carbon capture technology based on metal oxide has plausible extendibility to capture atmospheric Xe. Free from steel industry capacity limit?

- Resolution better than 2% FWHM to fully reject $2\nu\beta\beta$ mode
- solar ν is not an issue for 100 kg-class $0\nu\beta\beta$ experiment

Quite challenging 100 kg-class $0\nu\beta\beta$ experiment
(zero background, large mass, long time measurement, very good energy resolution)



Leading limits in each $0\nu\beta\beta$ isotope

A monoenergetic peak at the Q-value is searched for.
Need a large amount of decay isotope and low radioactive environment

Experiment	Isotope	Exposure [kg yr]	$T_{1/2}^{0\nu}$ [10^{25} yr]	$m_{\beta\beta}$ [meV]
Gerda	^{76}Ge	127.2	18	79-180
Majorana	^{76}Ge	26	2.7	200-433
CUPID-0	^{82}Se	5.29	0.47	276-570
NEMO3	^{100}Mo	34.3	0.15	620-1000
CUPID-Mo	^{100}Mo	2.71	0.18	280-490
Amore	^{100}Mo	111	0.095	1200-2100
CUORE	^{130}Te	1038.4	2.2	90-305
EXO-200	^{136}Xe	234.1	3.5	93-286
KamLAND-Zen	^{136}Xe	970	23	36-156

Zero background experiment

$$T_{1/2}^{0\nu}(\text{exp}) = (\ln 2) N_a \frac{a}{A} \varepsilon \sqrt{\frac{MT}{b\Delta E}}$$

M - mass of detector in kg,
A - mass number of candidate
 isotope

a - $\beta\beta$ isotope concentration
 (enrichment)

T - measured time in years

ε – detection efficiency

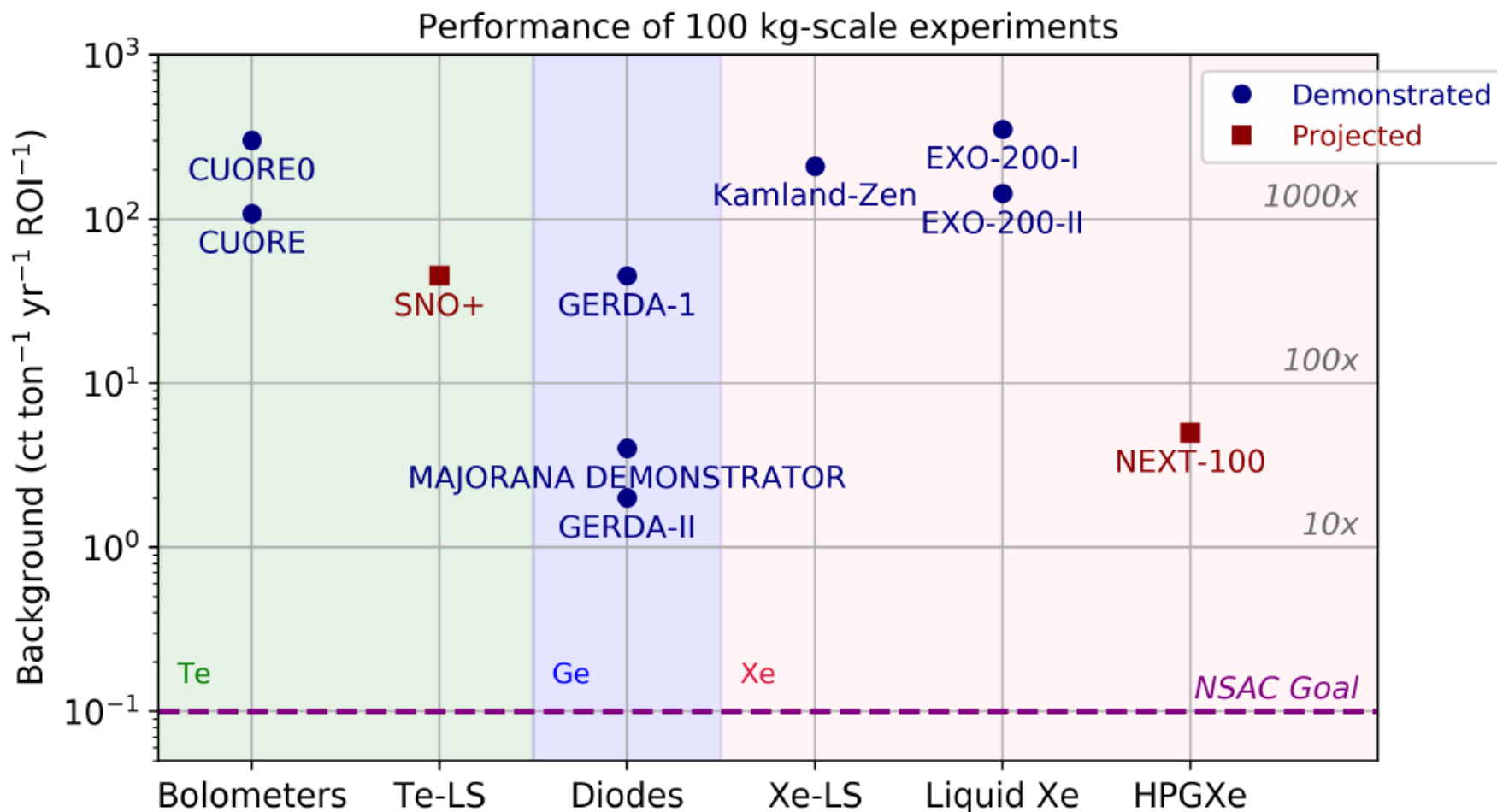
b - events in ROI

ΔE - detector performance

When $b = 0$

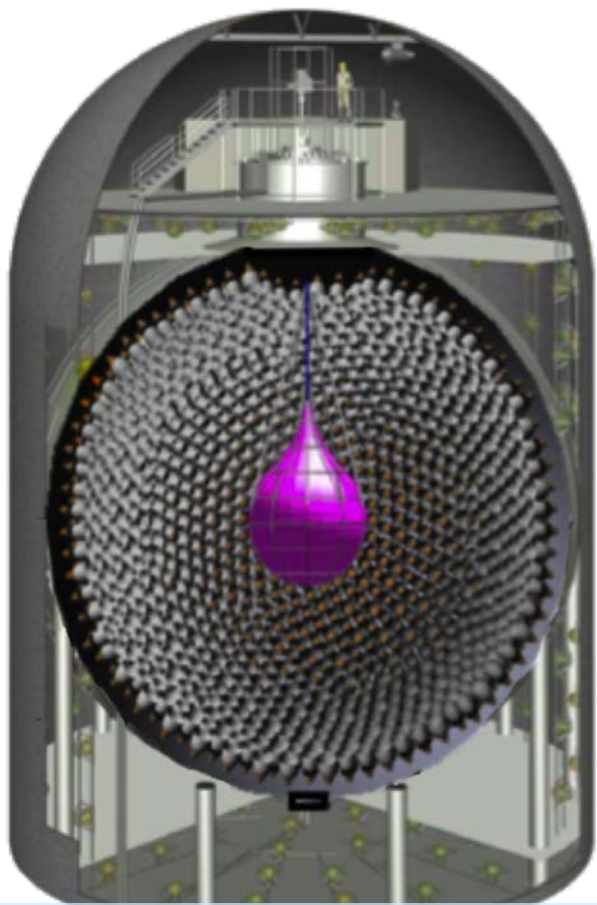
$$T_{1/2}^{0\nu}(\text{exp}) = (\ln 2) N_a \frac{a}{A} \varepsilon \frac{MT}{n_{CL}}$$

“100kg-class” experiments:



KamLAND-Zen

1 ton-class ^{136}Xe $0\nu\beta\beta$ experiment
reaching IH region



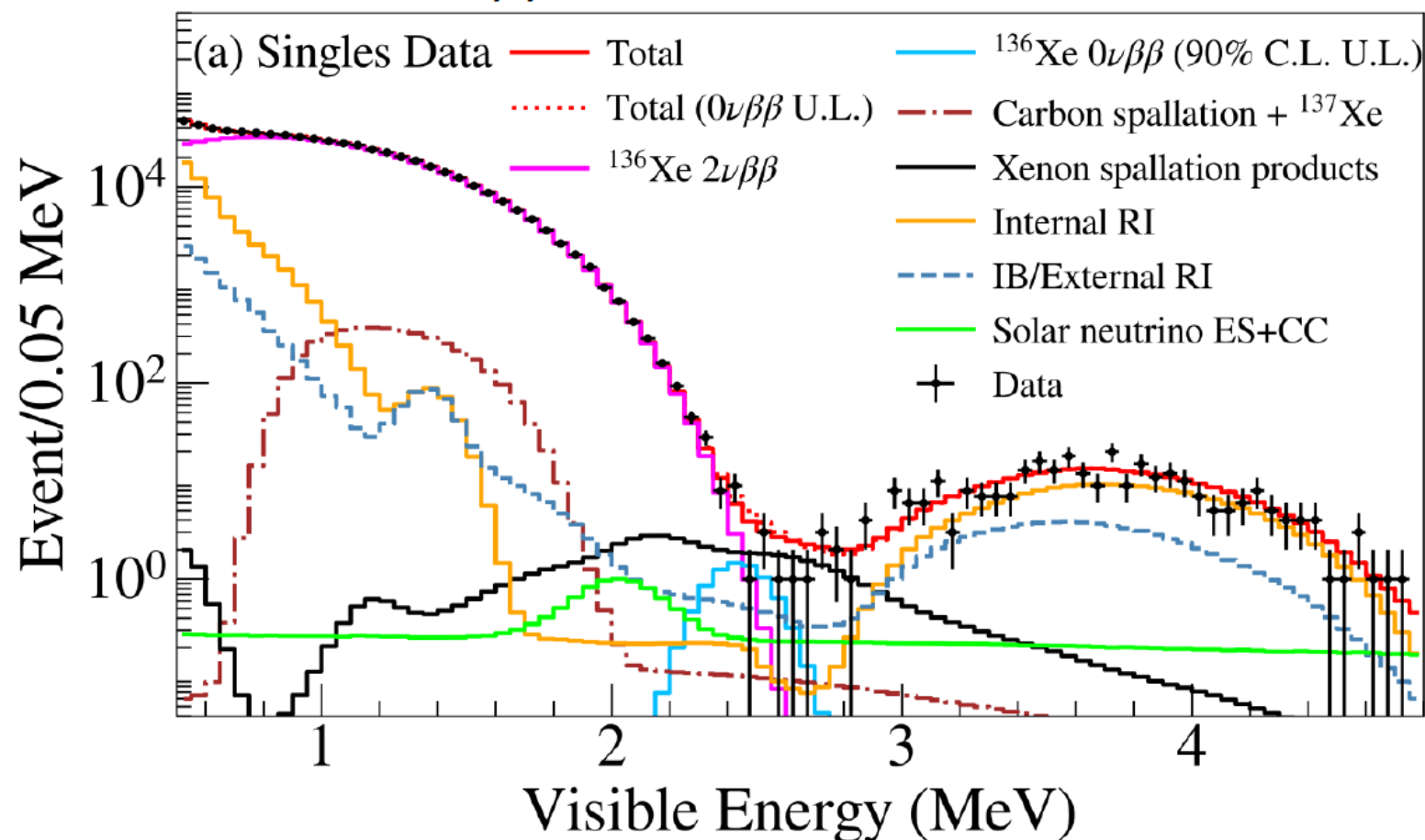
Large volume
liquid scintillator detector
LS: 30.5 m³, ^{136}Xe : 677 kg

KamLAND-Zen 400 and KamLAND-Zen 800 combined results

Limit: $T^{0\nu}_{1/2} > 2.3 \times 10^{26}$ year (90% C.L.), $m_{\beta\beta} < 36\text{--}156$ meV
($g_A=1.27$, NME = 1.11-4.77 are assumed)

Currently, the most strict $0\nu\beta\beta$ limit

$0\nu\beta\beta$ candidate data set



LEGEND

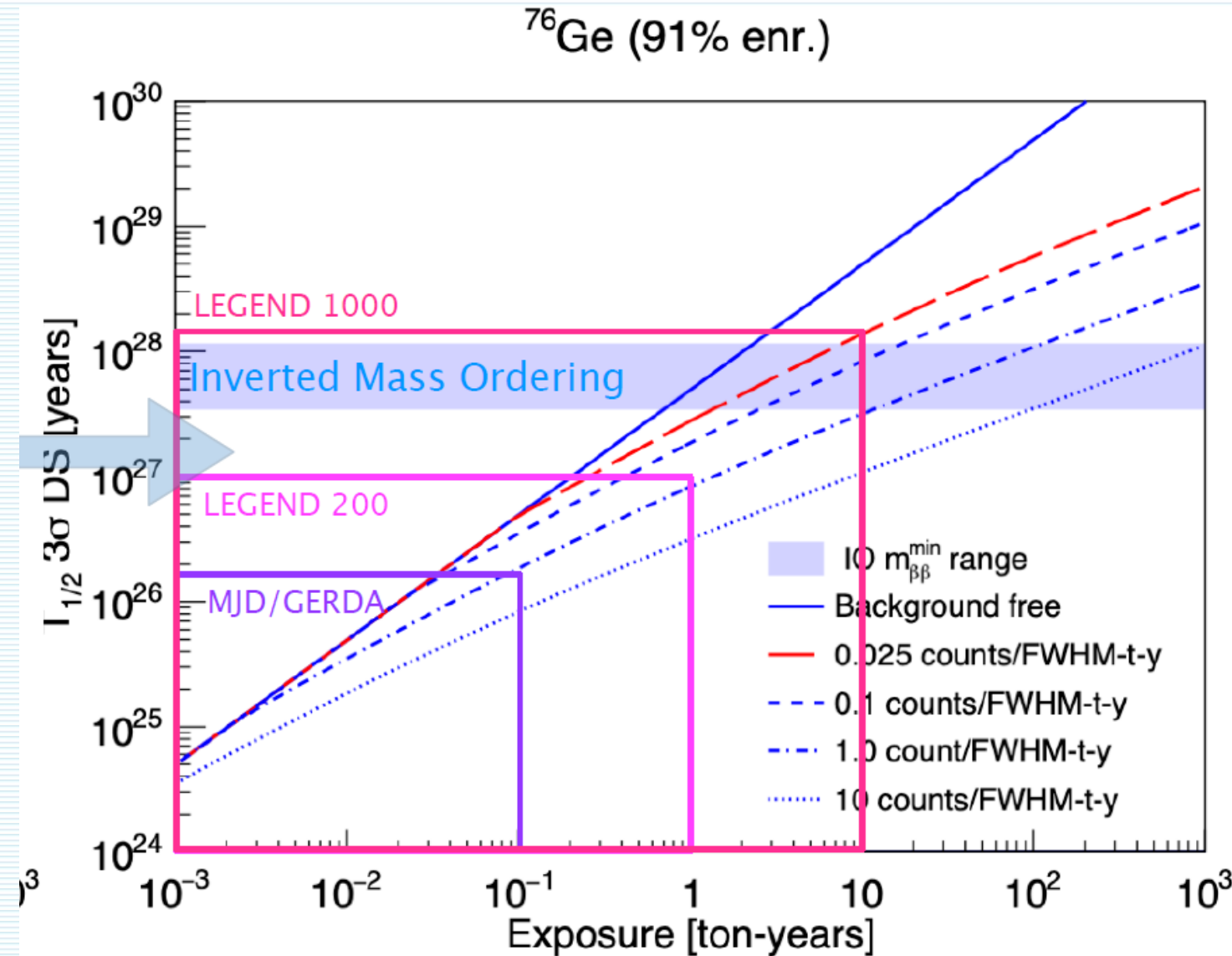
1 ton-class ^{76}Ge $0\nu\beta\beta$ experiment

LEGEND-200

- Builds on the past successes of the MAJORANA DEMONSTRATOR and GERDA
- Low-risk approach to meeting background and sensitivity goals
- LEGEND-200: start data taking in 2022

LEGEND-1000 is a next-generation Experiment aiming for unambiguous discovery of $0\nu\beta\beta$ with 10^{28} years of sensitivity targeting 10 years of exposure

- in Conceptual Design phase
- 20x reduction to LEGEND-200 background goal
- Next-generation R&D efforts including Germanium Machine learning in progress



Small scale demonstrators

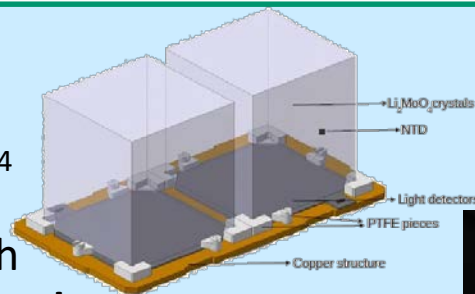
CUPID-Mo
CUPID-0
CUORE

→ CUPID → CUPID Reach / CUPID-1T

CUORE + CUPID-Mo

→ CUPID: 1 ton-class ^{100}Mo $0\nu\beta\beta$ experiment

- Single module: $\text{Li}_2^{100}\text{MoO}_4$
45×45×45 mm – ~280 g
- 57 towers of 14 floors with
2 crystals each - 1596 crystals
- ~240 kg of ^{100}Mo with
>95% enrichment
- ~ 1.6×10^{27} ^{100}Mo atoms
- Bolometric Ge light detectors
as in CUPID-Mo, CUPID-0



prototype tower



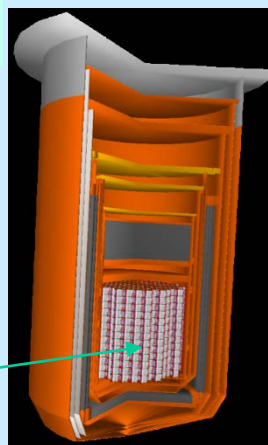
arXiv:1907.09376

J. Ouellet, TAUP 2021

CUPID is built on successful CUPID-Mo + CUORE

Li_2MoO_4 scintillating bolometer technology, with demonstration of energy resolution, crystal radiopurity and α rejection

Ton-scale bolometric experiment is possible
Electronics and data analysis tools
Reuse CUORE infrastructure



CUPID sensitivity

Data driven background model

- Information from CUPID-Mo, CUPID-0
- CUORE background model (same infrastructure!)

Projected background index: 1×10^{-4} c/(keV kg y)

Critical background component: random coincidence of $2\nu\beta\beta$ events (^{100}Mo fastest $2\nu\beta\beta$ emitter: $T_{1/2} = 7.1 \times 10^{18}$ y)

10 y discovery sensitivity

1.1×10^{27}

$m_{\beta\beta} < 12 - 20$ meV

Possible follow-up of CUPID

CUPID-reach - Same sensitive mass and cryostat as CUPID

Background improvement by factor 5

2.3×10^{27} y → $m_{\beta\beta} < 7.9 - 14$ meV

CUPID-1T - 1 ton isotope → new cryostat

Background improvement by factor 20

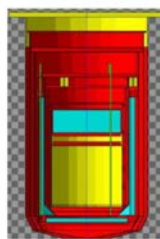
9.2×10^{27} y → $m_{\beta\beta} < 4.0 - 6.9$ meV

Surface events

Phased approach

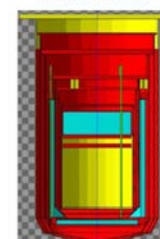
Technically ready

CUPID Baseline



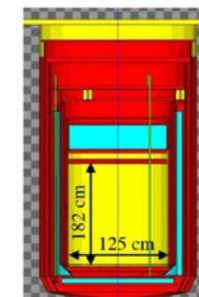
250 kg of ^{100}Mo

CUPID-reach



250 kg of ^{100}Mo

CUPID-1T



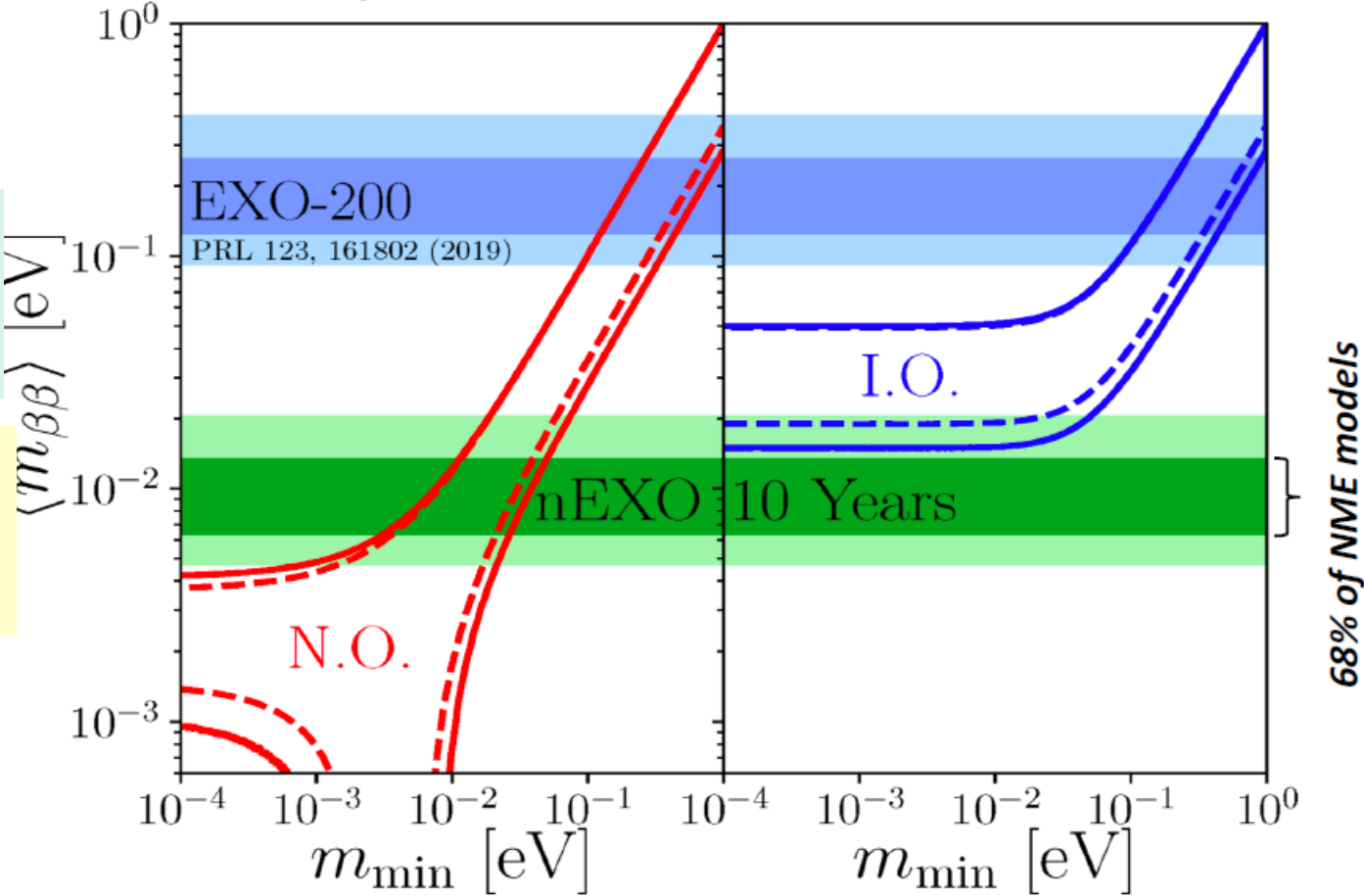
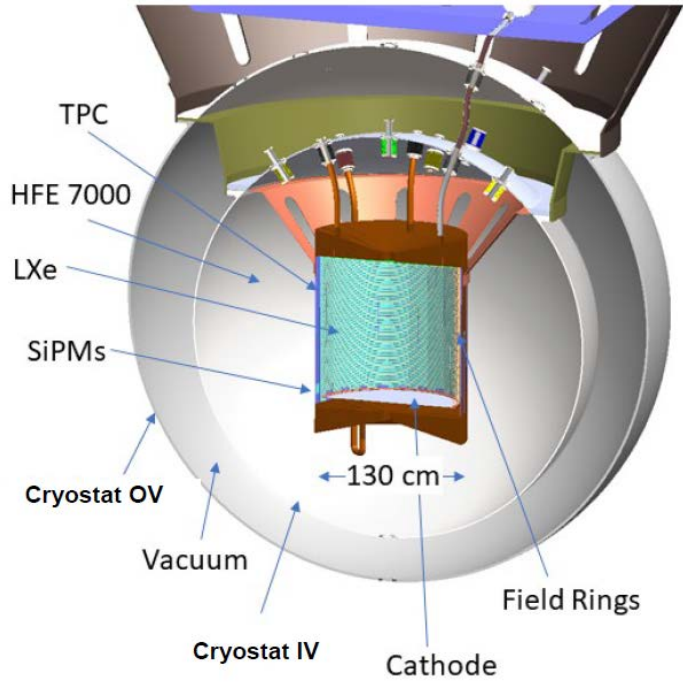
1000 kg of ^{100}Mo

nEXO

5 ton-class ^{136}Xe $0\nu\beta\beta$ experiment

EXO-200, 1st 100 kg-class $0\nu\beta\beta$ -experiment, excellent background-essential for nEXO design, Sensitivity increased linearly with exposure.

nEXO, discovery $0\nu\beta\beta$ experiment, reaches sensitivity of 10^{28} yr in 6.5 yr data taking, probes $m_{\beta\beta}$ down to 15 meV, scalable experiment.

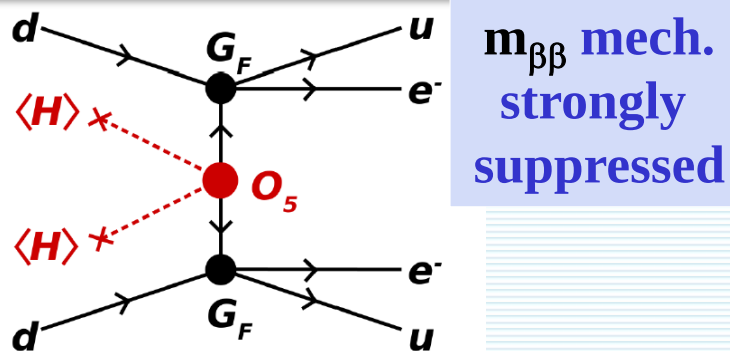
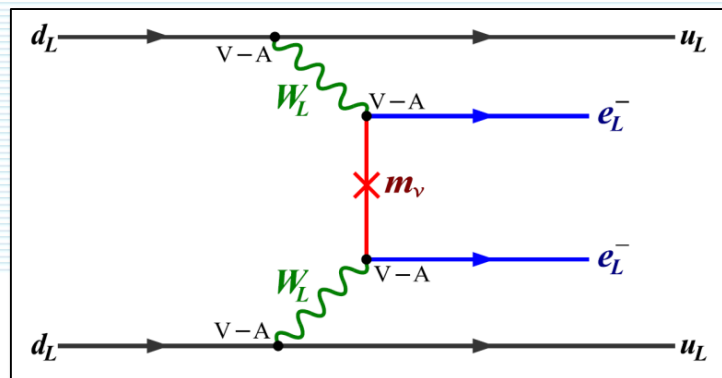


	isotope	$m_{\beta\beta}$ [meV] 90% excl. sensitivity	$m_{\beta\beta}$ [meV] 3 σ discovery potential
Legend	^{76}Se	8.2	11.1
CUPID	^{100}Mo	11.1	12.0
nEXO	^{136}Xe	12.9	15.0

$0\nu\beta\beta$ governed by exotic mechanisms

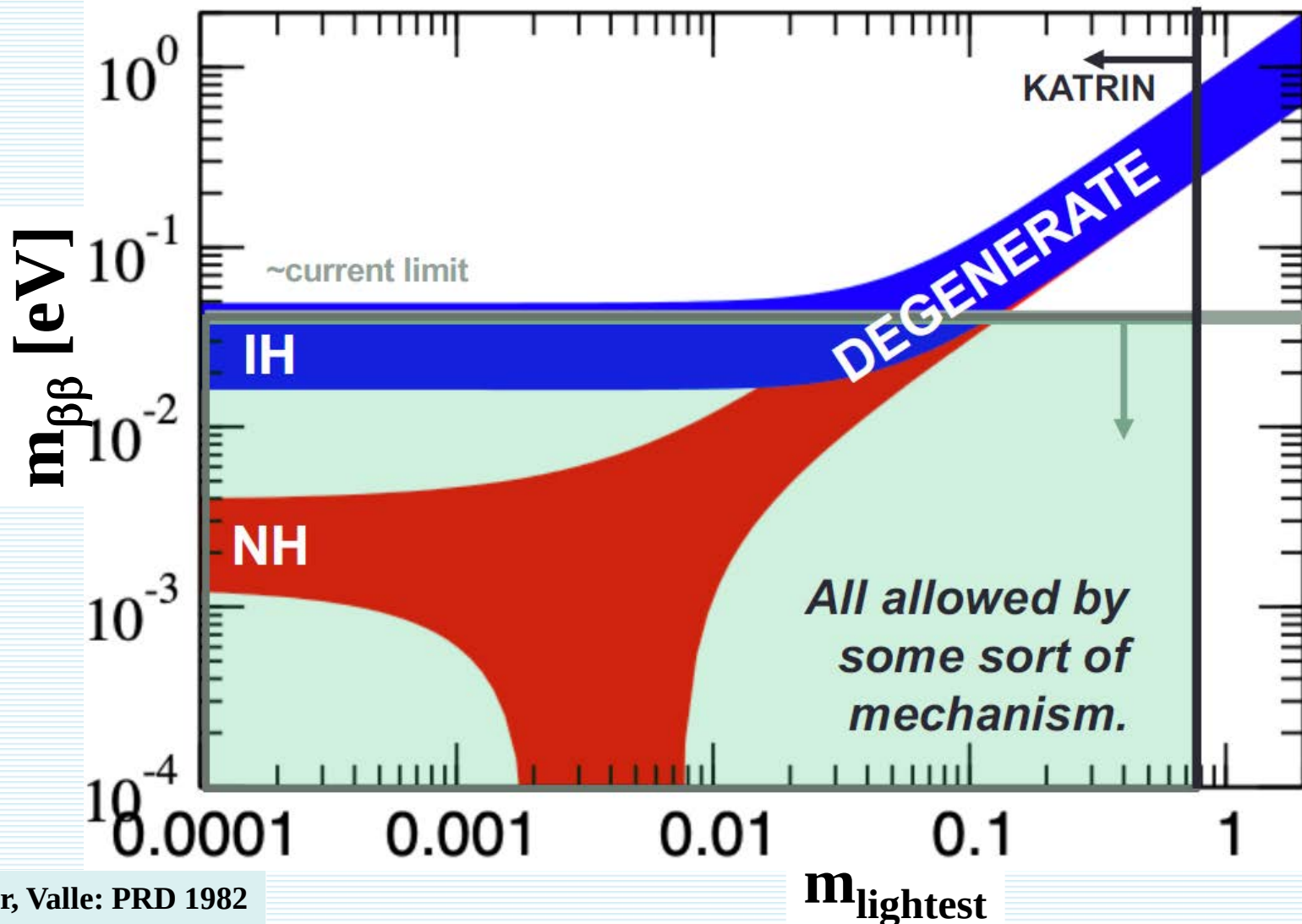
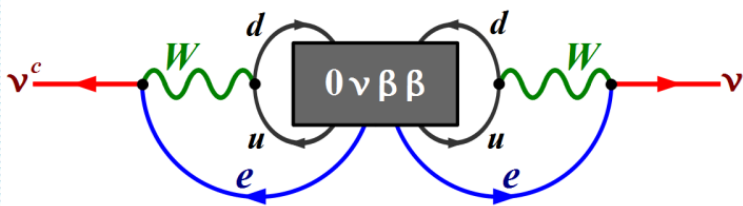
Light ν -mass mechanism can be strongly suppressed: $m_{\beta\beta} < 1$ meV

- It is not possible to discover $0\nu\beta\beta$ with 10-100 ton-class experiment
- It should be a **subject of theory** to justify it
- There might be a dominance of other $0\nu\beta\beta$ mechanisms



$m_{\beta\beta}$ mech. strongly suppressed

Any $0\nu\beta\beta$ mech. generates a small correction to ν -mass

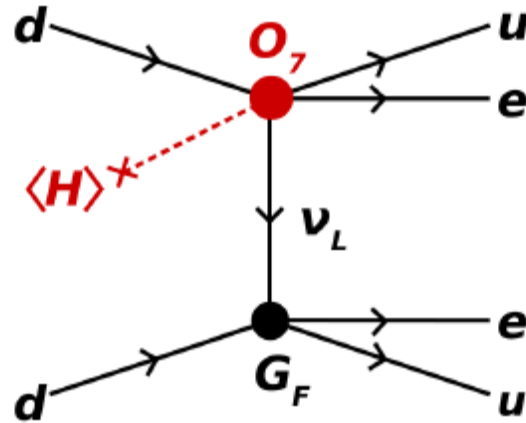


Beyond the SM physics

$$\mathcal{L} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} \sum_i c_i^{(5)} \mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2} \sum_i c_i^{(6)} \mathcal{O}_i^{(6)} + O\left(\frac{1}{\Lambda^3}\right)$$

Amplitude for
 $(A,Z) \rightarrow (A,Z+2) + 2e^-$
can be divided into:

long range: d=7



$$\mathcal{O}_2 \propto LLLe^c H$$

$$\mathcal{O}_3 \propto LLQd^c H$$

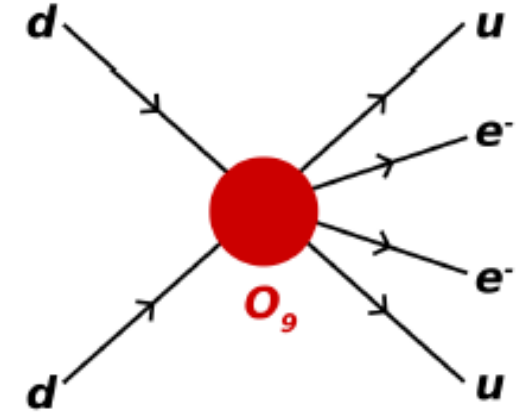
$$\mathcal{O}_4 \propto LL\bar{Q}\bar{u}^c H$$

$$\mathcal{O}_8 \propto L\bar{e}^c \bar{u}^c d^c H$$

Babu, Leung: 2001

de Gouvea, Jenkins: 2007

short range: d=9 (d=11)



$$\mathcal{O}_5 \propto LLQd^c H H H^\dagger$$

$$\mathcal{O}_6 \propto LL\bar{Q}\bar{u}^c H H^\dagger H$$

$$\mathcal{O}_7 \propto LQ\bar{e}^c \bar{Q} H H H^\dagger$$

$$\mathcal{O}_9 \propto LLLe^c Le^c$$

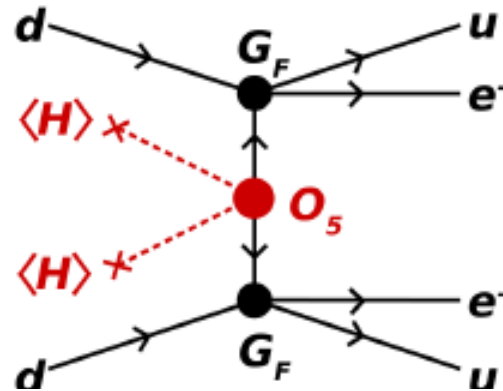
$$\mathcal{O}_{10} \propto LLLe^c Qd^c$$

$$\mathcal{O}_{11} \propto LLQd^c Qd^c$$

.....

Valle

mass mechanism: d=5



$$\mathcal{O}_W \propto \frac{c_{ij}}{\Lambda} (L_i H)(L_j H)$$

Weinberg, 1979

Quark Condensate Seesaw Mechanism for Neutrino Mass

PRD 103, 015007 (2021).

The SM gauge-invariant effective operators

$$\mathcal{O}_7^{u,d} = \frac{\tilde{g}_{\alpha\beta}^{u,d}}{\Lambda^3} \overline{L}_\alpha^C L_\beta H \left\{ (\overline{Q} u_R), (\overline{d}_R Q) \right\}$$

After the **EWSB** and **ChSB** one arrives at the Majorana mass matrix of active neutrinos

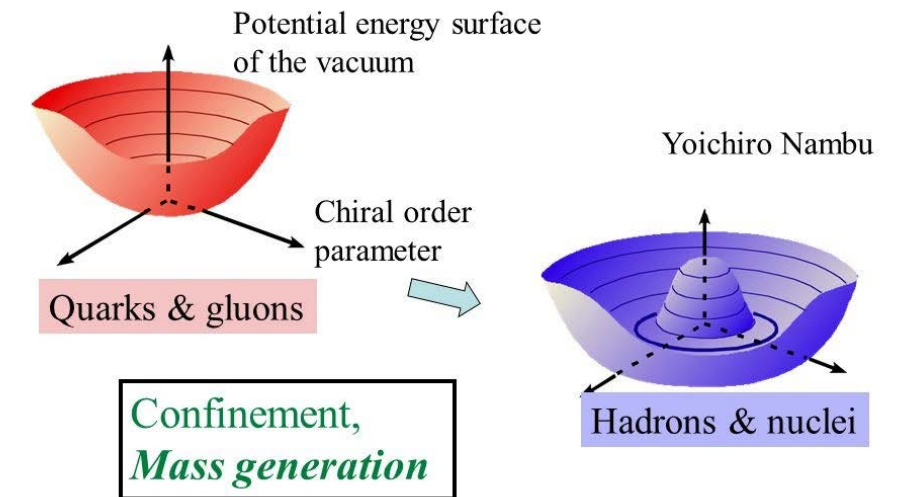
$$\begin{aligned} m_{\alpha\beta}^\nu &= g_{\alpha\beta} v \frac{\langle \overline{q} q \rangle}{\Lambda^3} \\ &= g_{\alpha\beta} v \left(\frac{\omega}{\Lambda} \right)^3 \end{aligned}$$

$$g_{\alpha\beta} = g_{\alpha\beta}^u + g_{\alpha\beta}^d, \quad v/\sqrt{2} = \langle H^0 \rangle$$

$$\omega = -\langle \overline{q} q \rangle^{1/3}, \quad \langle \overline{q} q \rangle^{1/3} \approx -283 \text{ MeV}_{\text{vic}}$$

This operator contributes to the **Majorana-neutrino mass matrix** due to chiral symmetry breaking via the **light-quark condensate**.

Spontaneous breaking of *chiral* (χ) *symmetry*



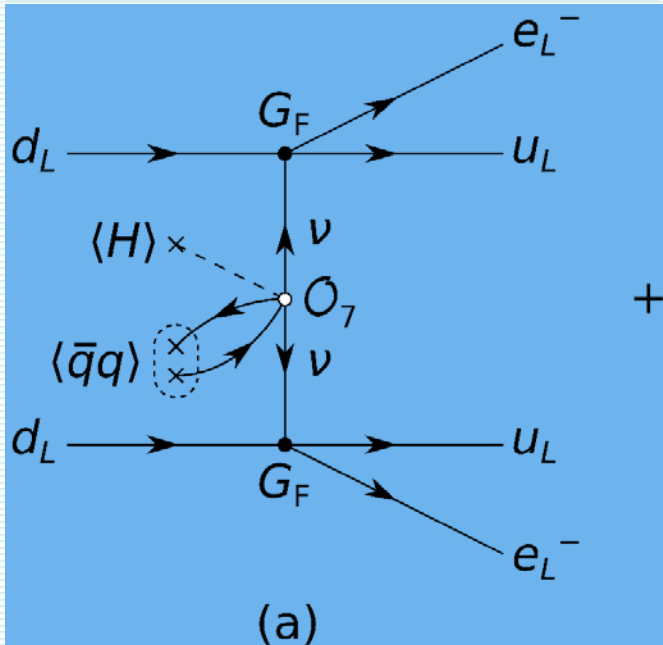
$\Lambda \sim$ **a few TeV**
we get the neutrino mass
in the **sub-eV ballpark**

The genuine QCSS scenario (predicts NH and ν -mass spectrum)

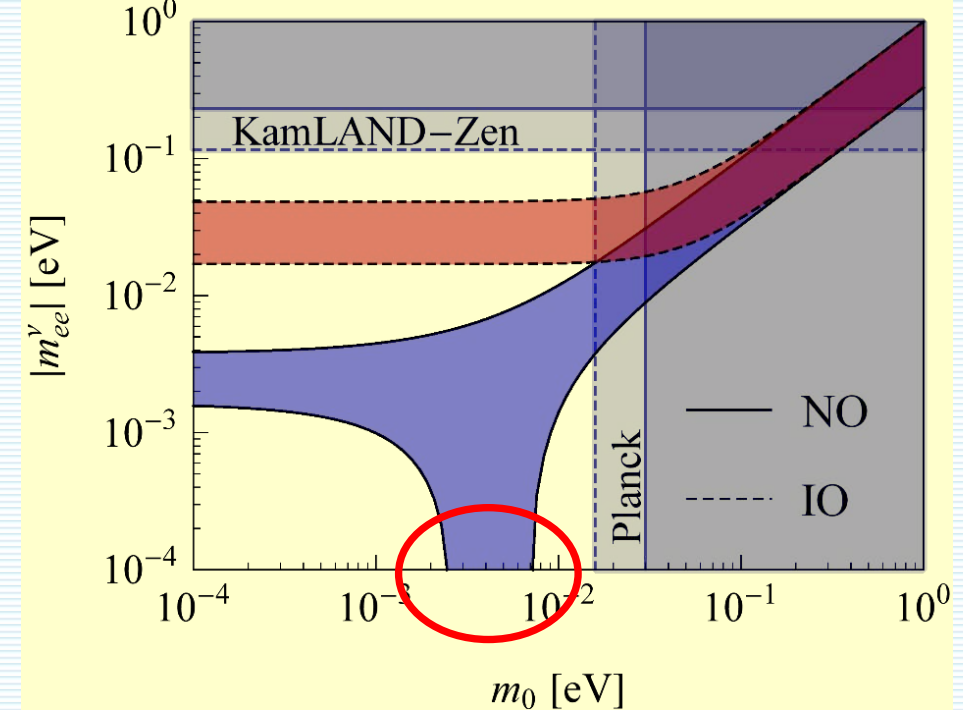
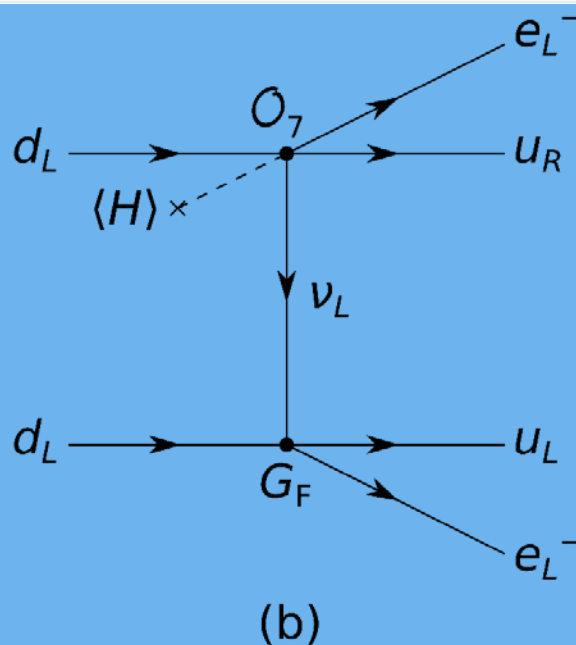
$$\mathcal{L}_7 = \frac{1}{\sqrt{2}} \sum_{\alpha\beta} \frac{v}{\Lambda^3} \bar{\nu}_{\alpha L}^C \nu_{\beta L} (g_{\alpha\beta}^u \bar{u}_L u_R + g_{\alpha\beta}^d \bar{d}_R d_L) + \text{H.c.}$$

$$m_{\alpha\beta}^\nu = -\frac{g_{\alpha\beta}}{\sqrt{2}} v \frac{\langle \bar{q}q \rangle}{\Lambda^3} = \frac{g_{\alpha\beta}}{\sqrt{2}} v \left(\frac{\omega}{\Lambda} \right)^3$$

(a) PRL 112, 142503 (2014).



(b) PLB 453, 194 (1999).



Neutrino spectrum (NH) !!!

$$2 \text{ meV} < m_1 < 7 \text{ meV}$$

$$9 \text{ meV} < m_2 < 11 \text{ meV}$$

$$50 \text{ meV} < m_2 < 51 \text{ meV}$$

Prediction for m_β

$$9 \text{ meV} < m_\beta < 12 \text{ meV}$$

Prediction for cosmology (Σ)

$$62 \text{ meV} < m_1 + m_2 + m_3 < 69 \text{ meV}$$

Majorana neutrino mass eigenstate N with arbitrary mass m_N mixed with 3 active neutrinos (U_{eN})

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} g_A^4 \left| \sum_N (U_{eN}^2 m_N) m_p M'^{0\nu}(m_N, g_A^{\text{eff}}) \right|^2$$

General case

light ν exchange

$$M'^{0\nu}(m_N, g_A^{\text{eff}}) = \frac{1}{m_p m_e} \frac{R}{2\pi^2 g_A^2} \sum_n \int d^3x d^3y d^3p \quad M'^{0\nu}(m_N \rightarrow 0, g_A^{\text{eff}}) = \frac{1}{m_p m_e} M_\nu'^{0\nu}(g_A^{\text{eff}})$$

$$\times e^{ip \cdot (x-y)} \frac{\langle 0_F^+ | J^{\mu\dagger}(\mathbf{x}) | n \rangle \langle n | J_\mu^\dagger(\mathbf{y}) | 0_I^+ \rangle}{\sqrt{p^2 + m_N^2} (\sqrt{p^2 + m_N^2} + E_n - \frac{E_I - E_F}{2})} M'^{0\nu}(m_N \rightarrow \infty, g_A^{\text{eff}}) = \frac{1}{m_N^2} M_N'^{0\nu}(g_A^{\text{eff}})$$

Particular cases

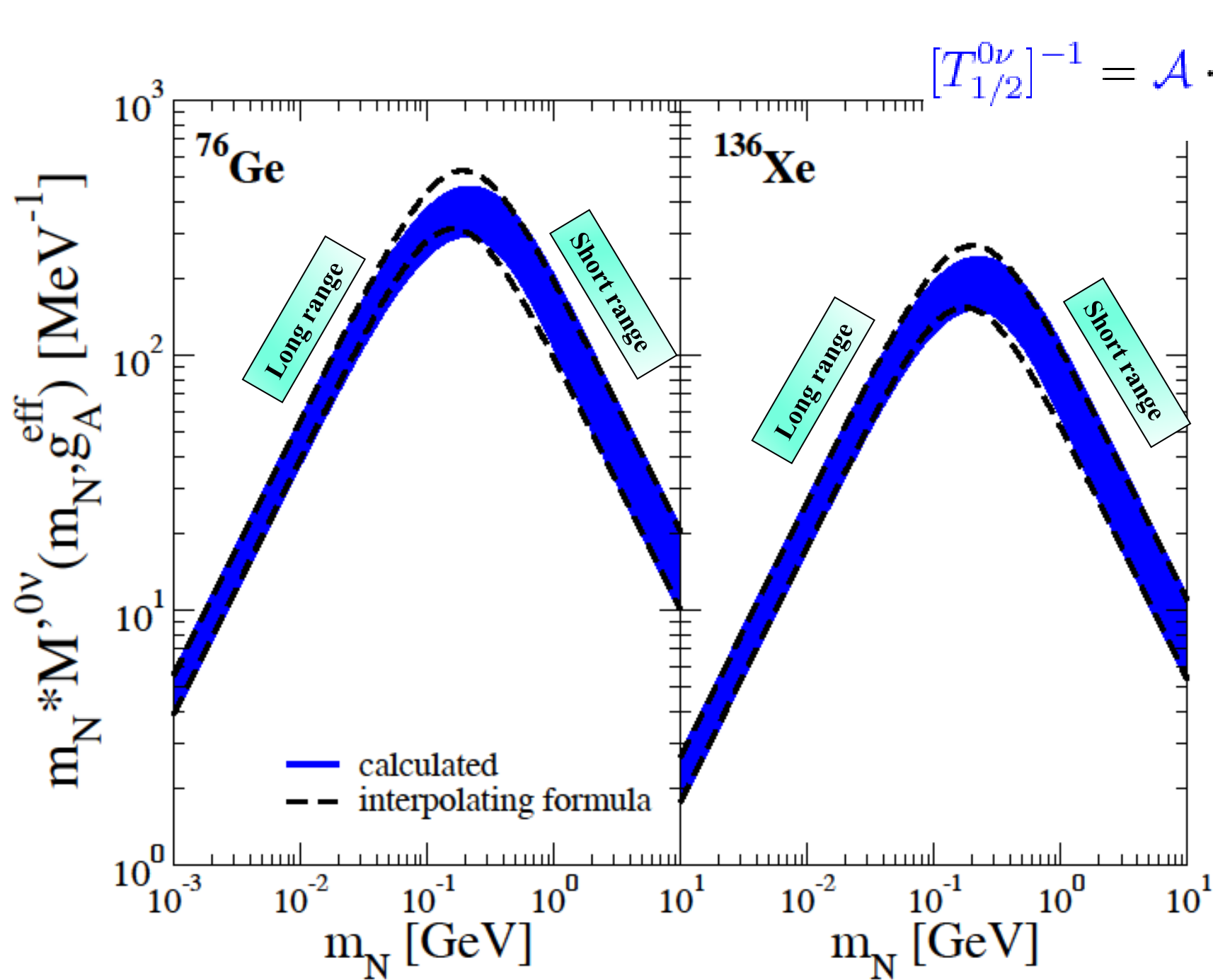
heavy ν exchange

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} g_A^4 \times$$

$$\times \begin{cases} \left| \frac{\langle m_\nu \rangle}{m_e} \right|^2 \left| M_\nu'^{0\nu}(g_A^{\text{eff}}) \right|^2 & \text{for } m_N \ll p_F \\ \left| \langle \frac{1}{m_N} \rangle m_p \right|^2 \left| M_N'^{0\nu}(g_A^{\text{eff}}) \right|^2 & \text{for } m_N \gg p_F \end{cases} \text{ or Simkovic}$$

$$\langle m_\nu \rangle = \sum_N U_{eN}^2 m_N$$

$$\left\langle \frac{1}{m_N} \right\rangle = \sum_N \frac{U_{eN}^2}{m_N}$$



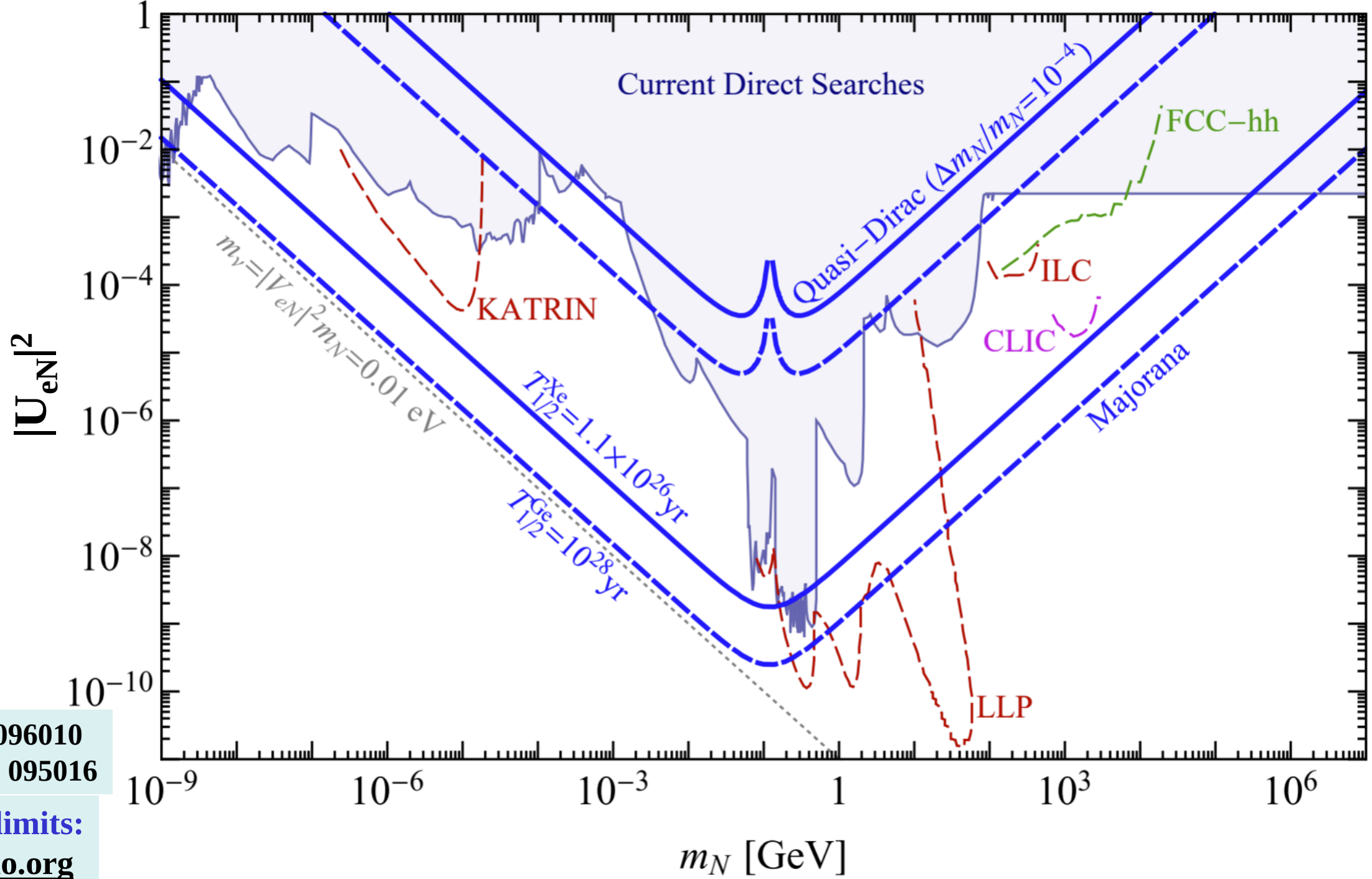
$$[T_{1/2}^{0\nu}]^{-1} = \mathcal{A} \cdot \left| m_p \sum_N U_{eN}^2 \frac{m_N}{\langle p^2 \rangle + m_N^2} \right|^2,$$

$$\langle p^2 \rangle = m_p m_e \frac{M_N'^{0\nu}}{M_\nu'^{0\nu}}$$

**Interpolating
formula**

Sterile ν (U_{eN} , m_N)

Constraints from Direct Searches are less Stringent as those from $0\nu\beta\beta$



PRD 90 (2014) 096010
PRD 102 (2020) 095016

Direct search limits:
sterile-neutrino.org

The $0\nu\beta\beta$ -decay within L-R symmetric theories (interpolating formula)

(D-M mass term, see-saw, V-A and V+A int., exchange of heavy neutrinos)

$$[T_{1/2}^{0\nu}]^{-1} = \eta_{\nu N}^2 C_{\nu N}$$

$$C_{\nu N} = g_A^4 |M_{\nu}^{'0\nu}|^2 G^{0\nu}$$

$$\nu_{eL} = \sum_{j=1}^3 \left(U_{ej} \nu_{jL} + S_{ej} (N_{jR})^C \right),$$

$$\nu_{eR} = \sum_{j=1}^3 \left(T_{ej}^* (\nu_{jL})^C + V_{ej}^* N_{jR} \right)$$

Mixing of 3 light
and 3 heavy neutrinos
generally parametrized
with 15 angles + 15 phases

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

Effective LNV parameter within LRS model
(due interpolating formula)

$$\langle p^2 \rangle = m_p m_e \frac{M_N^{'0\nu}}{M_{\nu}^{'0\nu}}$$

$$\eta_{\nu N}^2 = \left| \sum_{j=1}^3 \left(U_{ej}^2 \frac{m_j}{m_e} + S_{ej}^2 \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M_j^2} \frac{M_j}{m_e} \right) \right|^2 + \lambda^2 \left| \sum_{j=1}^3 \left(T_{ej}^2 \frac{m_j}{m_e} + V_{ej}^2 \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M_j^2} \frac{M_j}{m_e} \right) \right|^2$$

The dominance of
light and heavy
v-mass contributions to
 $0\nu\beta\beta$ -decay rate can not be
established by observing this
process at different nuclei.

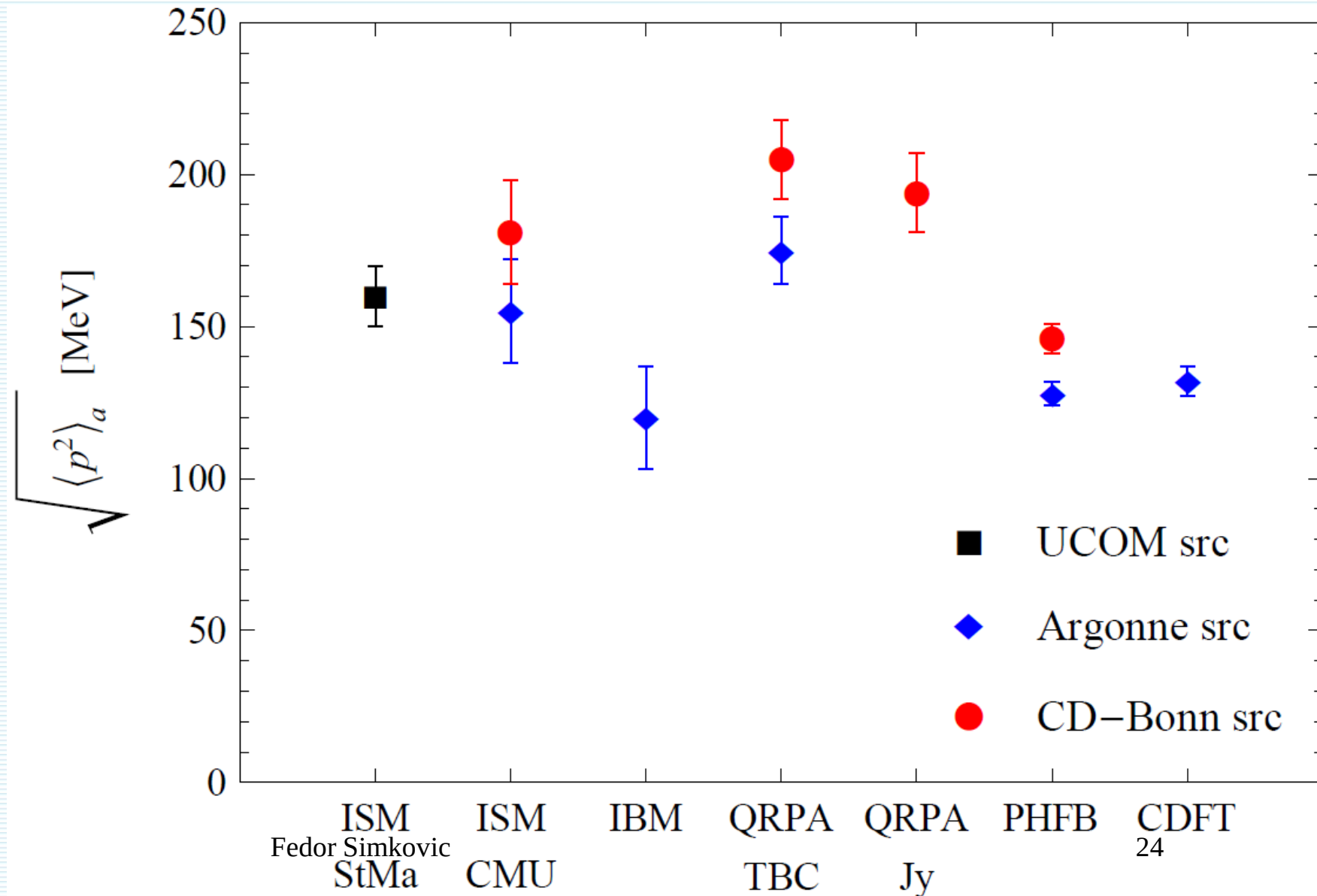
$\langle p^2 \rangle$ only weakly depends on A

$$\langle p^2 \rangle = m_p m_e \frac{M_N'^{0\nu}}{M_\nu'^{0\nu}}$$

Supported by detailed analysis:
**Degeneracies of particle
 and nuclear physics
 uncertainties in $0\nu\beta\beta$ -decay**
 PRD 92, 093004 (2018)

As a consequence
 contributions to
 $0\nu\beta\beta$ decay rate
 from **light and heavy
 neutrino mass
 mechanisms**
**can not be
 distinguished.**

PRD 98, 015003 (2018)



6x6 PMNS see-saw ν -mixing matrix (the most economical one)

$$\mathcal{U} = \begin{pmatrix} U_0 & \zeta \mathbf{1} \\ -\zeta \mathbf{1} & V_0 \end{pmatrix} \quad \zeta \text{ - see-saw parameter}$$

$$U_0 = U_{\text{PMNS}}$$

PRD 98, 015003 (2018)

The mixing for heavy ν follows from the unitarity of U

$$\eta_{\nu N}^2 = \frac{1}{m_e^2} \left(m_{\beta\beta}^2 + (M_{\beta\beta}^R)^2 \right)$$

$$V_0 = U_{\text{PMNS}}^\dagger =$$

$$\begin{pmatrix} c_{12} c_{13} e^{-i\alpha_1} & \left(-s_{12} c_{23} - c_{12} s_{13} s_{23} e^{-i\delta} \right) e^{-i\alpha_1} & \left(s_{12} s_{23} - c_{12} s_{13} c_{23} e^{-i\delta} \right) e^{-i\alpha_1} \\ s_{12} c_{13} e^{-i\alpha_2} & \left(c_{12} c_{23} - s_{12} s_{13} s_{23} e^{-i\delta} \right) e^{-i\alpha_2} & \left(-c_{12} s_{23} - s_{12} s_{13} c_{23} e^{-i\delta} \right) e^{-i\alpha_2} \\ s_{13} e^{i\delta} & c_{13} s_{23} & c_{13} c_{23} \end{pmatrix}$$

Assumption about heavy neutrino masses M_i (by assuming see-saw)

Inverse
proportional

$$m_i M_i \simeq m_D^2$$

$$M_{\beta\beta}^R = \lambda \frac{\langle p^2 \rangle_a}{m_D^2} \left| \sum_{j=1}^3 (U_0^\dagger)_{ej}^2 m_j \right|$$

$M_{\beta\beta}^R$ depends on
“Dirac” CP phase δ
unlike “Majorana”
CP phases α_1 and α_2

Proportional

$$m_i \simeq \zeta^2 M_i$$

$$M_{\beta\beta}^R = \lambda \zeta^2 \left| \sum_{j=1}^3 (U_0^\dagger)_{ej}^2 \frac{\langle p^2 \rangle_a}{m_j} \right|$$

Contribution from exchange of heavy neutrino to $0\nu\beta\beta$ -decay rate might be large

Inverse proportional

$$V_0 = U_{PMNS}^\dagger$$

$$M_i = m_D^2 / m_i \quad m_D \simeq 5 \text{ MeV}$$

$$\lambda = 7.7 \times 10^{-4}$$

$$m_i M_i \simeq m_D^2$$

$$M_{\beta\beta}^R = \lambda \frac{\langle p^2 \rangle_a}{m_D^2} \left| \sum_{j=1}^3 (U_0^\dagger)_{ej}^2 m_j \right|$$

Proportional

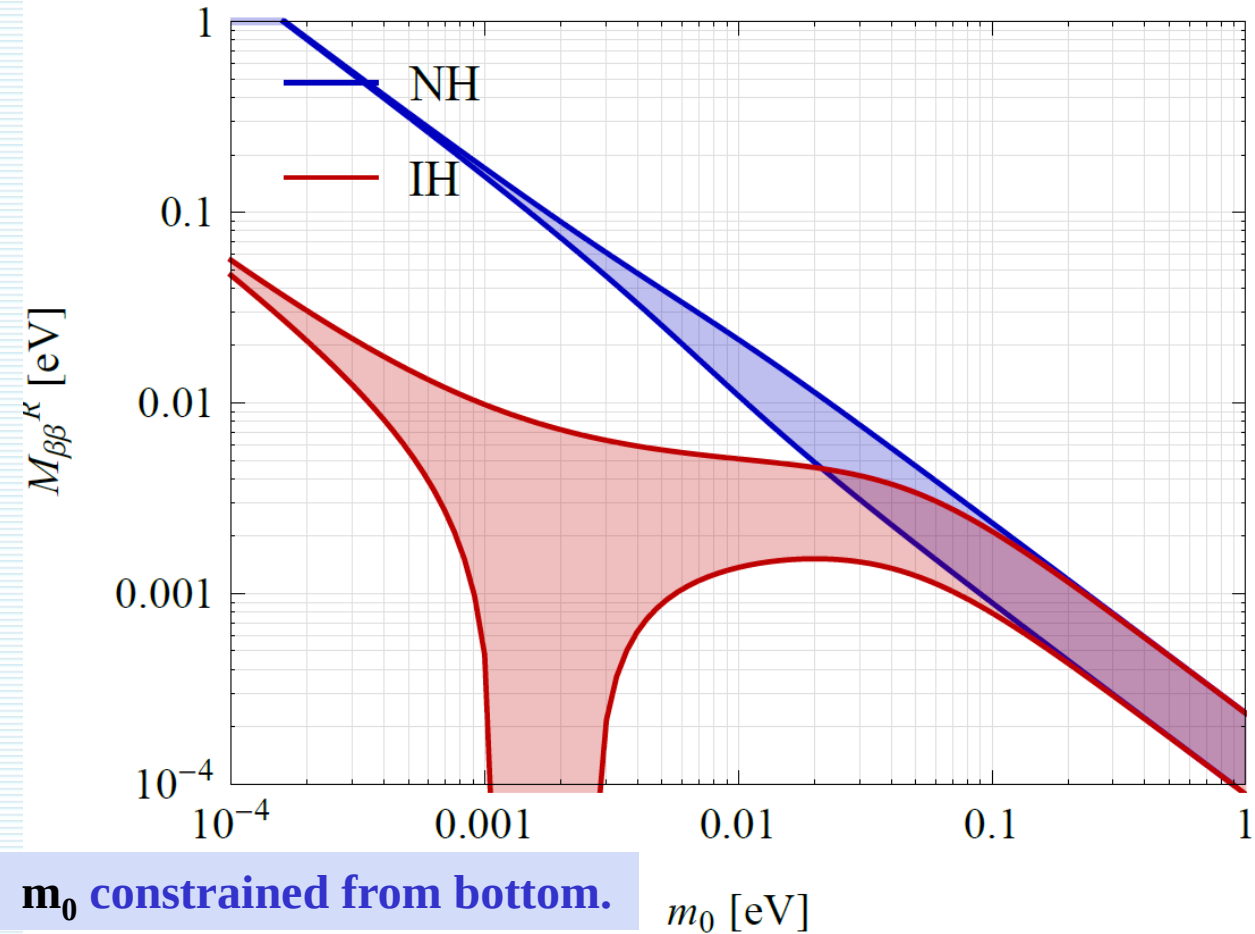
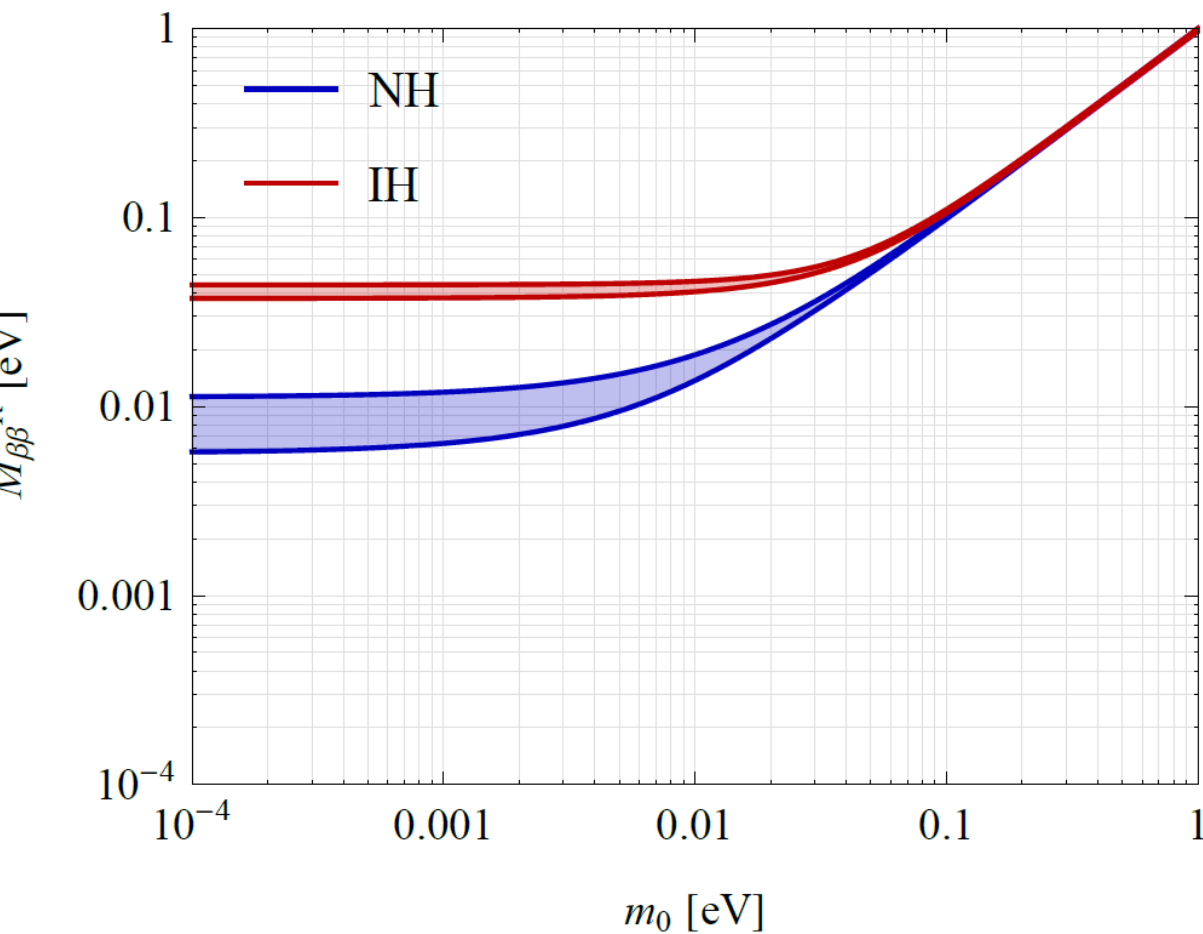
$$m_i \simeq \zeta^2 M_i$$

$$M_{\beta\beta}^R = \lambda \zeta^2 \left| \sum_{j=1}^3 (U_0^\dagger)_{ej}^2 \frac{\langle p^2 \rangle_a}{m_j} \right|$$

$$V_0 = U_{PMNS}^\dagger$$

$$\zeta = m_i / M_i \quad \zeta^2 \simeq 5 \times 10^{-17}$$

$$\lambda = 7.7 \times 10^{-4}$$



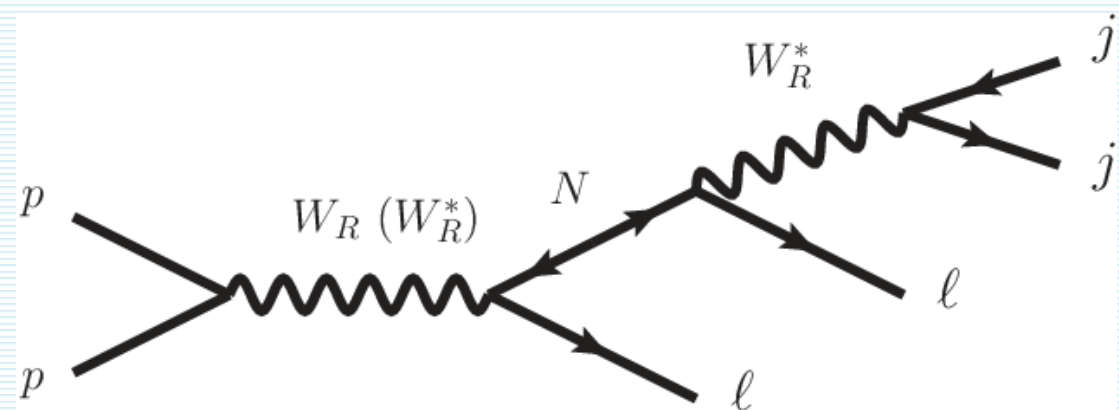
Direct probe of Majorana nature of heavy neutrino (or HNL) at LHC

Keung-Senjanovic processes:

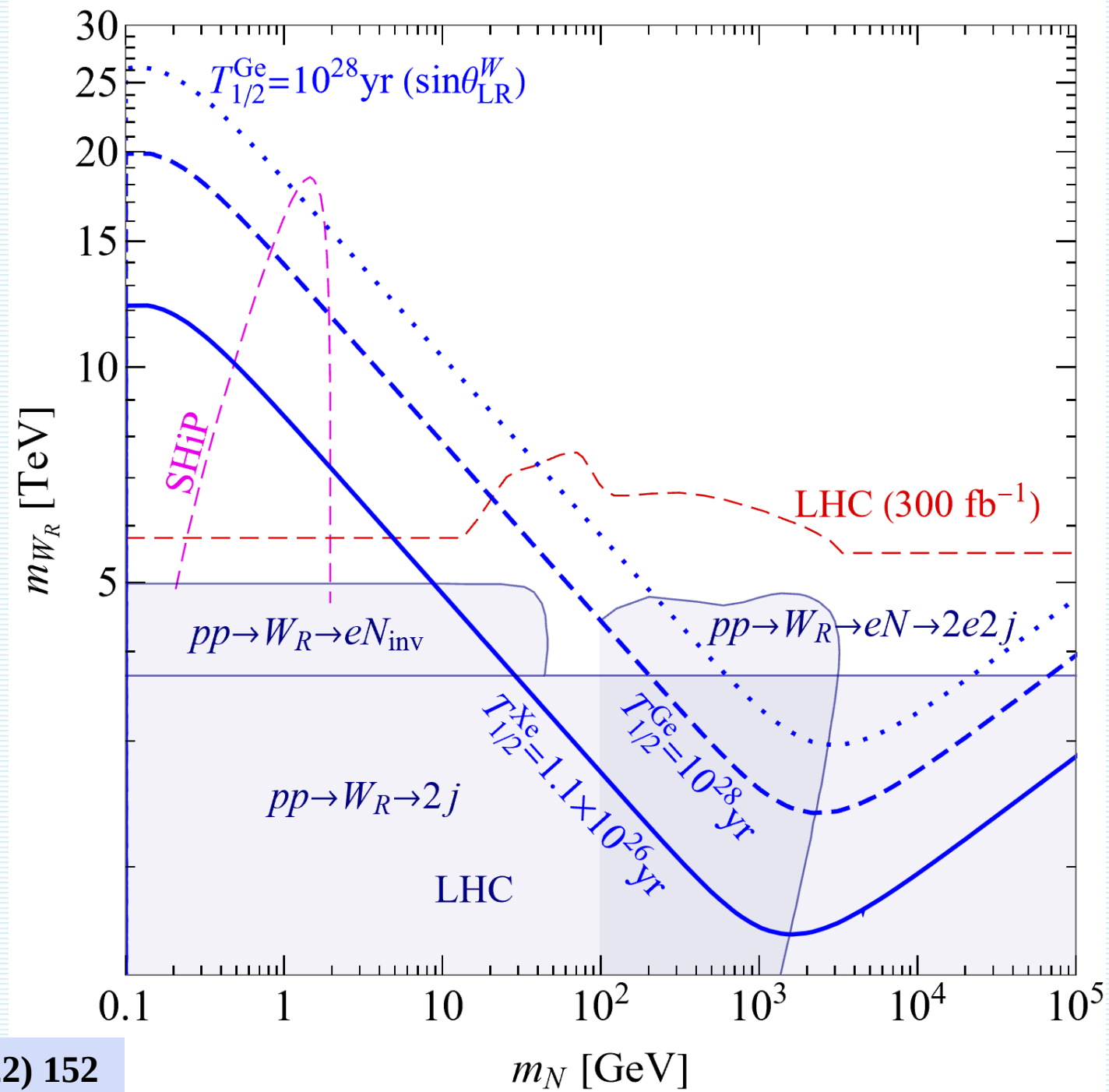
$$pp \rightarrow W_R \rightarrow eN \rightarrow 2e2j$$

or

$$pp \rightarrow W_R \rightarrow \mu N \rightarrow 2\mu 2j$$



JHEP 03 (2022) 152



**$2\nu\beta\beta$ is sensitive
to New Physics as well**

Common subjects:

Majoron(s) emission
(partly)bosonic neutrinos,
Lorentz invariance
violation

Recent subjects:

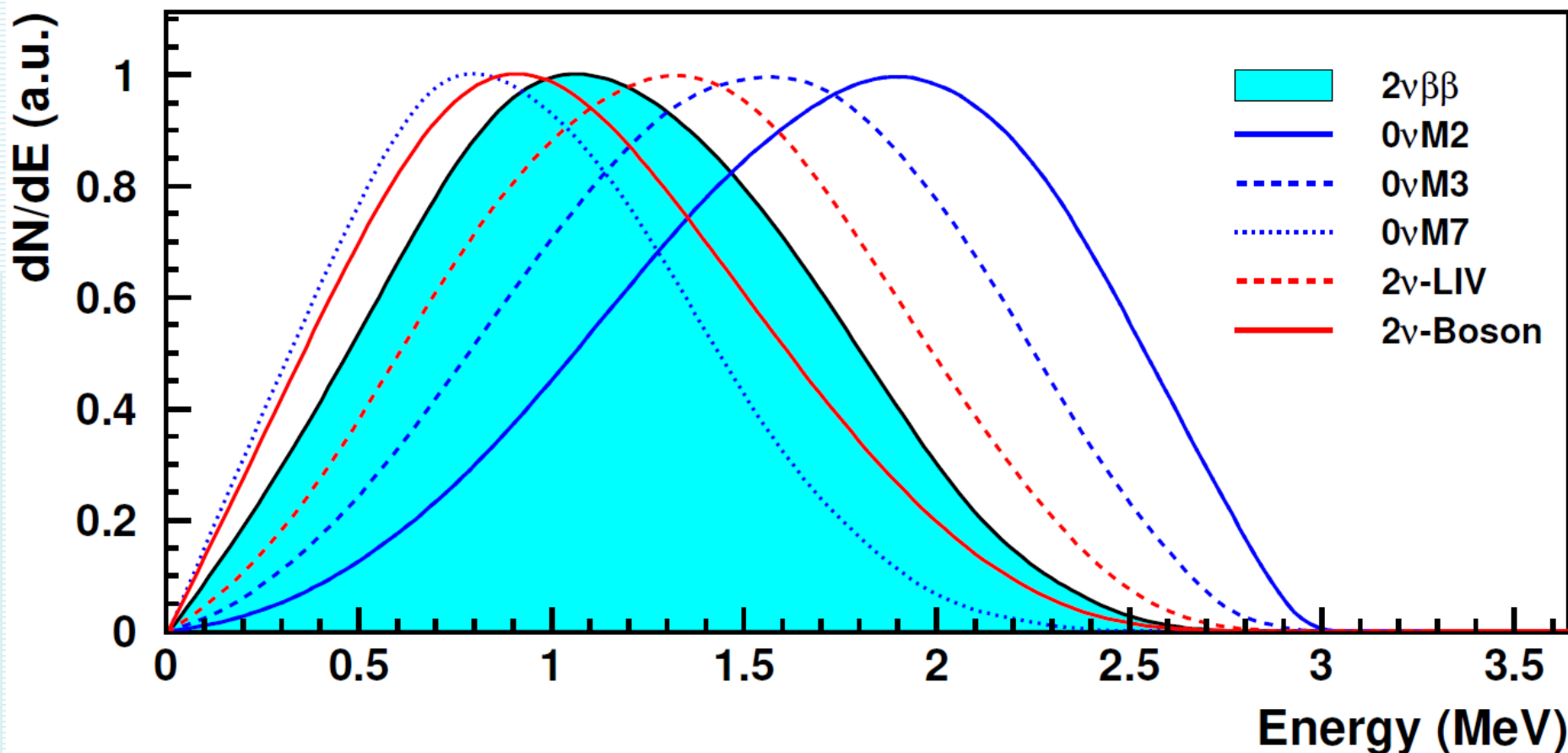
Lepton-number conserving
right-handed currents
(PRL 125 (2020) 17, 171801)

Neutrino self-interactions
(PRD 102 (2020) 5, 051701)

Sterile neutrino and light
fermion searches through
energy end point

(PRD 103 (2021) 5, 055019;
PLB 815 (2021) 136127)

All 100 kg- and ton-class $0\nu\beta\beta$ experiments can also study a diverse
range of **exotic phenomena**, e.g. through **spectral distortion** in $2\nu\beta\beta$.
Future searches will probe the $2\nu\beta\beta$ with **high statistics** about 10^5 - 10^6 events.



$$\frac{d\Gamma}{d\varepsilon_1 d\varepsilon_2} = C (Q - \varepsilon_1 - \varepsilon_2)^n [p_1 \varepsilon_1 F(\varepsilon_1)] [p_2 \varepsilon_2 F(\varepsilon_2)]$$

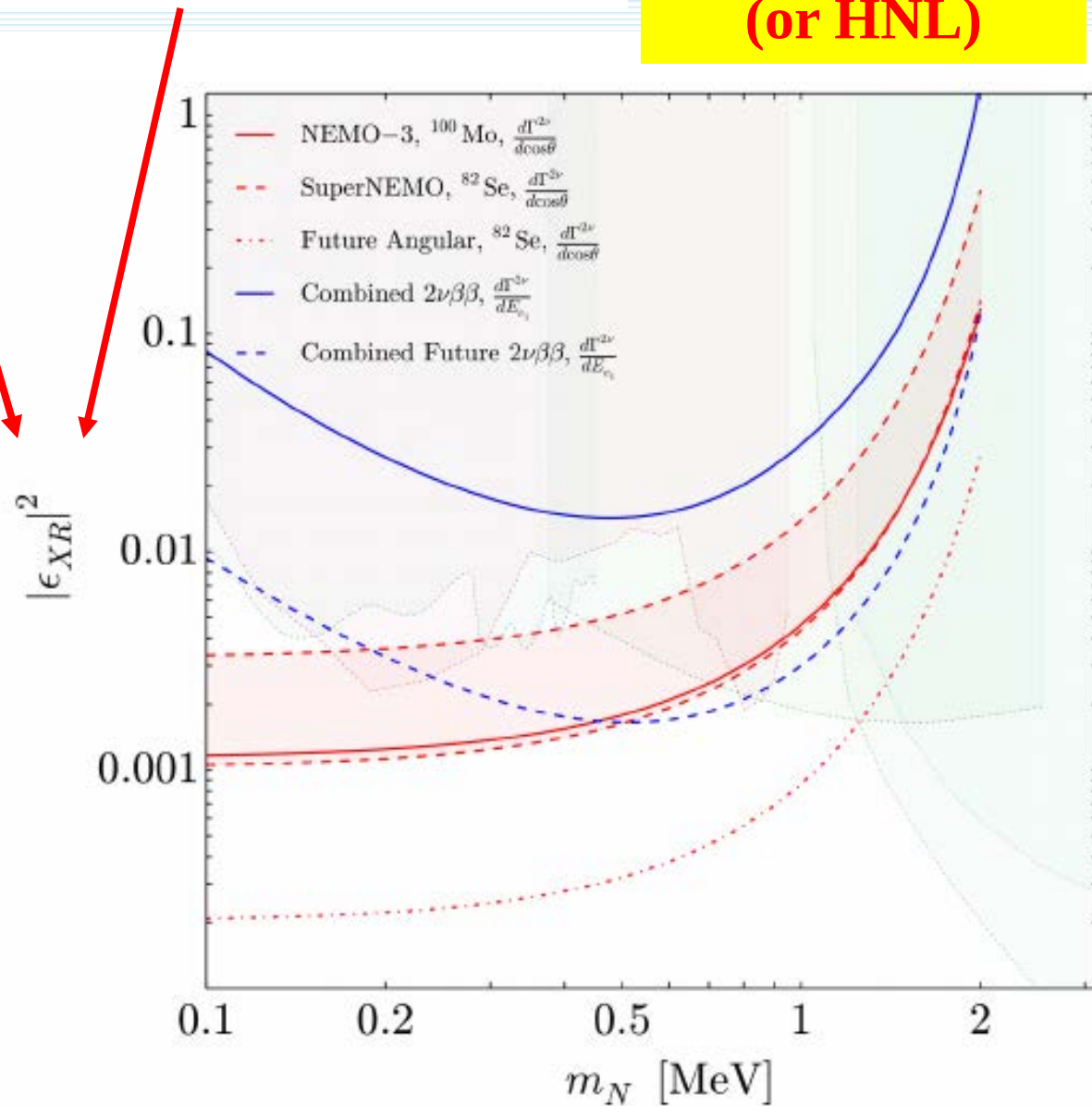
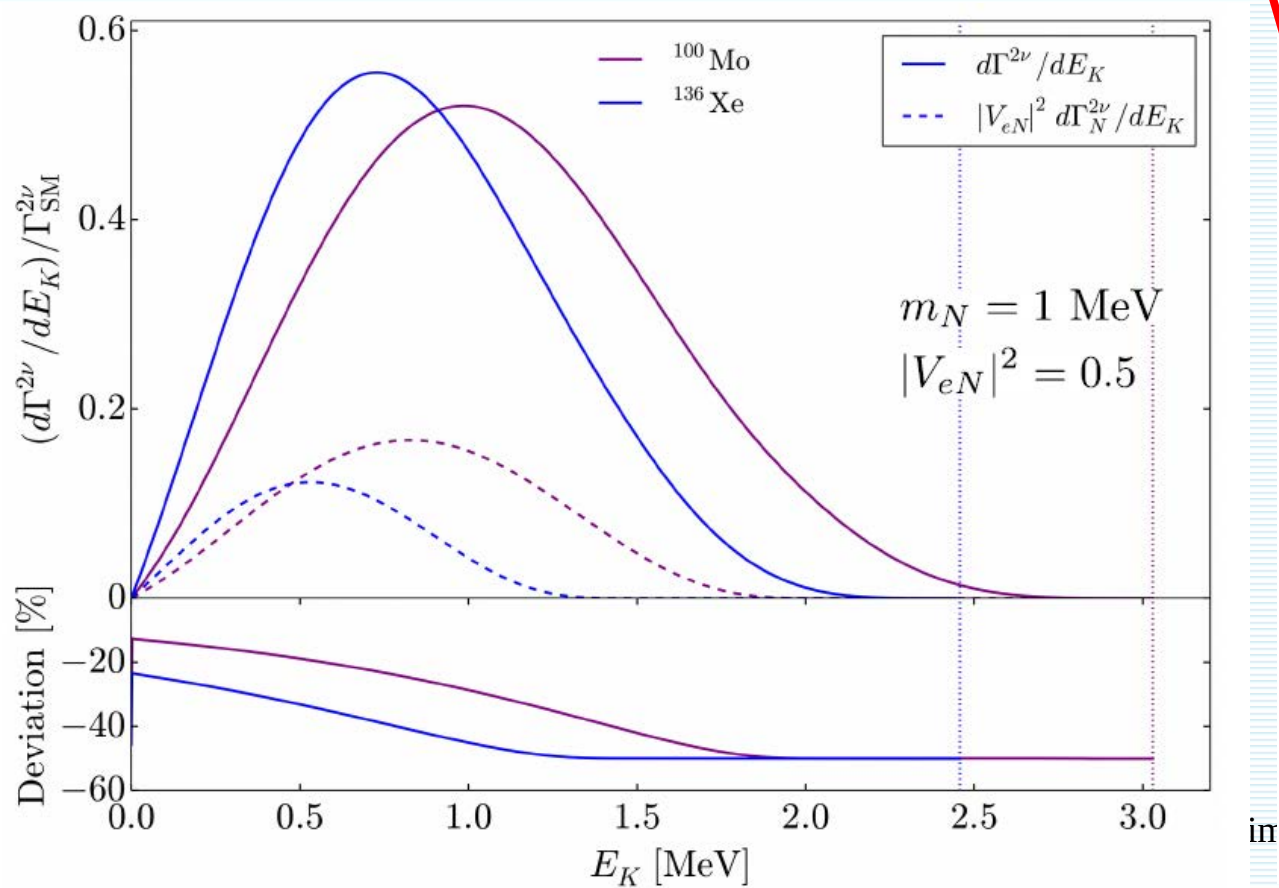
Spectral index n

$$\mathcal{L} = \frac{G_F \cos \theta_C}{\sqrt{2}} \left[(1 + \delta_{\text{SM}}) j_L^\mu J_{L\mu} + V_{eN} j_L^{N\mu} J_{L\mu} + \epsilon_{LR} j_R^{N\mu} J_{L\mu} + \epsilon_{RR} j_R^{N\mu} J_{R\mu} \right] + \text{h.c.}$$

**Exotic $2\nu\beta\beta$
Sterile Neutrino
(or HNL)**

Consider either mixing of **sterile ν** with **active ν** ,
or right-handed currents

Phys.Rev.D 103 (2021) 055019

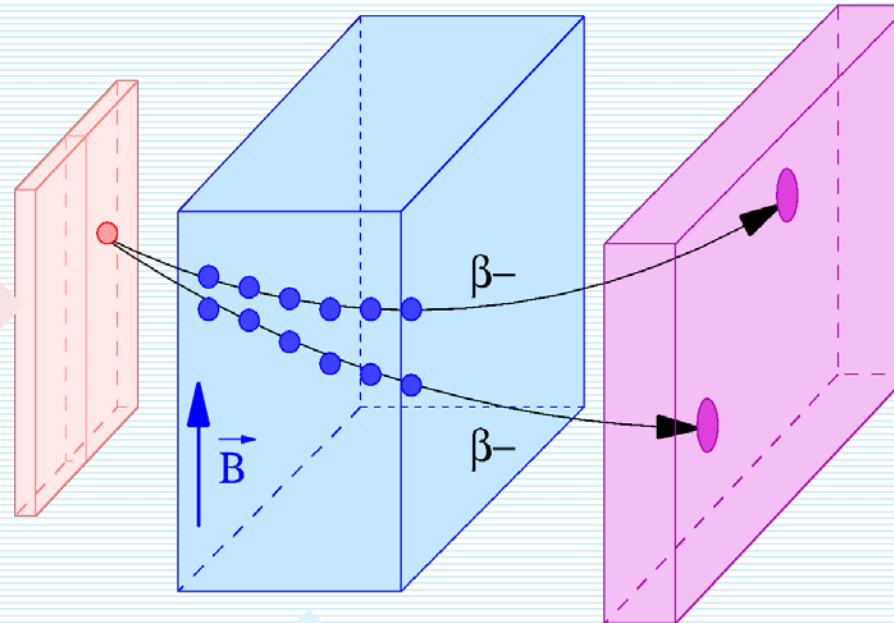
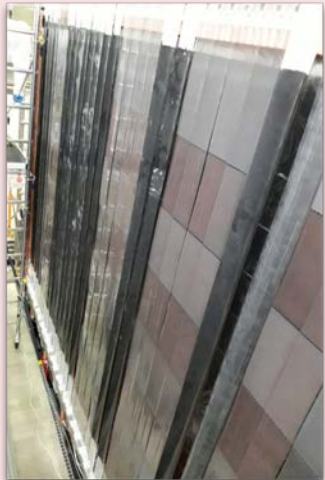


SuperNEMO Double-Beta Decay Experiment

Unique tracker-calorimeter technology

Source foil

^{82}Se or any $\beta\beta$ isotope



Segmented calorimeter

Individual e^- & γ energies



Tracker

Identification of e^- , e^+ , γ and α
Full $\beta\beta$ kinematics & topology

2034 Geiger cells



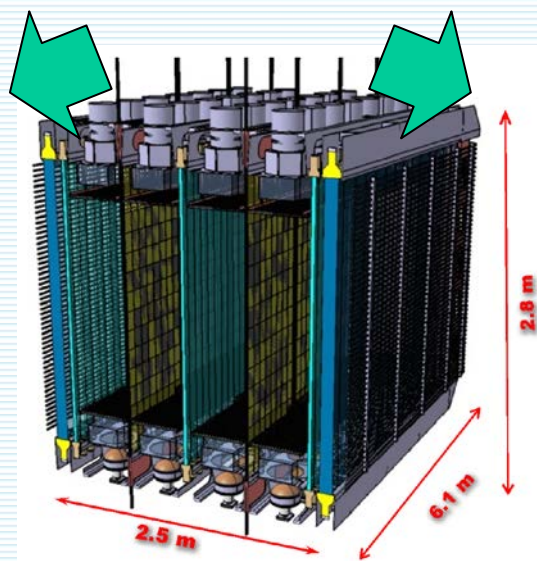
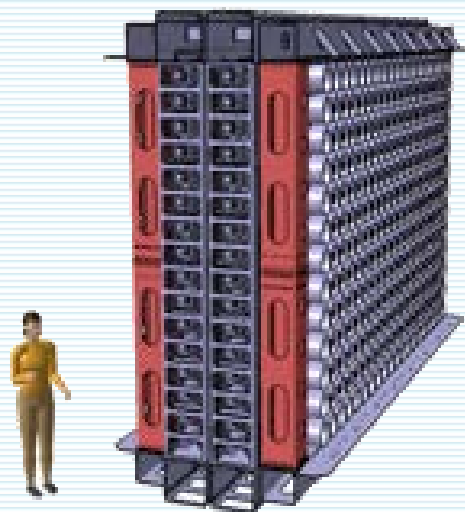
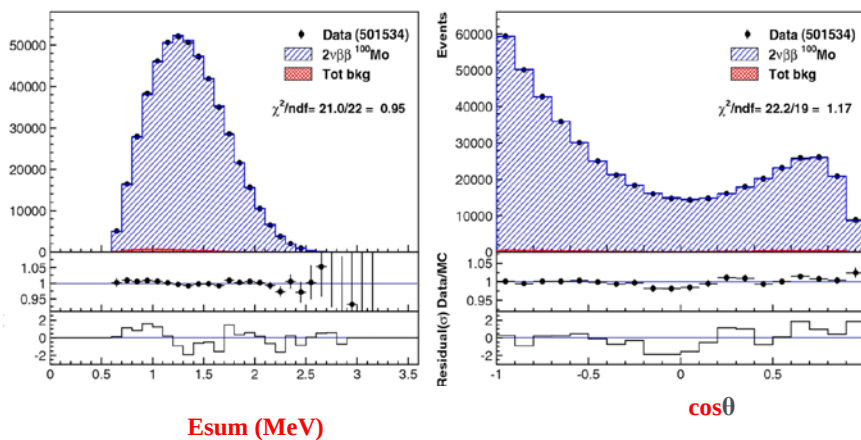
- Excellent background rejection
- Disentangle $0\nu\beta\beta$ mechanisms: $V+A$, sterile ν ...
- Nuclear physics: constrain g_A in $2\nu\beta\beta$
- e - γ separation probes decays to excited states

Role of SuperNEMO

Full kinematics and precision measurements of $2\nu\beta\beta$

- Nuclear model constraints
- g_A quenching constraints
- Sterile neutrinos
- Right-handed currents
- $2\nu\beta\beta$ with emission of single e^- , etc
(NEMO-3 analysis in preparation)

NEMO3 100Mo total data



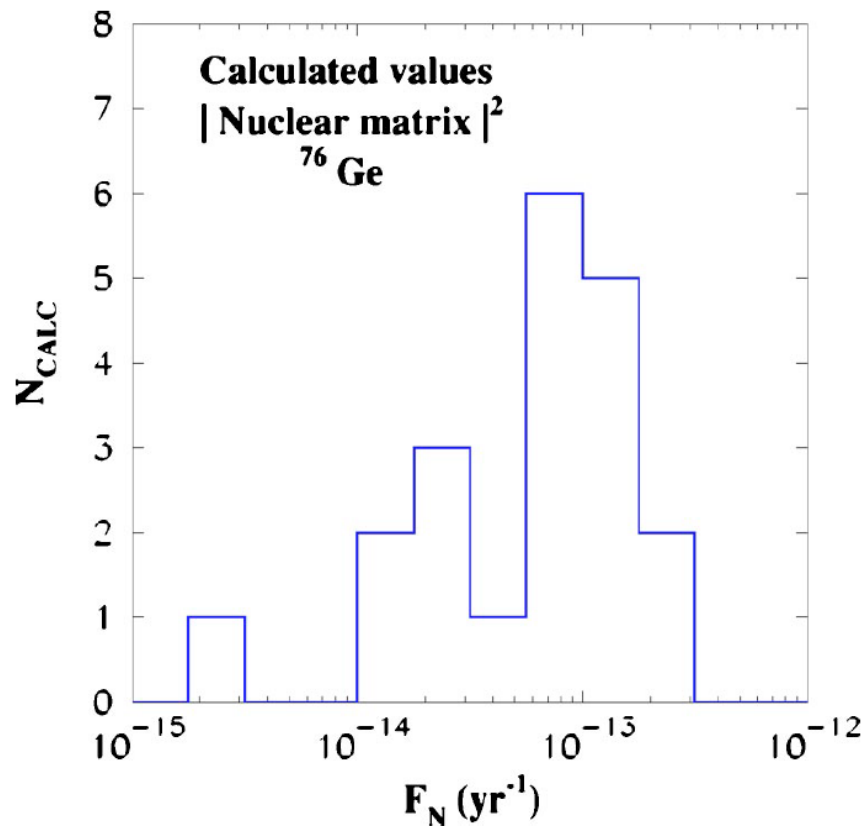
Understanding the Ultimate Reach of the Tracker-Calorimeter Technique

- Can the technique be used to **confirm & probe a signal** found in the next generation of $0\nu\beta\beta$ experiments?
- Explore **alternative tracker-calorimeter** technologies & **different isotopes**

$0\nu\beta\beta$ decay NMEs

Before: 2004 (factor 10 uncertainty)

few groups, 2 nuclear structure methods:
Nuclear Shell Model, QRPA



PRC 70, 033012 (2004)

Present: 2022 (factor 2-3)

Recognized priority task in nuclear physics
many groups, many nuclear structure methods:
Nuclear Shell Model, QRPA, PHFB, IBM, EDF

Attempts (light nuclear systems):

Ab initio calculations by different approaches – No Core Shell Model, Green's Function Monte Carlo, Coupled Cluster Method, Lattice QCD etc

Additional problems:

+ Effective weak coupling constant

$g_A^{\text{eff}} \approx 0.4 - 1.27$ (factor ≈ 3)

+ contact term (factor ≈ 2)

Evaluation of the $0\nu\beta\beta$ -decay NMEs calculation - Approximations needed

Nuclear Shell Model (Madrid-Strasbourg, Michigan, Tokyo): Relatively small model space (1 shell), all correlations included, solved by direct diagonalization
QRPA (Tuebingen-Bratislava-Calltech, Jyvaskyla, Chapel Hill, Lanzhou): Several major shells, only simple correlations included
Interacting Boson Method (Yale-Concepcion): Small space, important proton-neutron Pairing correlations missing
Projected Hartree-Fock-Bogoliubov Method (Lucknow): Several major shells, missing GT proton-neutron residual interaction.
Energy Density Functional theory (Madrid, Beijing): >10 shells, important proton-neutron pairing missing
An initio approaches:

The nuclear w. f. of
(A,Z), (A,Z+1)*, (A,Z+2)
Many-body methods
of choice:

The $0\nu\beta\beta$ nuclear transition operators
(F, GT, and tensor type):

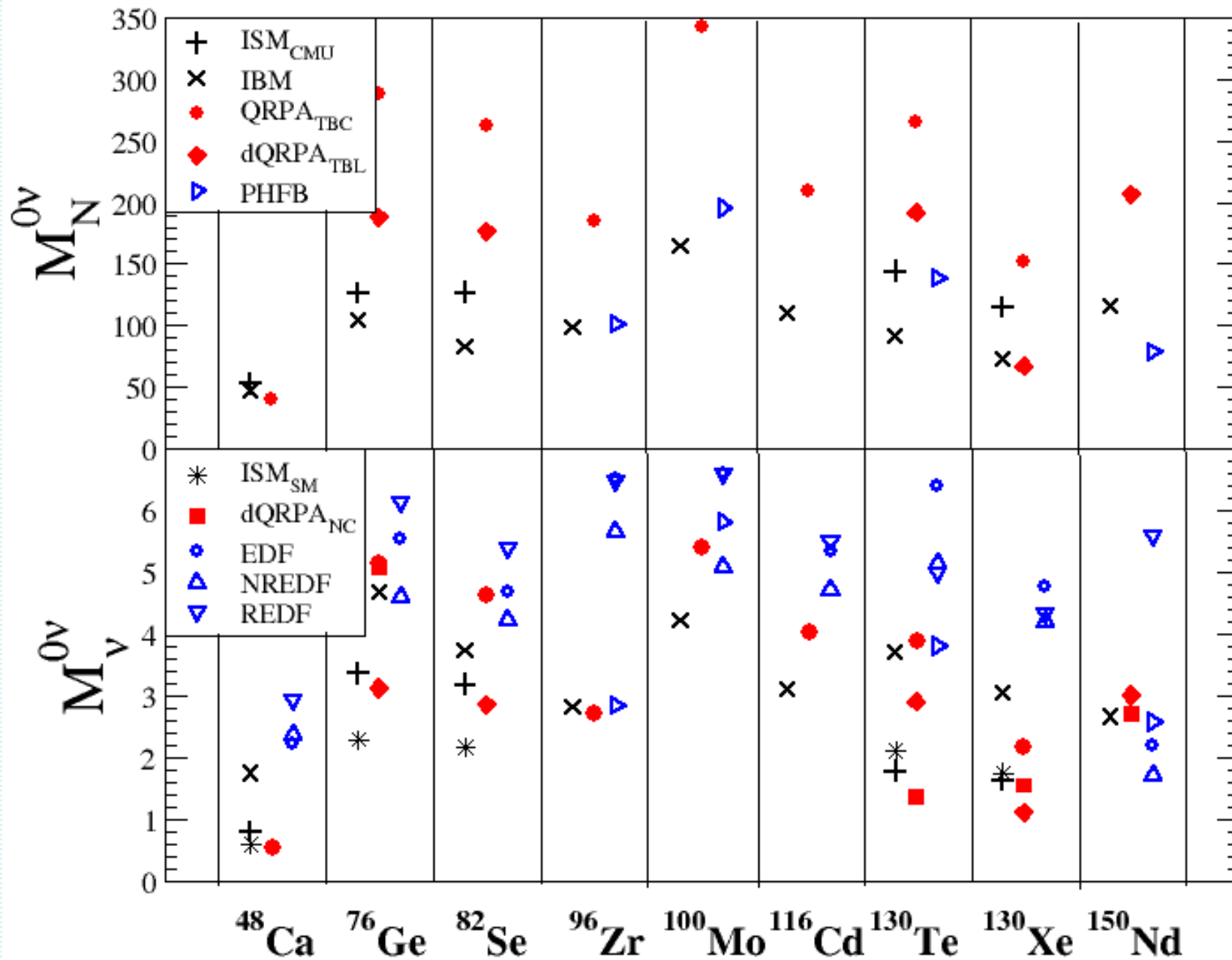
- + Transition operators involving complete set of states of intermediate nucleus
- + Transition operators in closure approximation – just two-body operators
- + Chiral effective field theory two-body transition operators with contact term
- +

! Isospin, and spin-isospin symmetries
($M_{Fcd} \approx 0$, M_{GTcd} strongly suppressed):

Initial 0^+ g.s.: (T, T)

Final 0^+ g.s.: (T-2, T-2) $\Rightarrow \Delta T = 2$ (!)

Till now, this issue addressed only within the QRPA
PRC 98, 064325 (2018)



**$0\nu\beta\beta$ -decay
NME
status 2022**

All
models
missing
essential
physics

Impossible
to assign
rigorous
uncertainties

Differences:

- Many-body approxim.
- Size of the m.s.
- Residual interactions

unquenched g_A

Assuming that the $0\nu\beta\beta$ process is mediated by a light-Majorana-neutrino exchange, a systematic analysis in chiral effective field theory shows that already at leading order **a contact operator is required to ensure renormalizability of the amplitude for $nn \rightarrow pp + ee$ process**. Without the strong 1S_0 short range interaction (which appears universally in all nuclear potentials) there would be no need of contact term.

Nuclear
matrix
element

$$M^{0\nu} = \langle \Psi_f | O^{0\nu} | \Psi_i \rangle$$

$$O_F^{0\nu} = (4\pi R_A) \sum_{a \neq b} V_F^{0\nu}(r_{ab}) \tau_a^+ \tau_b^+,$$

$$O_{GT}^{0\nu} = (4\pi R_A) \sum_{a \neq b} V_{GT}^{0\nu}(r_{ab}) \sigma_{ab} \tau_a^+ \tau_b^+,$$

$$O_T^{0\nu} = (4\pi R_A) \sum_{a \neq b} V_T^{0\nu}(r_{ab}) S_{ab} \tau_a^+ \tau_b^+.$$

Standard
type
contrib.

$$O_S^{0\nu} = (4\pi R_A) \sum_{a \neq b} V_S^{0\nu}(r_{ab}) \tau_a^+ \tau_b^+$$

Contact
term
contrib.

$$V_S^{0\nu}(r_{ab}) = 2 \frac{g_{\nu}^{\text{NN}}}{g_A^2} \delta_R^{(3)}(\mathbf{r}_{ab}),$$

PPNP 112, 1037 (2020)

$$V_{\alpha}^{0\nu}(r_{ab}) = \frac{1}{g_A^2} \int \frac{d^3 \mathbf{q}}{(2\pi)^3} e^{i\mathbf{q} \cdot \mathbf{r}_{ab}} V_{\alpha}^{0\nu}(\mathbf{q}^2)$$

v-potential ($\langle \mathbf{E}_{\text{aver.}} \rangle = 0$).

$$V_{\alpha}^{0\nu}(\mathbf{q}^2) = \frac{1}{\mathbf{q}^2} v_{\alpha}(\mathbf{q}^2)$$

**regularized with
a dipole form-factors**

$$v_F(\mathbf{q}^2) = -g_V^2(\mathbf{q}^2),$$

$$v_{GT}(\mathbf{q}^2) = g_A^2(\mathbf{q}^2) + \frac{2}{3} \frac{\mathbf{q}^2}{2m_N} g_A(\mathbf{q}^2) g_P(\mathbf{q}^2) + \frac{1}{3} \frac{\mathbf{q}^4}{4m_N^2} g_P^2(\mathbf{q}^2) + \frac{2}{3} \frac{\mathbf{q}^2}{4m_N^2} g_M^2(\mathbf{q}^2),$$

$$v_T(\mathbf{q}^2) = -\frac{2}{3} \frac{\mathbf{q}^2}{2m_N} g_A(\mathbf{q}^2) g_P(\mathbf{q}^2) - \frac{1}{3} \frac{\mathbf{q}^4}{4m_N^2} g_P^2(\mathbf{q}^2) + \frac{1}{3} \frac{\mathbf{q}^2}{4m_N^2} g_M^2(\mathbf{q}^2),$$

Nucleus	QRPA	NSM
	$M_S^{0\nu}/M_L^{0\nu} (\%)$	$M_S^{0\nu}/M_L^{0\nu} (\%)$
^{48}Ca		23 – 62
^{76}Ge	32 – 73	15 – 42
^{82}Se	30 – 70	15 – 41
^{96}Zr	29 – 69	
^{100}Mo	49 – 108	
^{116}Cd	26 – 61	
^{124}Sn	36 – 81	17 – 46
^{128}Te	35 – 77	17 – 46
^{130}Te	34 – 77	17 – 47
^{136}Xe	30 – 70	17 – 47

$$[t_{1/2}^{0\nu}]^{-1} = G_{0\nu} g_A^4 |M_L^{0\nu} + M_S^{0\nu}|^2 \frac{m_{\beta\beta}^2}{m_e^2}$$

Long-range

$$M_L^{0\nu} = M_{\text{GT}}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_{\text{F}}^{0\nu} + M_{\text{T}}^{0\nu}$$

Short-range

$$M_S^{0\nu} = \frac{2R}{\pi g_A^2} \langle 0_f^+ | \sum_{m,n} \tau_m^- \tau_n^- \int j_0(qr) h_S(q^2) q^2 dq | 0_i^+ \rangle$$

$$h_S(q^2) = -2g_{\nu}^{\text{NN}} e^{-q^2/(2\Lambda^2)}$$

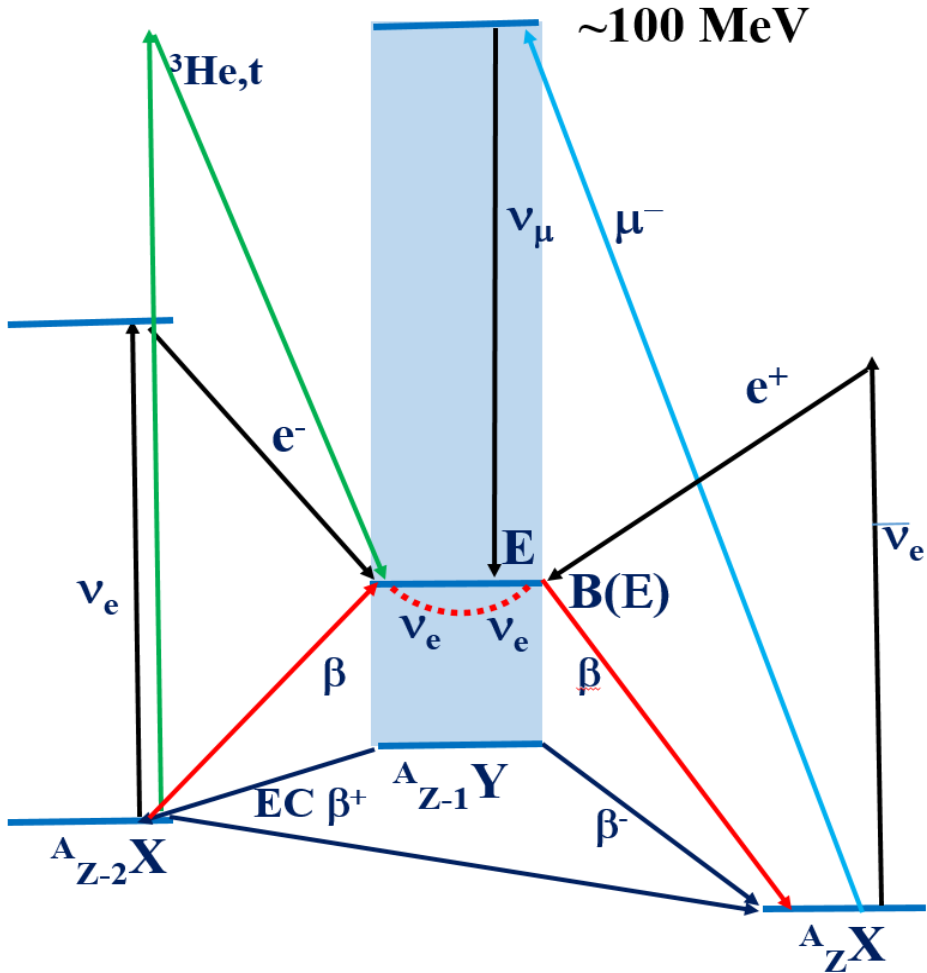
Regularized with Gaussian

(maybe better with dipole form-f. like weak magn. contr.)

Open urgent questions:

- A **correspondence** of the standard and the chiral field theory formalisms. Is contact term involved in the standard mechanism (completeness ...).
- What is the **magnitude** of the contact term NME? **Can it be large?** Justification with other phenomenology needed – pion and heavy-ion DCX, etc.

Supporting nuclear physics experiments
(Measurements still not conclusive for $0\nu\beta\beta$ NME)

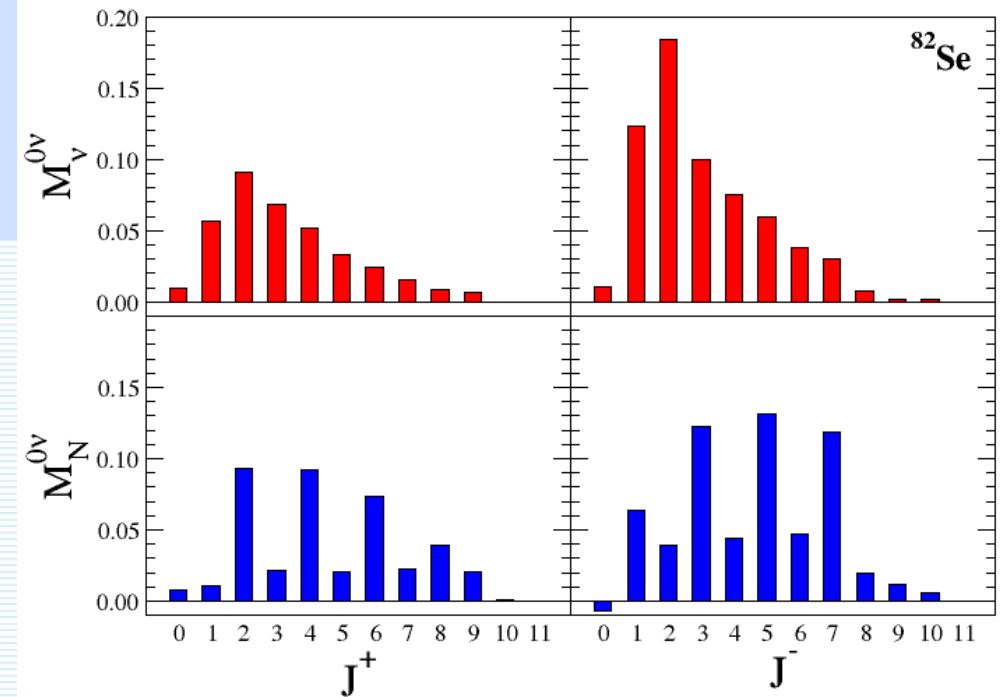


*Higher multipoles
are populated mostly due large v -
momenta transfer*

*Multipole
decomposition
of light and heavy
 $0\nu\beta\beta$ -decay NMEs
normalized to unity*

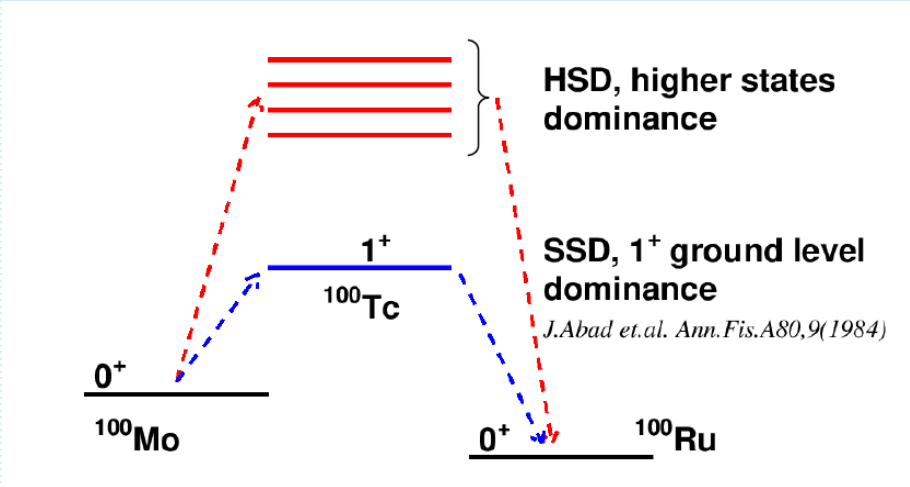
Fedor Simkovic

- ✓ β -decay, EC and $2\nu\beta\beta$ decay
- ✓ μ -capture
- ✓ (π^+, π^-) , single charge exchange
- ✓ $({}^3\text{He}, t)$, $(d, {}^2\text{He})$, transfer reactions
- ✓ γ -ray spectroscopy, $\gamma\gamma$ -decay
- ✓ A promising experimental tool:
Heavy-Ion Double Charge-Exchange



Understanding of the $2\nu\beta\beta$ -decay NMEs is of crucial importance for correct evaluation of the $0\nu\beta\beta$ -decay NMEs

There is no reliable calculation of the $2\nu\beta\beta$ -decay NMEs yet

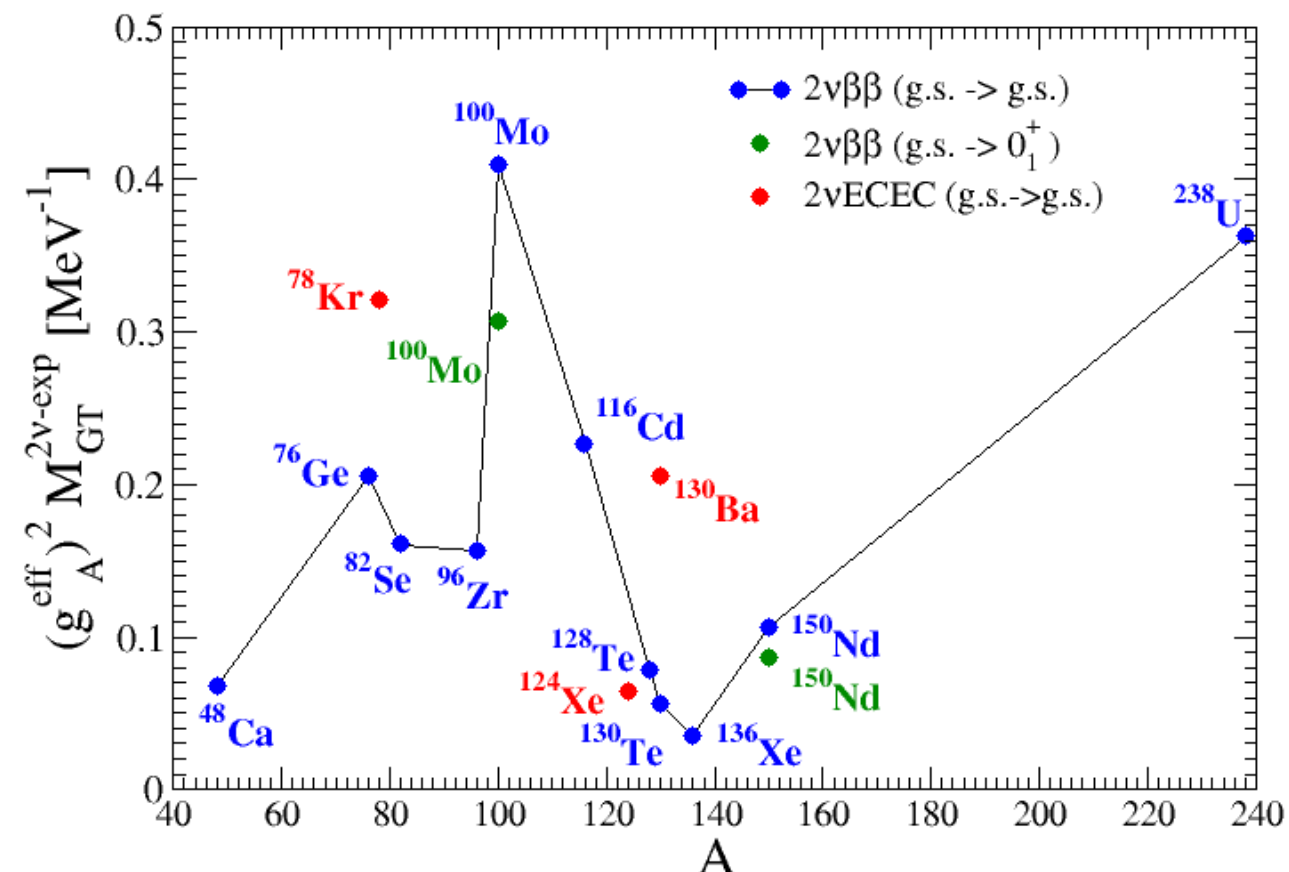


$$M_{GT}^{2\nu} = \sum_m \frac{\langle 0_f^+ || \tau^+ \sigma || 1_m^+ \rangle \langle 1_m^+ || \tau^+ \sigma || 0_i^+ \rangle}{E_m - E_i + \Delta}$$

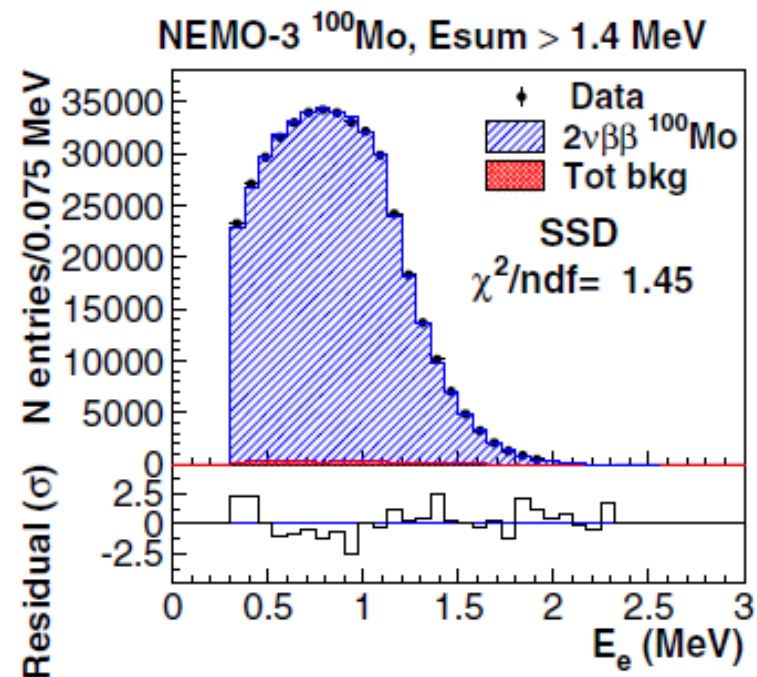
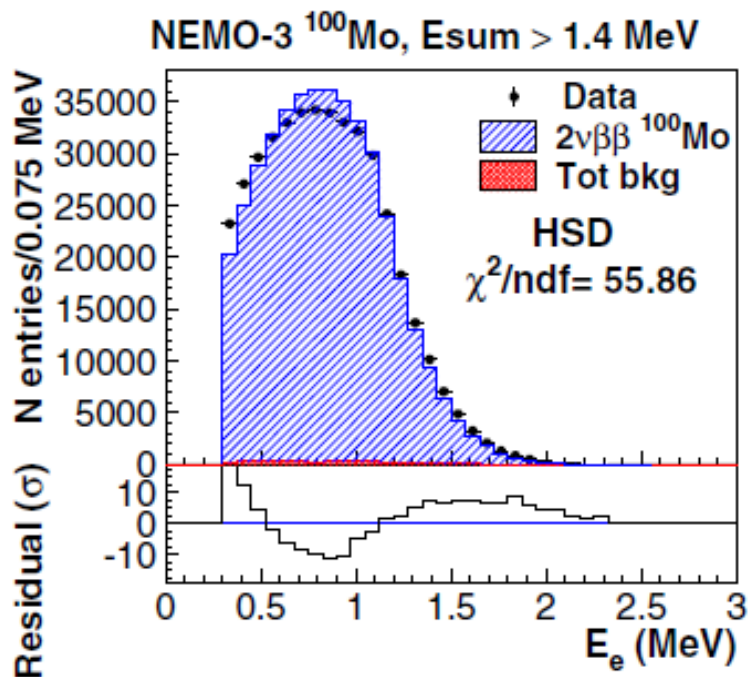
The **spread** of $2\nu\beta\beta$ and $0\nu\beta\beta$ NMEs is large and small, respectively.

Reasons:

- $2\nu\beta\beta$ NMEs governed by contribution from single (1^+) multipole unlike $0\nu\beta\beta$ NMEs by contributions from all **multipoles** (J^π) of int. nucl.
- Neutrino **potential** vs $2\nu\beta\beta$ potential



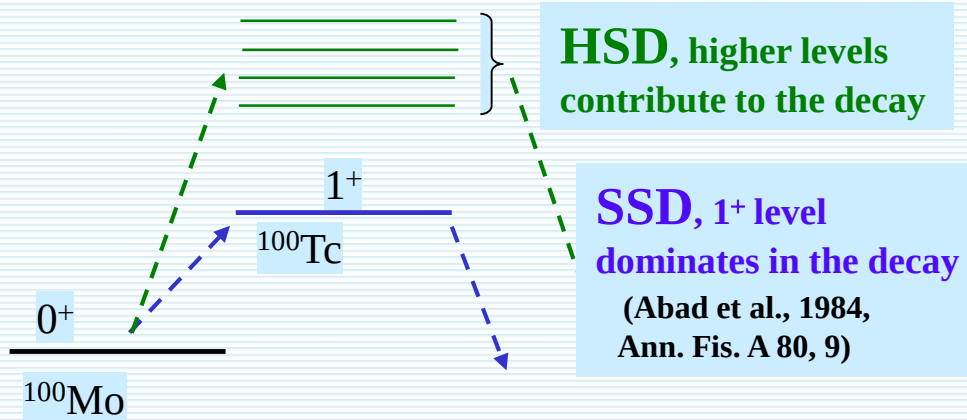
6/3/20 Both $2\nu\beta\beta$ and $0\nu\beta\beta$ operators connect the same states. Both change two neutrons into two protons. Explaining $2\nu\beta\beta$ -decay is necessary but not sufficient



**Looking for
SSD/HSD effect**

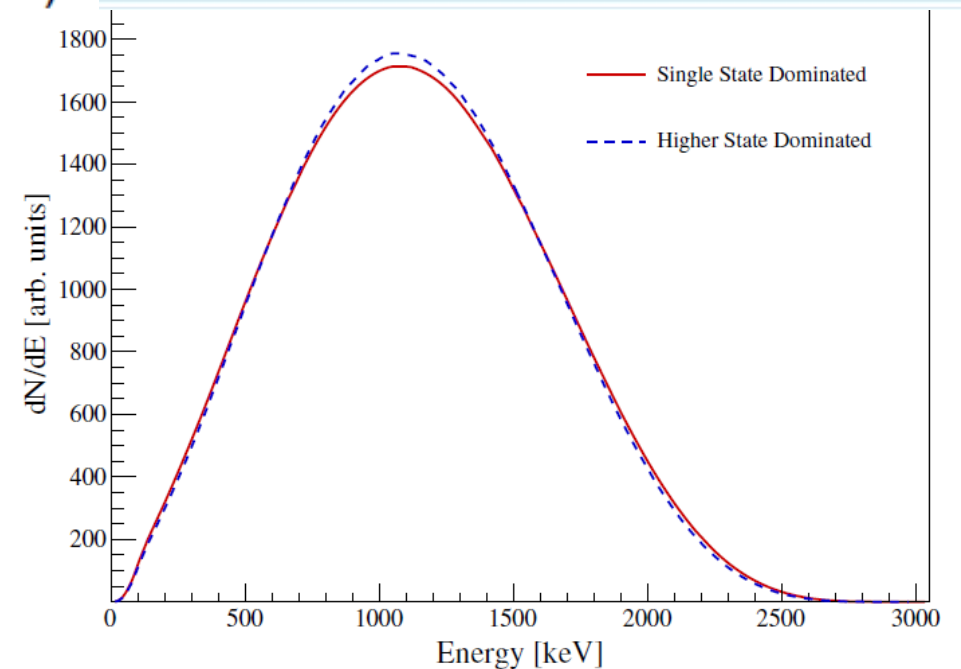
SSD favored
 $\Rightarrow$
 Strong trans.
 through low
 lying states
 of $(A, Z+1)$;
 $M_F^{2\nu} \approx 0$

NEMO3 Collaboration, Eur. Phys. J. C79, 440 (2019)



6/3/2022

Fedor Šimkovic



CUPID-0 Coll., PRL 123, 262501 (2019)

Improved description of the $0\nu\beta\beta$ -decay rate (a way to fix g_A^{eff})

PRC 97, 034315 (2018)

Taylor expansion

$$\frac{\epsilon_{K,L}}{E_n - (E_i + E_f)/2}$$

$$\epsilon_{K,L} \in \left(-\frac{Q}{2}, \frac{Q}{2}\right)$$

We get

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \simeq \left(g_A^{\text{eff}}\right)^4 \left|M_{GT-3}^{2\nu}\right|^2 \frac{1}{|\xi_{13}^{2\nu}|^2} \left(G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu}\right)$$

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \epsilon_{K,L}^2}$$

$$\epsilon_K = (E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1})/2$$

$$\epsilon_L = (E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1})/2$$

$$M_{GT-1}^{2\nu} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2)}$$

$$M_{GT-3}^{2\nu} = \sum_n M_n \frac{4 m_e^3}{(E_n - (E_i + E_f)/2)^3}$$

$$\xi_{13}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}$$

$$\xi_{15}^{2\nu} = \frac{M_{GT-5}^{2\nu}}{M_{GT-1}^{2\nu}}$$

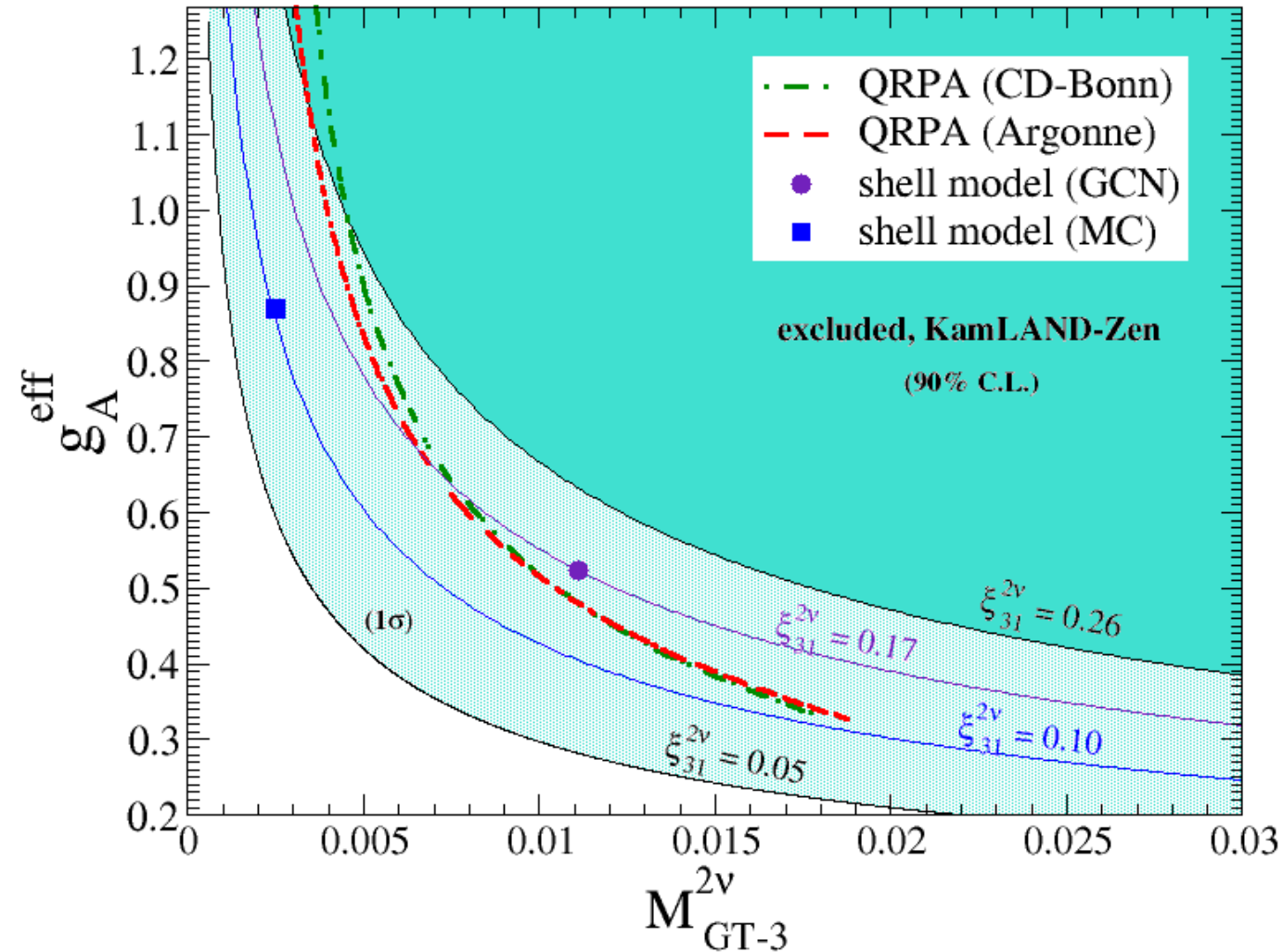
The g_A^{eff} can be determined with measured half-life and ratio of NMEs and calculated NME dominated by transitions through low lying states of the intermediate nucleus (ISM)

ξ_{13} can be determined phenomenologically
from the shape of energy
distributions of emitted electrons

The g_A^{eff} can be determined with measured
half-life, ratio of NMEs ξ_{31} and calculated NME,
dominated by transitions through
low lying states of the intermediate nucleus.

$$\left(g_A^{\text{eff}}\right)^2 = \frac{1}{\left|M_{GT-3}^{2\nu}\right|} \frac{\left|\xi_{13}^{2\nu}\right|}{\sqrt{T_{1/2}^{2\nu-\text{exp}} \left(G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu}\right)}}$$

M_{GT-3} have to be calculated
by nuclear theory - ISM

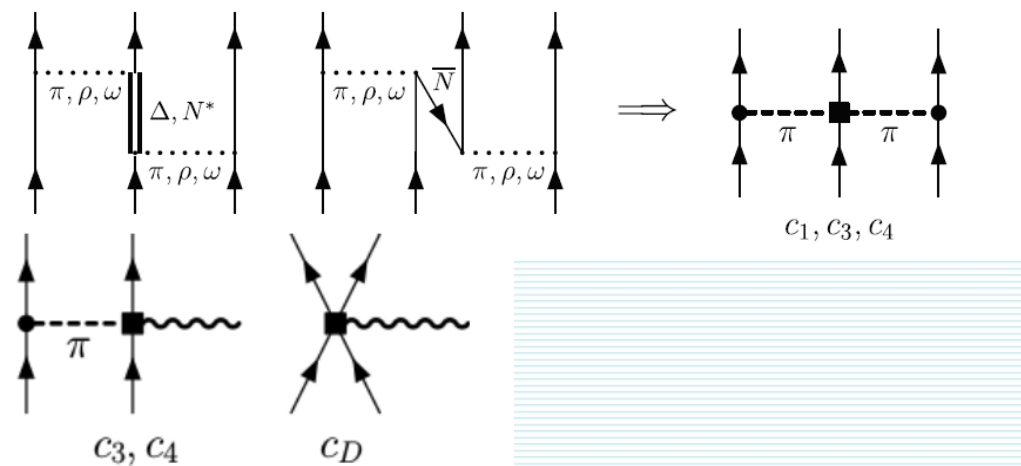


KamLAND-Zen Coll.
Phys.Rev.Lett. 122, 192501 (2019)

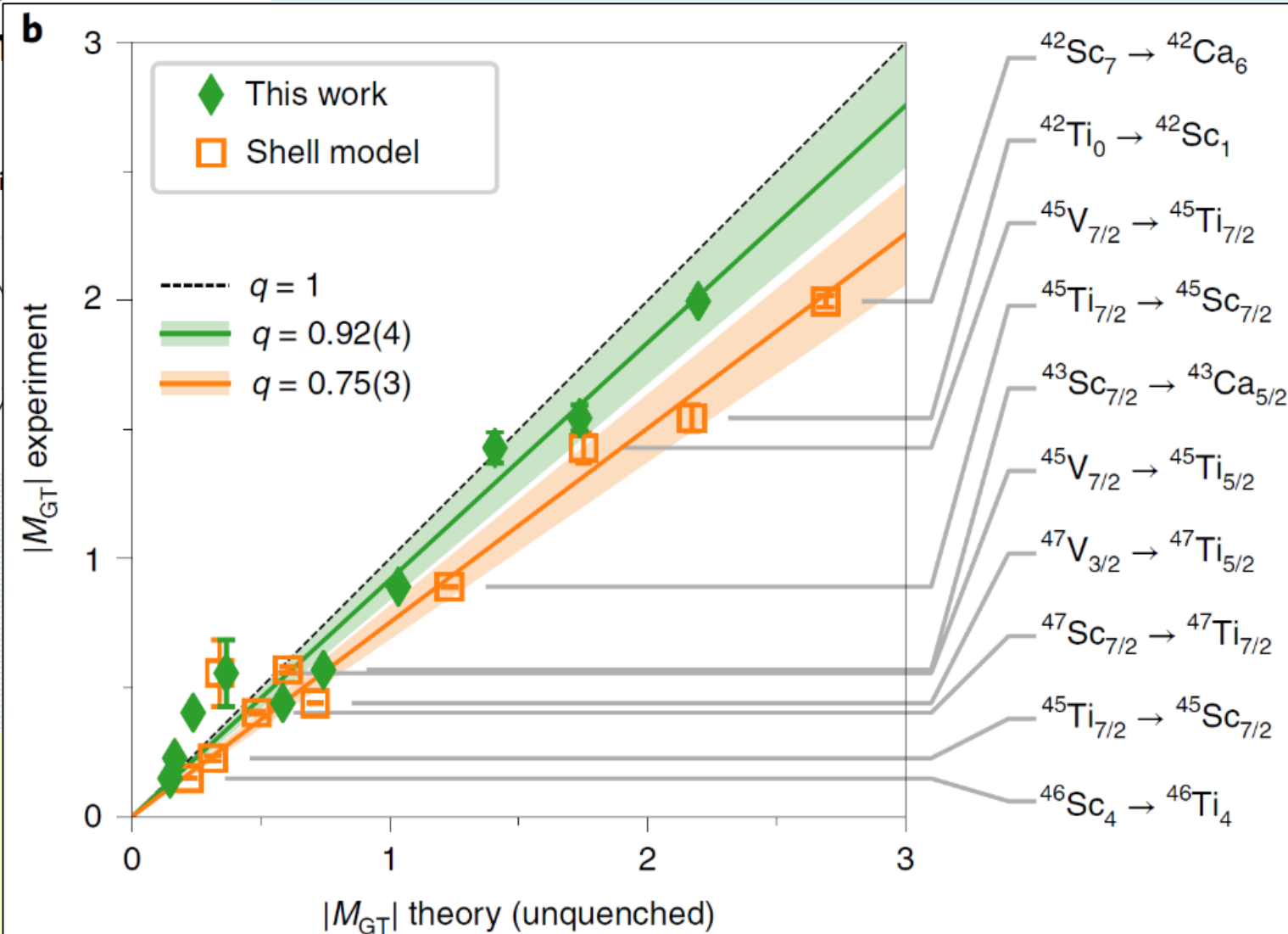
Ab initio β -decay study: g_A is unquenched (light nuclear systems)

Discrepancy between experimental and theoretical β -decay rates resolved from first principles

P. Gysbers^{1,2}, G. Hagen^{3,4*}, J. D. Holt¹, G. R. Jansen^{3,5}, T. D. Morris^{3,4,6}, P. Navrátil⁷, S. Quaglioni⁷, A. Schwenk^{8,9,10}, S. R. Stroberg^{1,11,12} and K. A. Wendt⁷



Once meson-exchange currents and 3-body forces are considered there is no need for any “quenching”



Measurement of GT strength via μ -capture

$$\mu^-_b + (A, Z) \rightarrow (A, Z-1) + \nu_\mu$$

Contradicting results:

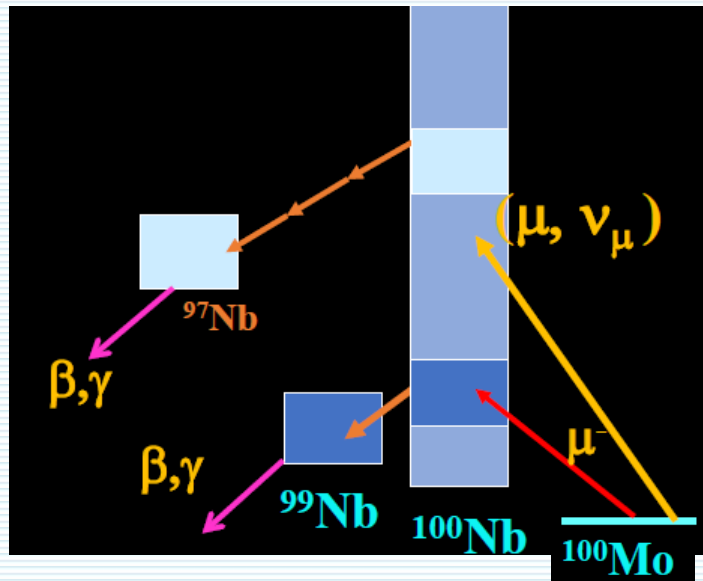
- Strong quenching ($g_A \approx 0.6$)

PRC 100, 014619 (2019)

- Weak quenching ($g_A \approx 1.1$)

PRC 74, 024326 (2006)

PRC 79, 054323 (2009)



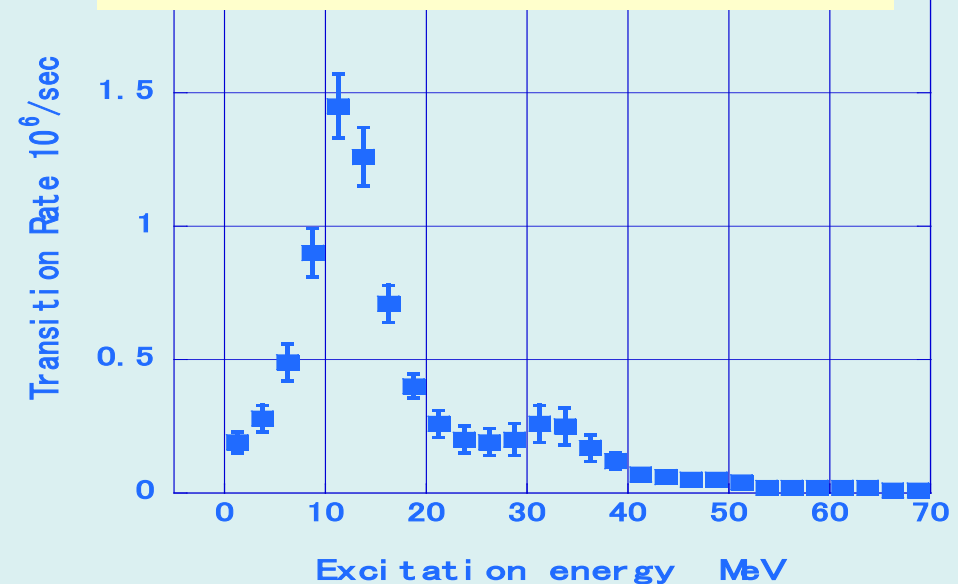
⇒ Small basis nuclear structure calculations (NSM, IBM) are disfavored. ⇒

J-PARC 3-50 GeV p, ν , μ



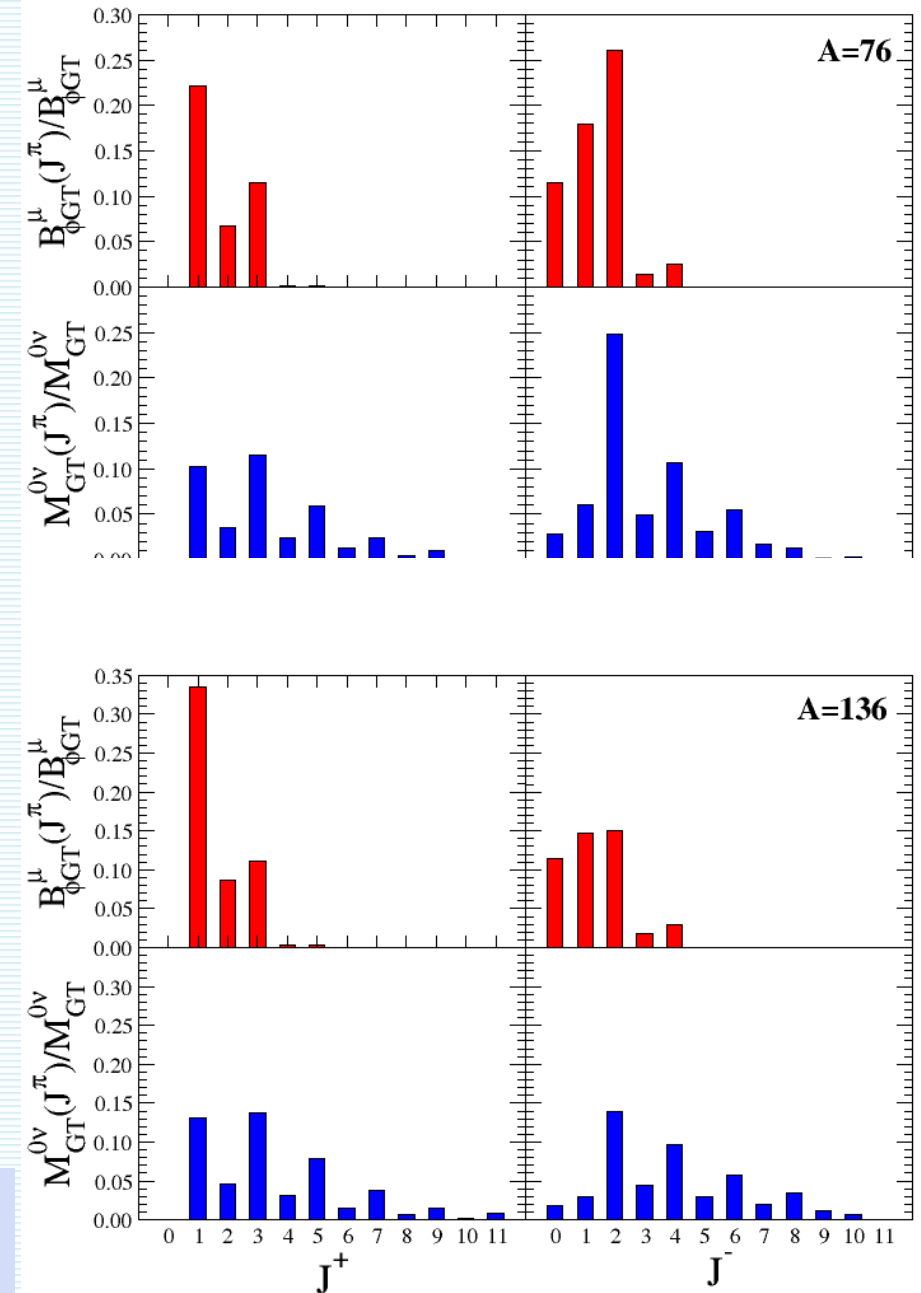
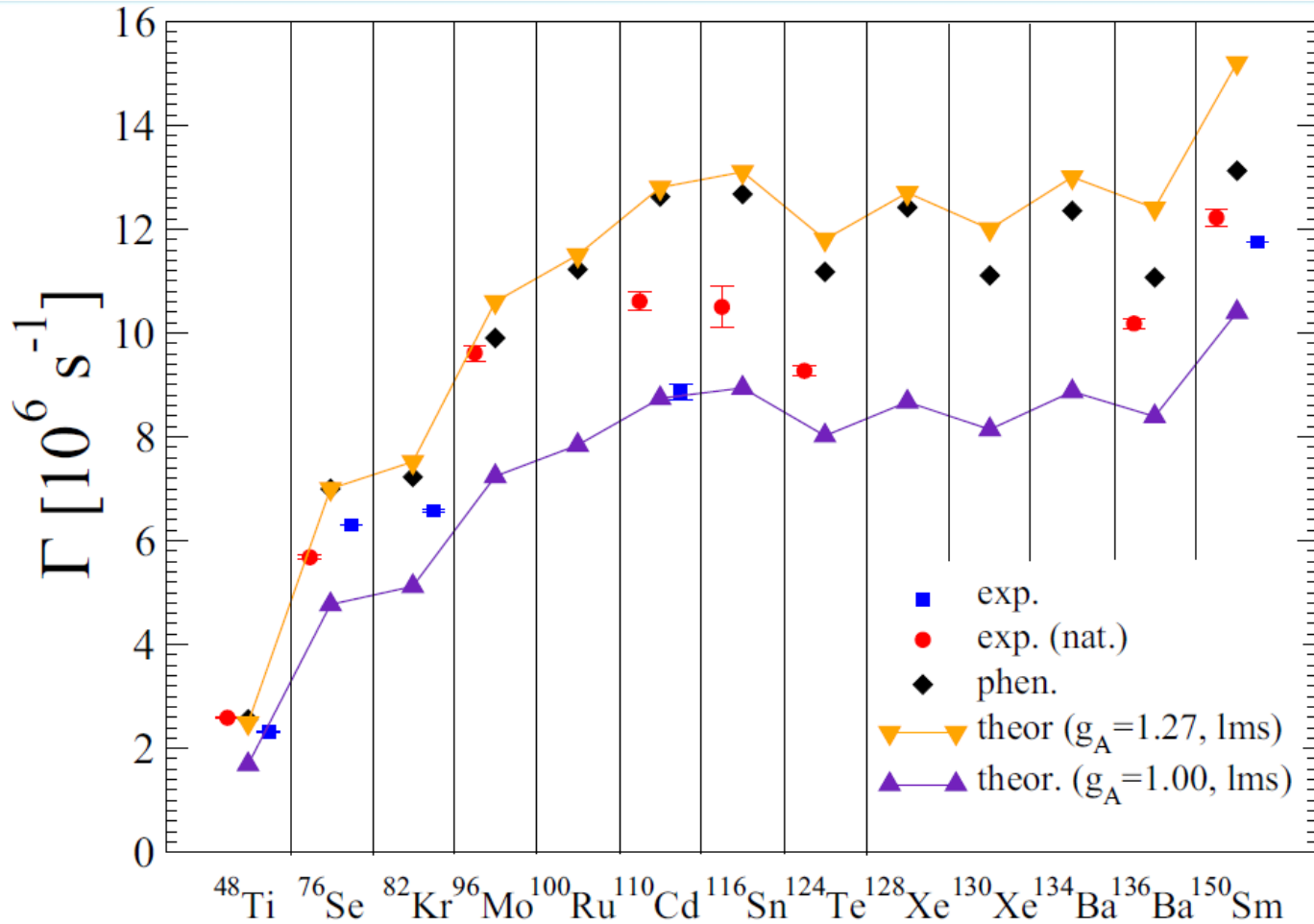
I. Hashim H. Ejiri, MXG16, PR C 97 2018

Momentum transfer $q \sim 80$ MeV



Muon capture rates evaluated within QRPA

In agreement with soft quenching ($g_A \approx 1.1$)

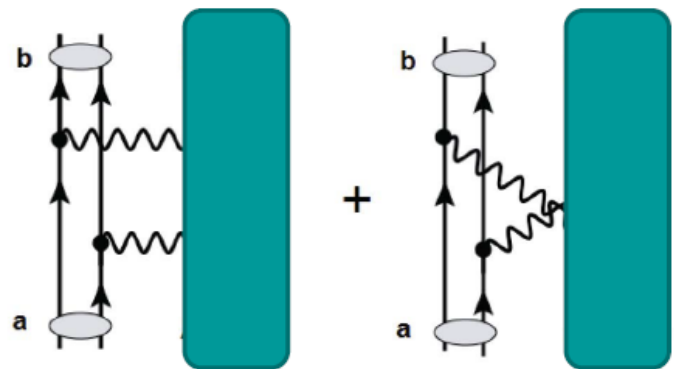


Multipole decomposition of $B_{\mu\text{GT}}^\mu$ and $M_{\mu\text{GT}}^{0v}$

PRC 102, 034301 (2020).

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Experiment Monument at PSI
will study contributions from all multipoles

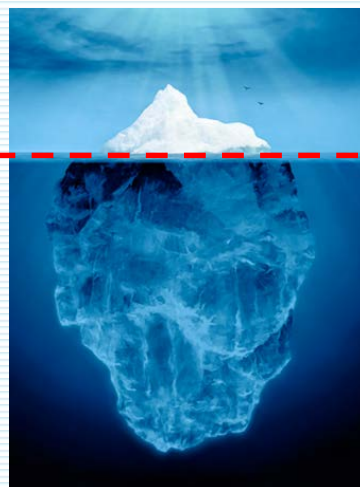


Heavy-ion DCE as surrogate processes of $\beta\beta$ -decay

- ✓ Induced by strong interaction
- ✓ Sequential nucleon transfer mechanism 4th order: Kinematical matching
- ✓ Meson exchange mechanism 1st or 2nd order
- ✓ Possibility to go in both directions
- ✓ Low cross section

Tiny amount of
DGT strenght for
low lying states

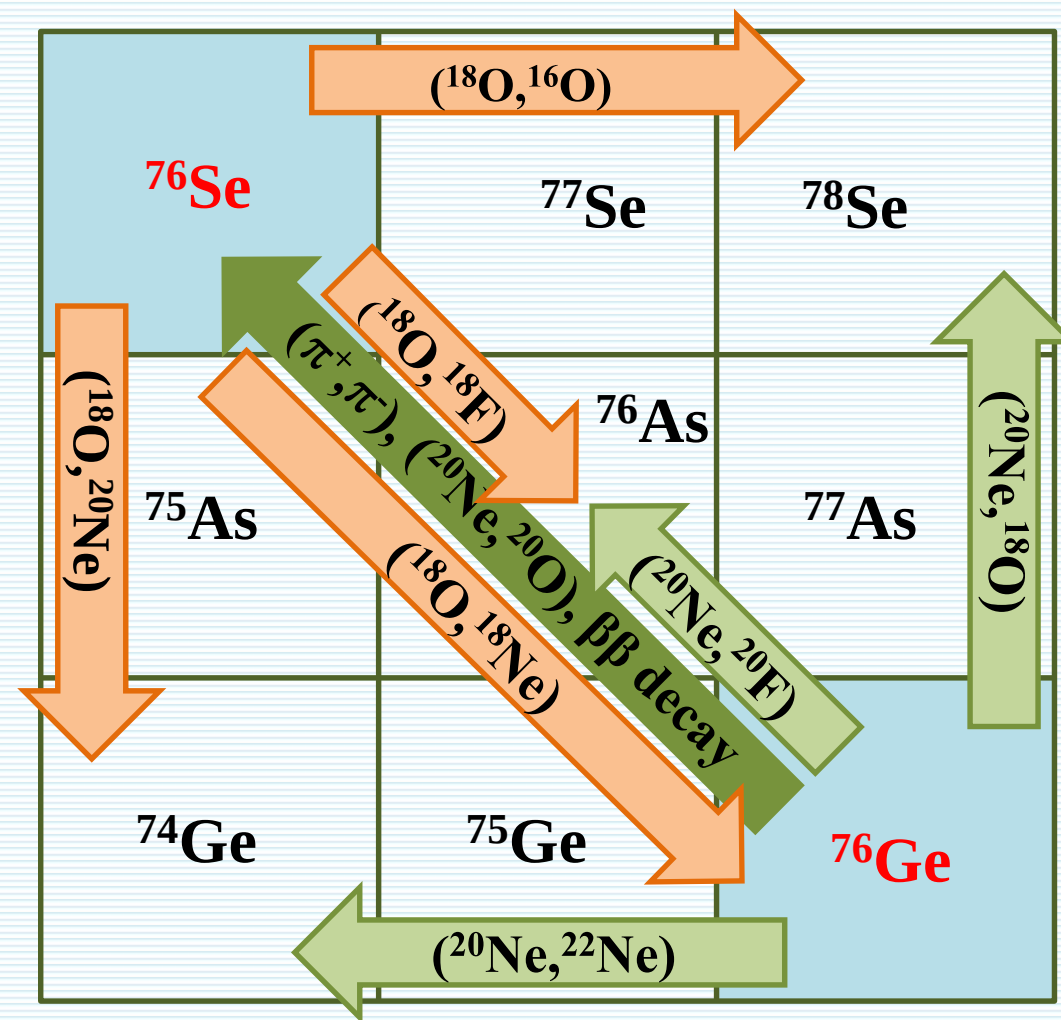
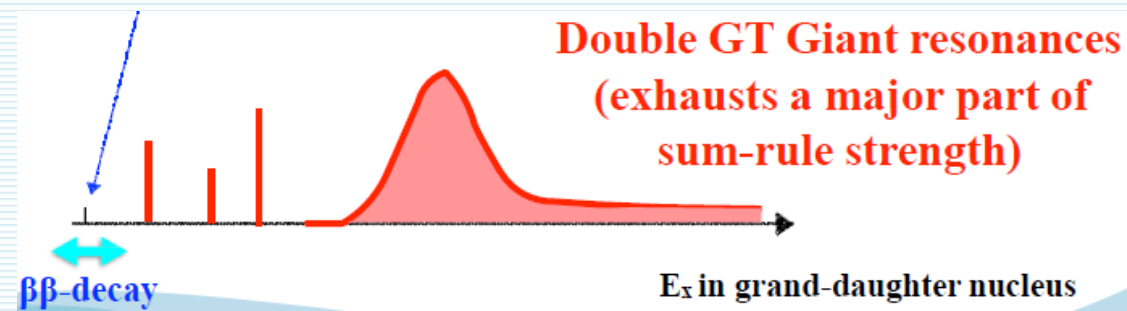
Sum rule almost
exhausted by
DGT Giant Mode,
still not observed

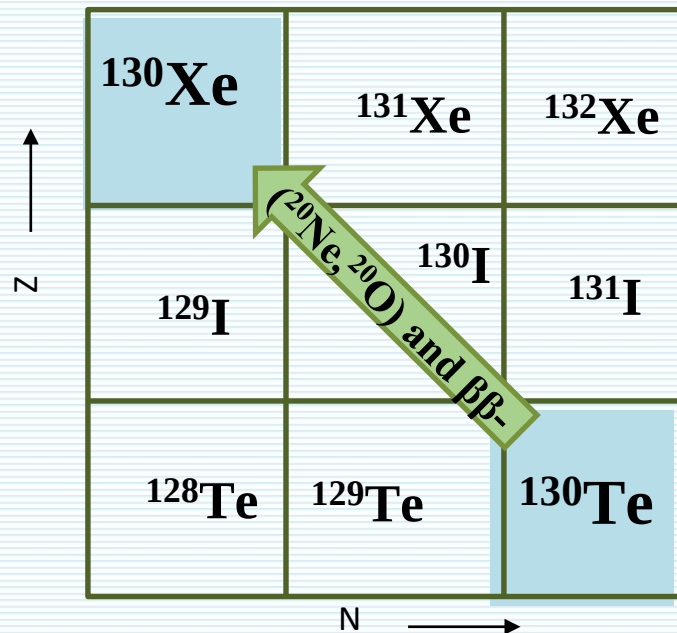
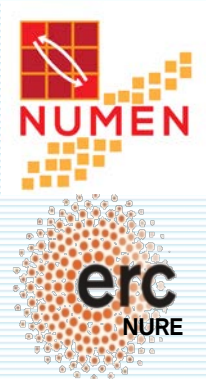


RIKEN

RCNP

Future:
INFN-LNS



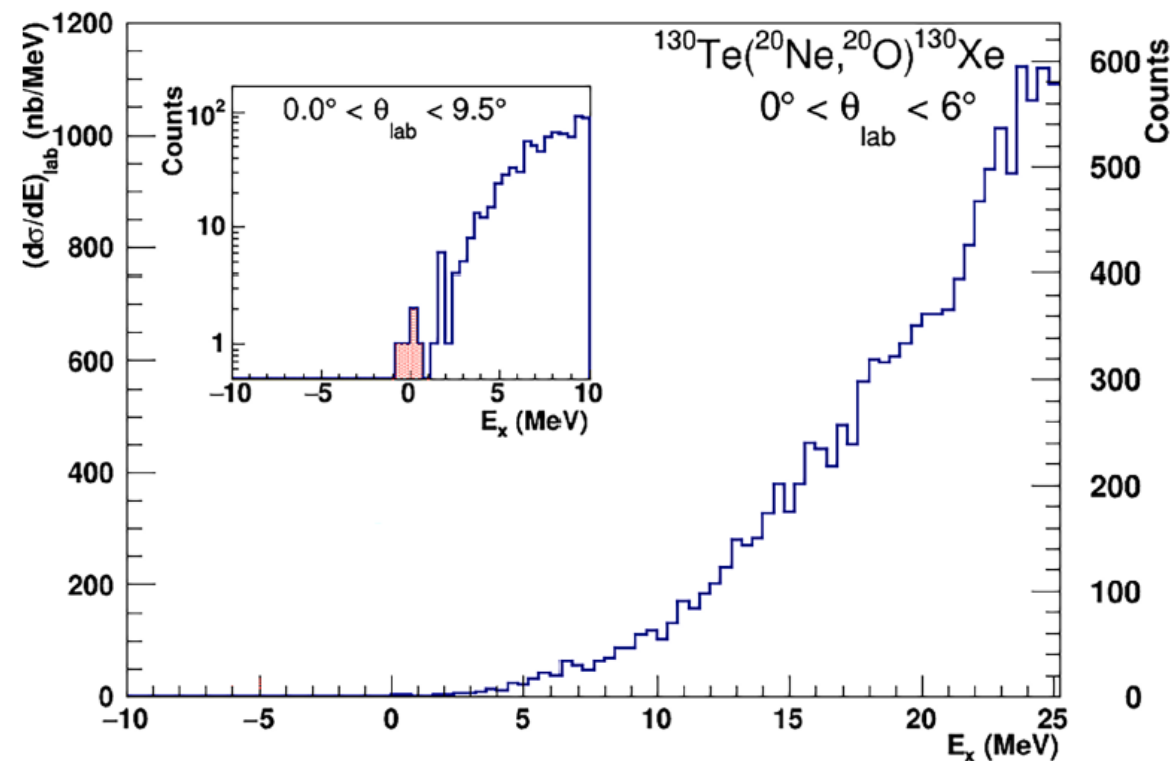


- g.s. $\rightarrow$ g.s. transition maybe isolated
- Absolute cross section measured

Resolution ~ 500 keV FWHM

No spurious counts at
 $-10 < E_x < -2$ MeV

The $^{130}\text{Te}(^{20}\text{Ne}, ^{20}\text{O})^{130}\text{Xe}$ DCE reaction



State (MeV)	Counts	Absolute cross section (nb)	Cross section 95% limit (nb)
g.s. (0^+) + 2^+ (536 keV)	5	13	[3--18]

Analysis of cross-section sensitivity < 0.1 nb in the Region Of Interest

**There is still
some time
to complete
the job**
Waiting on
observation of $0\nu\beta\beta$

*$0\nu\beta\beta$ - NMEs
must be evaluated using
tools of nuclear theory*

6/3/2022



**To complete
information
on $0\nu\beta\beta$
see reviews:**

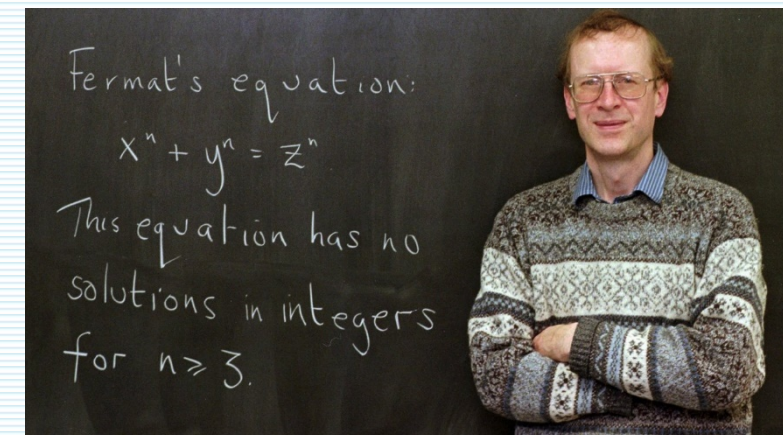
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- M. J. Dolinski, A. W. P. Poon and W. Rodejohann, **Neutrinoless Double-Beta Decay: Status and Prospects**, Ann. Rev. Nucl. Part. Sci. **69** (2019), 219-251
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- J. D. Vergados, H. Ejiri and F. Simkovic, **Theory of Neutrinoless Double Beta Decay**, Rept. Prog. Phys. **75** (2012), 106301
- A. Giuliani and A. Poves, **Neutrinoless Double-Beta Decay**, Adv. High Energy Phys. **2012** (2012), 857016
- S. M. Bilenky and C. Giunti, **Neutrinoless double-beta decay: A brief review**, Mod. Phys. Lett. A **27** (2012), 1230015



Around 1637, Pierre de Fermat wrote
in the margin of a book that
the more general equation

$$a^n + b^n = c^n$$

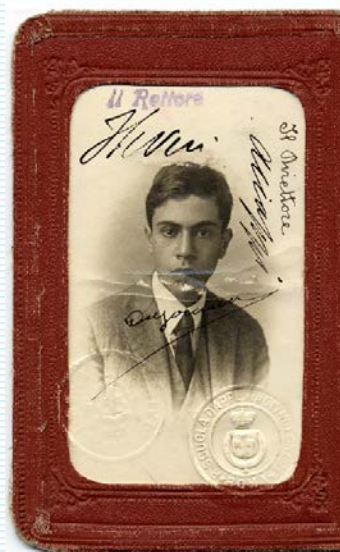
had no solutions in positive integers if n
is an integer greater than 2.



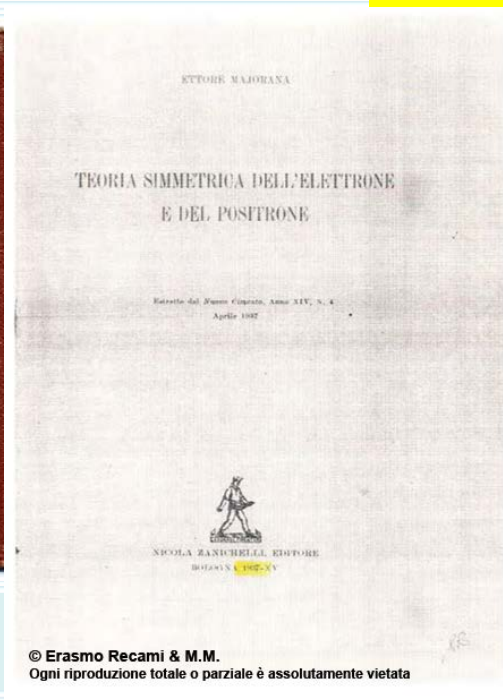
The proof was published by
Andrew Wiles in 1995.

After 358 years

Some long-standing tasks of humanity ...



1937



After
85
years

n-ton-class $0\nu\beta\beta$ exp.
with discovery potential

KamLAND-Zen 800

SNO+

LEGEND

nEXO

NEXT

CUPID

etc

After
?
years

If $m_{\beta\beta} < 1$ meV,
what
technology is
needed for
observation
of $0\nu\beta\beta$?