Lepton and quark mixing patterns using generalized CP symmetries

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Abstract

In this work, we have modified a scenario, originally proposed by Grimus and Lavoura, in order to obtain maximal values for atmospheric mixing angle and CP violating Dirac phase of the lepton sector. To achieve this, we have employed CP and some discrete symmetries in a type II seesaw model. In order to make predictions about neutrino mass ordering and the smallness of the reactor angle, we have obtained some conditions on the elements of the neutrino mass matrix of our model. Finally, within the framework of our model, we have studied quark masses and mixing pattern.

Introduction

Neutrinos are massless in the standard model(SM) of particle physics but solar and atmospheric neutrino oscillation dictates tiny neutrino mass and the mixing among them. So, many models have been constructed beyond SM, one of them is $\mu-\tau$ symmetric mass matrix that predicts maximal atmospheric angle (θ_{23}) and $\delta_{\rm CP}$, and non-zero reactor angle (θ_{13}). A type II based seesaw model is built here.

A model for lepton mixing

Three Higgs doublets $\phi_i=(\phi_i^+,\phi_i^0)^T$ and one Higgs triplet Δ have been proposed.

$$\Delta=\begin{pmatrix}\Delta^+/\sqrt{2} & \Delta^{++}\\ \Delta^0 & -\Delta^+/\sqrt{2}\end{pmatrix}$$

Three lepton doublets and charged lepton singlets are denoted as $D_{\alpha L}=(\nu_{\alpha L},\alpha_L)^T$ and α_R, where $\alpha=e,\mu,\tau$.The symmetries used are

- Lepton number symmetry $U(1)_{L\alpha}$.
- Generalized CP symmetries defined as

$$D_{\alpha L}\to iS_{\alpha\beta}\gamma^0C\bar{D}_{\beta L}^T,\quad \alpha_R\to iS_{\alpha\beta}\gamma^0C\bar{\beta}_R^T,\quad S=\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},$$

$$\phi_{1,2}\to\phi_{1,2}^*,\quad \phi_3\to-\phi_3^*,\quad \Delta\to-\Delta^*.$$

- One Z_2 symmetry under which ϕ_1 and e_R change sign.
- One Z_3 symmetry: $\phi_{2,3}\to\Omega\phi_{2,3}$, $\mu_R,\tau_R\to\Omega^2\mu_R,\tau_R$.
- K symmetry: $\mu_R\to-\mu_R$, $\phi_2\leftrightarrow\phi_3$.

The charged lepton Lagrangian can be written as

$$\mathcal{L}_Y=-y_e\bar{D}_{eL}\phi_1e_R-\sum_{j=2}^{3}\sum_{\alpha=\mu,\tau}g_{j\alpha}\bar{D}_{\alpha L}\phi_j\alpha_R+h.c..$$

Then the mass of the muon and tau are found as

$$\frac{m_\mu}{m_\tau}=\left|\frac{v_2-v_3}{v_2+v_3}\right|,\quad v_2,v_3 \text{ are complex.}$$

Now, due to K symmetry $v_2=v_3$ as a result mass of muon will be zero. So, small violation of K symmetry is required to explain the hierarchy between muon and tau masses.

The neutrino mass generation Lagrangian is written by breaking the lepton number by a small amount as

$$\mathcal{L}_Y=\frac{1}{2}\sum_{\alpha,\beta=e,\mu,\tau}Y^\nu_{\alpha\beta}\bar{D}^c_{\alpha L}i\sigma_2\Delta D_{\beta L}+h.c..$$

So, the mass matrix is

$$M_\nu=Y^\nu v_\Delta,\quad SY^\nu S^*=(Y^\nu)^*.$$

The form of mass matrix is then $\mu-\tau$ symmetric under the condition v_Δ real. Due to breaking of the lepton number symmetry $Y^\nu\sim10^{-3}$. To satisfy the neutrino oscillation data for atmospheric and solar mass squared differences, $v_\Delta=1-10$ eV.

Analysis of scalar potential

- We write the full scalar potential and parameterize the VEV of scalars as

$$\langle\phi_1\rangle=v_1,\quad\langle\phi_2\rangle=v\cos\sigma e^{i\alpha},\quad\langle\phi_3\rangle=v\sin\sigma e^{i\beta},\quad\langle\Delta\rangle=v'e^{i\theta}$$

  We find that $\sigma=\frac{\pi}{4}$, $\zeta=\alpha-\beta=0$ and $\theta=0$ is a minima that makes the triplet VEV real.
- Then, we write all possible terms in the scalar potential that violates K symmetry explicitly by a small amount taking all parameters small. So, as a result the above minima will get shifted to small amount as follows.

$$\sigma=\frac{\pi}{4}-\frac{\delta_0}{2},\quad\zeta=0+\delta_\zeta,\quad\theta=0+\delta_\theta.$$

- The minimization of the full scalar potential will give $\delta_0,\delta_\zeta\neq0$ and $\delta_\theta=0$. Then the muon to tau mass ratio is calculated to be

$$\frac{m_\mu}{m_\tau}=\frac{1}{2}|\delta_0+i\delta_\zeta|.$$

  which explain the the required hierarchy between muon and tau masses, and $\delta_\theta=0$ explained that VEV of triplet is always real that was required for the mass matrix to be mu-tau symmetric.

Quark mass and mixing

The quark mass and mixing is explained by introducing a singlet X field by proposing certain texture of quark mass matrix having the form

$$M_u=\begin{pmatrix}h^u_{11}\epsilon^6 & h^u_{12}\epsilon^4 & h^u_{13}\epsilon^4\\ h^u_{21}\epsilon^4 & h^u_{22}\epsilon^2 & h^u_{23}\epsilon^2\\ h^u_{31}\epsilon^4 & h^u_{32}\epsilon^2 & h^u_{33}\end{pmatrix}v_1,\quad M_d=\begin{pmatrix}h^d_{11}\epsilon^6 & h^d_{12}\epsilon^6 & h^d_{13}\epsilon^6\\ h^d_{21}\epsilon^9 & h^d_{22}\epsilon^4 & h^d_{23}\epsilon^4\\ h^d_{31}\epsilon^{10} & h^d_{32}\epsilon^4 & h^d_{33}\epsilon^2\end{pmatrix}v_1.$$

where $\epsilon=\langle X\rangle/M$. The masses and mixing angles are calculated to be

$$(m_t,m_c,m_u)\approx(|h^u_{33}|,|h^u_{22}||\epsilon|^2,|h^u_{11}-h^u_{12}h^u_{21}/h^u_{22}||\epsilon|^6)v_1,$$
$$(m_b,m_s,m_d)\approx(|h^d_{33}||\epsilon|^2,|h^d_{22}||\epsilon|^4,|h^d_{11}||\epsilon|^6)v_1,$$
$$|V_{us}|\approx\left|\frac{h^d_{12}}{h^d_{22}}-\frac{h^u_{12}}{h^u_{22}}\right||\epsilon|^2,$$
$$|V_{cb}|\approx\left|\frac{h^d_{23}}{h^d_{33}}-\frac{h^u_{23}}{h^u_{33}}\right||\epsilon|^2,$$
$$|V_{ub}|\approx\left|\frac{h^d_{13}}{h^d_{33}}-\frac{h^u_{12}h^d_{23}}{h^u_{22}h^d_{33}}-\frac{h^u_{13}}{h^u_{33}}\right||\epsilon|^4,$$
$$\arg(V_{ub})\approx4\arg(\epsilon).$$

The higher-dimensional terms of the Lagrangian are motivated from UV completion is discussed in the main article in detail.

References

J. Ganguly and R. S. Hundi, Lepton and quark mixing patterns with generalized CP transformations," [arXiv:2107.07275 [hep-ph]].