



Present knowledges of neutrino



안양환 (Yang-Hwan Ahn)

OUTLINE

Present knowledges of 3ν SM

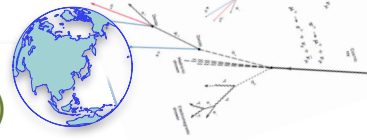
Twists and turns of a light sterile neutrino

A possible scenario for anomalous phenomena via

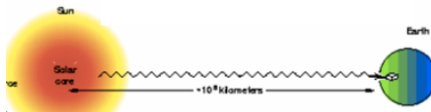
a new parameterization of neutrino mixing (2016 Ahn, SK Kang, CS Kim)

One of the most important recent discoveries in particle physics is
the experimental evidence of **neutrino oscillations**

Atmospheric- ν exp. Super-Kamiokande (1998)



Confirmed by K2K (2004) and Minos (2006)
accelerator neutrino exp



Confirmed by Kamland (2004)
reactor neutrino exp

Solar- ν exp. SNO (2002)

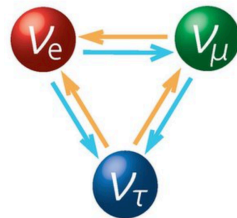
+

fully confirmed
in different channels
by other experiments
(NO ν A, Super-K, T2K,...)



Reactor- ν exp. (2012)

A neutrino in flight exists as a **superposition** of the three different “**flavors**” of neutrinos. Its flavor can be different when detected than when it was created.



= mixtures (θ) of neutrinos with definite masses (m_{ν_i})

Nonzero neutrino mass $\Rightarrow$ Signals physics beyond the Standard Model !!

But, still we do not know the origin of neutrino mass !!

Neutrinos have Mass....super-light... ν SM

Higgs gives masses to the SM particles

Fermion mass terms describe transitions between R- and L-handed states

In the SM, only L-handed ν fields couple with W and Z bosons

R-handed ν s can not be created in weak interactions.

So, ν s do not participate in the Higgs mechanism:

in the SM ν s are massless !!

Add more particles + Abandon renormalizability

SM

→

ν SM

Gauge symmetry + Lorentz invariance

economical Particle content

No R-handed $\nu \rightarrow$ No Dirac neutrino mass term

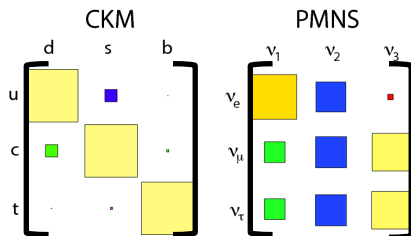
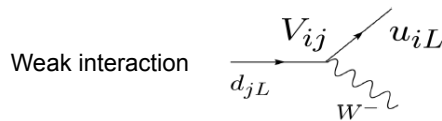
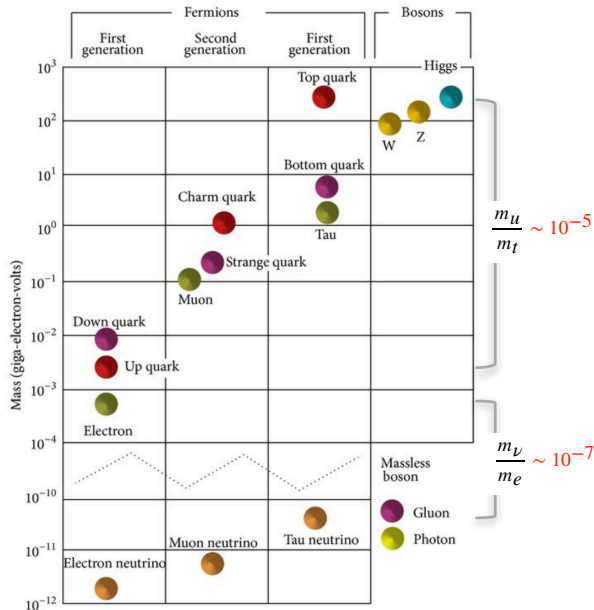
One Higgs doublet $\rightarrow$ No Majorana neutrino mass term

new particle
(sterile neutrinos,

mandatory Renormalizability

B and L conservations $\rightarrow$ No Majorana mass term

Higher-dimensional operator



What is the origin of the quark and lepton mass and mixing hierarchies ?

Dirac or Majorana neutrinos ?

Leptonic CP violation ?

How many neutrinos ?

Absolute neutrino mass ?

Dirac vs Majorana

$$\mathcal{L}_D = Y_\nu (\bar{\nu} \bar{e})_L \begin{pmatrix} \varphi^0 \\ -\varphi^- \end{pmatrix} N_R + Y_e (\bar{\nu} \bar{e})_L \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} e_R + \text{h.c.}$$

$$\frac{m_\nu}{m_e} \sim 10^{-7}$$

Charged-lepton (& quarks)

$$\text{how about? } \frac{m_\mu}{m_t} \sim 10^{-5}$$

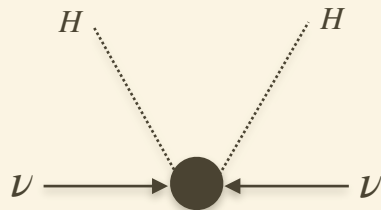
→ unnatural ? but Possible

Fermion (quark and lepton) mass hierarchy problem !!

theoretically,
Lepton number conservation (Dirac neutrino) $\Leftrightarrow$ global $U(1)_L$ symmetry
Global symmetry is not protected from violations by quantum gravity effects (PRL62,1221)

$\Rightarrow U(1)_L$ Majoran neutrino ?

Weinberg Dimension-5 operator



$$\mathcal{L}_M = \frac{1}{\Lambda} HLHL$$

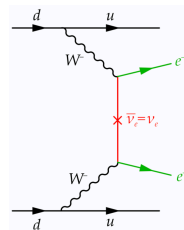
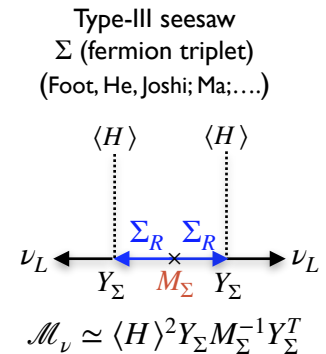
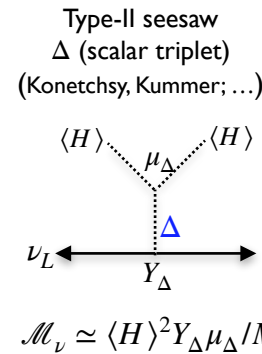
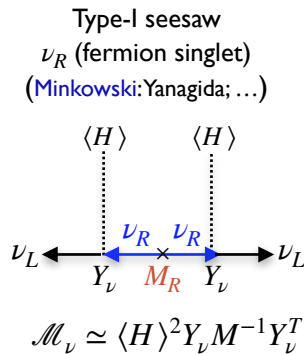
Majorna neutrino mass (< 0.1 eV)

$$\Rightarrow \Lambda \sim 10^{14-15} \text{ GeV}$$

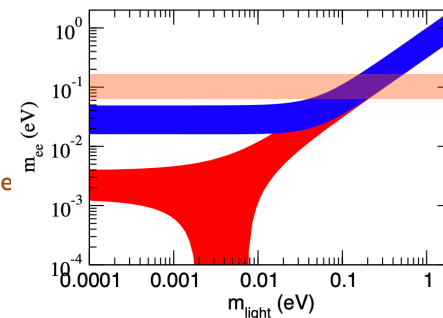
Prediction: super-heavy particle
(← the early universe)

Leptogenesis

$0\nu\beta\beta$ -decay



Particle = Anti-particle



$m_{ee} < 61 - 165 \text{ meV}$ PDG (2021)

GERDA
EXO-200
KamLAND-Zen
AMoRE

Seesaw has two appealing features.....

Smallness of ν mass in a very nice way

SM + ν_R

10^{14} GeV
a new mass scale of nature



100 GeV

$$\mathcal{M}_\nu \simeq \langle H \rangle^2 Y_\nu M_R^{-1} Y_\nu^T$$

should reproduce Δm^2 and θ in agreement with ν oscillation experiments

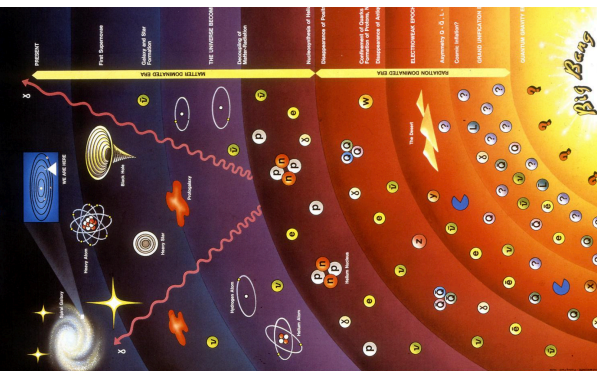
should satisfy the upper bounds on m_ν (from cosmology) and $|(\mathcal{M}_\nu)_{ee}|$ (from $0\nu\beta\beta$ decay)

should reproduce the observed BAU $\sim 10^{-10}$

Why is there only Matter in universe but no anti-matter ?
(sakharov condition)

Leptogenesis (built-in mechanism for generating BAU)

Yanagida; ...



$$Br(\nu_R \rightarrow \ell + \phi) \neq Br(\nu_R \rightarrow \bar{\ell} + \phi^\dagger)$$

Matter

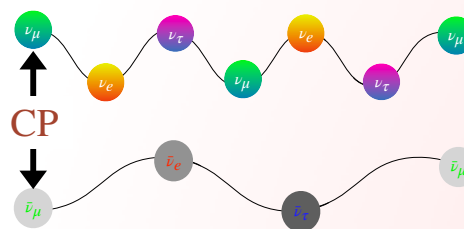
anti-matter

$$\nu_R = \nu_R^c : \cancel{L} \ \& \ \cancel{CP}$$

Too small $\cancel{CP}$ with CKM

Leptonic $\cancel{CP}$ can be responsible for the Baryon Asymmetry of the Universe

ν oscillation and
 $0\nu\beta\beta$ -decay
experiments



Neutrino Oscillation

$\cancel{CP}$

Antineutrino oscillation

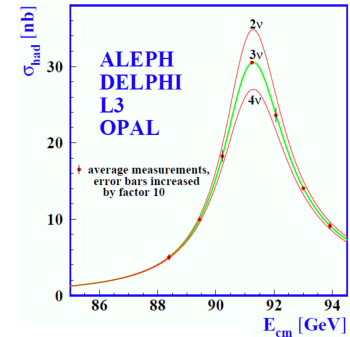
PMNS
 δ_{CP}

Seemingly, a robust 3-neutrino mixing scheme.....

Neutrinos are produced in weak interactions together with their charged lepton

$$-\mathcal{L}_\nu = \left(\frac{1}{2} \bar{\nu}_L^c \mathcal{M}_\nu \nu_L \right) + \frac{g}{\sqrt{2}} W_\mu^- \bar{\ell}_L \gamma^\mu \nu_L + \text{h.c.} + \frac{g}{2 \cos \theta_W} Z_\mu \bar{\nu}_L \gamma^\mu \nu_L$$

e, μ, τ ν_e, ν_μ, ν_τ



From LEP $N_\nu = 2.984 \pm 0.008$: only 3 “active” neutrinos ($m_\nu < M_Z/2$) couple with the $Z \rightarrow 3$ lepton flavor

3 × 3 neutrino mass matrix
 $\Rightarrow$ 3 × 3 Unitary leptonic mixing matrix

Flavor eigenstates

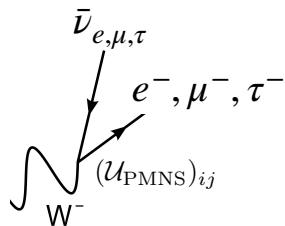
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Mass eigenstates

Unitary condition is assumed

Lepton mixings

Diagonal phase matrix
 (Majorana neutrino)



Pontecorvo; Maki,
 Nakagawa, Sakata

$$\mathcal{U}_{\text{PMNS}} = \mathcal{R}_1(\theta_{23}) \mathcal{R}_2(\theta_{13}, \delta_{CP}) \mathcal{R}_3(\theta_{12}) \mathcal{P}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \begin{pmatrix} \cos \theta_{13} & 0 & \sin \theta_{13} e^{i\delta_{CP}} \\ 0 & 1 & 0 \\ -\sin \theta_{13} e^{-i\delta_{CP}} & 0 & \cos \theta_{13} \end{pmatrix} \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathcal{P}$$

Atm
 Accelerator

Reactor
 Accelerator

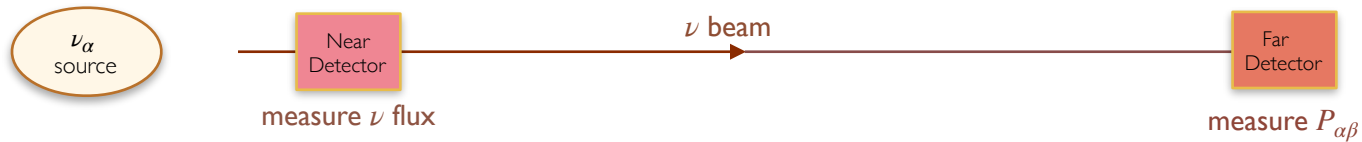
Solar
 Reactor (KamLAND)

$$\Delta m_{\text{Atm}}^2$$

$$\Delta m_{\text{Sol}}^2$$

Seemingly, a robust 3-neutrino mixing scheme.....

PDG (2021)



$|\nu(0)\rangle = |\nu_\alpha\rangle$
 ν produced with a defined flavor

$$|\nu(t)\rangle = e^{ip \cdot r} \sum_k U_{\alpha k} e^{-iE_k t} |\nu_k\rangle$$

The complex phases $e^{-iE_k t}$
 are different if $m_j \neq m_k$

$$E_k = \sqrt{p^2 + m_k^2}$$

Neutrinos propagate with different frequencies due to their different masses
 and different phases develop with distance travelled

For two-neutrino mixing

$$|\nu(t)\rangle = e^{i(p \cdot r - E_1 t)} \left\{ \cos \theta |\nu_1\rangle + e^{-i(E_2 - E_1)t} \sin \theta |\nu_2\rangle \right\}$$

Probability to detect ν_β
 at distance $L \simeq t$

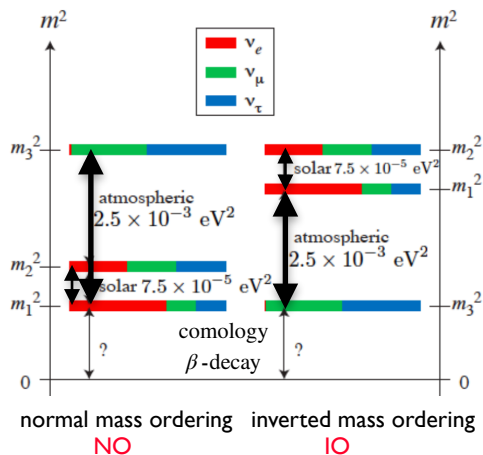
$$P_{\alpha\beta} = |\langle \nu_\beta | \nu(t) \rangle|^2 = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

Global-fit of 3-flavor ν -oscillations:

best-fit and (1σ) 3σ level w/o SK atmospheric data

NuFIT 5.1 (2021)

$$\Delta m_{\text{Sol}}^2 \equiv m_{\nu_2}^2 - m_{\nu_1}^2 \text{ and } \Delta m_{\text{Atm}}^2 \equiv m_{\nu_3}^2 - m_{\nu_1}^2 \text{ for NO } (\Delta m_{\text{Atm}}^2 \equiv m_{\nu_2}^2 - m_{\nu_3}^2 \text{ for IO})$$



	$\theta_{13} [^\circ]$	$\delta_{CP} [^\circ]$	$\theta_{12} [^\circ]$	$\theta_{23} [^\circ]$	$\Delta m_{\text{Sol}}^2 [10^{-5} \text{eV}^2]$	$\Delta m_{\text{Atm}}^2 [10^{-3} \text{eV}^2]$
NO	$8.57^{+(0.13)0.40}_{-(0.12)0.37}$	$194^{+(52)211}_{-(25)89}$	$33.44^{+(0.77)2.42}_{-(0.74)2.17}$	$49.2^{+(1.0)2.8}_{-(1.3)9.7}$	$7.42^{+(0.21)0.62}_{-(0.20)0.60}$	$2.515^{+(0.028)0.084}_{-(0.031)0.084}$
IO	$8.60^{+(0.12)0.38}_{-(0.12)0.36}$	$287^{+(27)74}_{-(32)95}$	$33.45^{+(0.77)2.42}_{-(0.74)2.18}$	$49.5^{+(1.0)2.6}_{-(1.2)9.7}$		$2.498^{+(0.028)0.086}_{-(0.029)0.085}$

$$\text{cosmology } \sum m_\nu = m_{\nu_1} + m_{\nu_2} + m_{\nu_3}$$

$$\text{CMB PLANCK} < (0.340 \sim 0.715) \text{ eV}$$

$$\beta\text{-decay } m_{\bar{\nu}_e} = \left(\sum |U_{ek}|^2 m_{\nu_k}^2 \right)^{1/2}$$

$$\text{KATRIN} < 0.2 \text{ eV}$$

Seemingly, a robust 3-neutrino mixing scheme.....

Hints for normal mass ordering: SK,T2K, NO ν A

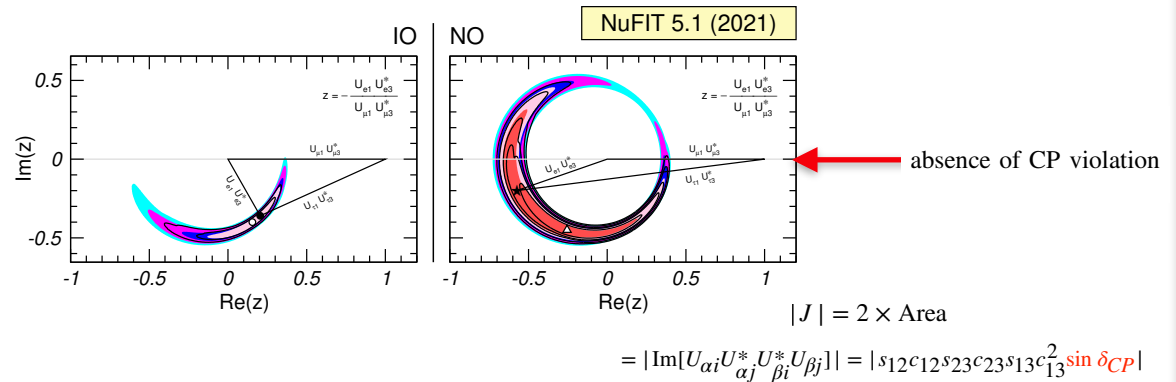
Hint for non-maximal atmospheric mixing: NO ν A

Hint for CP violation:T2K, NO ν A

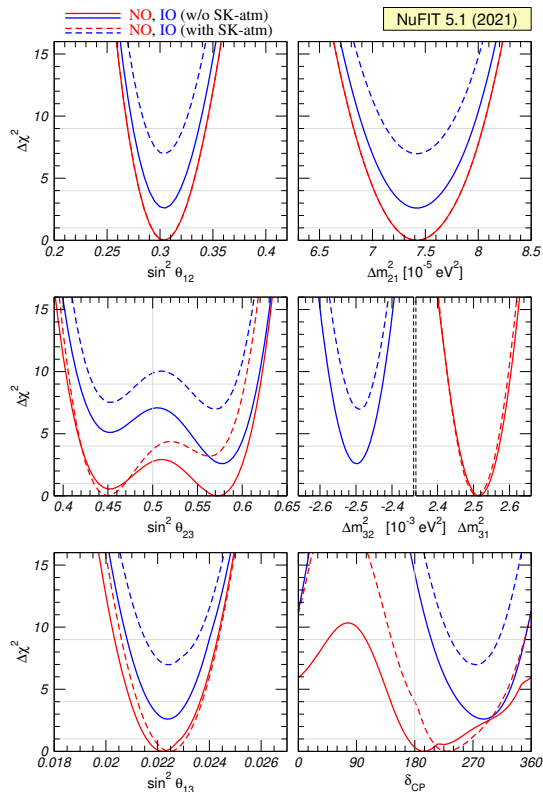
Unitarity Triangle in the Lepton sector: **unitarity is assumed**

Triangle $\Leftrightarrow$ unitarity conditions on the 1st and 3rd columns

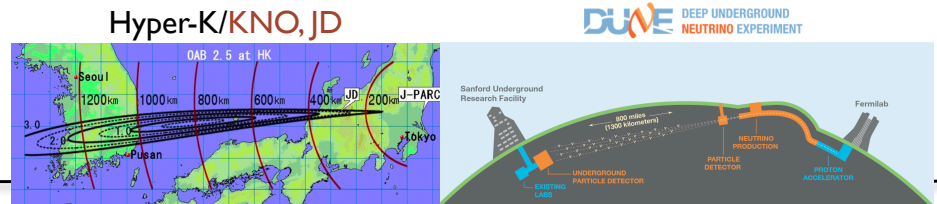
$$U_{e1}U_{e3}^* + U_{\mu 1}U_{\mu 3}^* + U_{\tau 1}U_{\tau 3}^* = 0$$



Test flavor symmetry model !!



Definite discoveries are still to come !!



The upcoming high-precision neutrino oscillation experiments are expected to determine the neutrino mass ordering, mixing angles, and CP violation at high C.L. and to provide a rigorous test of the 3- ν oscillation framework at various baselines (L) and energies (E) in the presence of Earth's matter effect

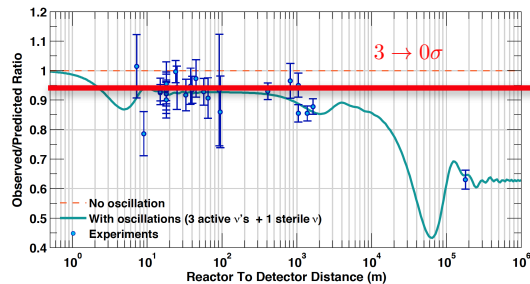
In spite of the huge success of 3-flavor oscillations,

SBL anomalies and Solar neutrino tension.....

- (i) unlikely statistical fluctuations in the current data
- (ii) unknown physical phenomena

SBL anomalies

reactor anti-neutrino($\bar{\nu}_e$) anomaly

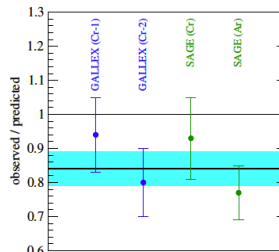


$$R_g = 0.945 \pm 0.007(\text{exp}) \pm 0.023(\text{model})$$

arXiv :1808.1083

Gallium solar anomaly

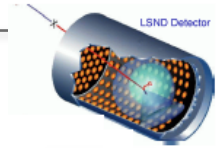
Measuring in the channel $\nu_e \rightarrow \nu_e$
 a very SBL ($\langle L \rangle_{\text{GALLEX}} = 1.9 \text{ m}$, $\langle L \rangle_{\text{SAGE}} = 0.6 \text{ m}$)



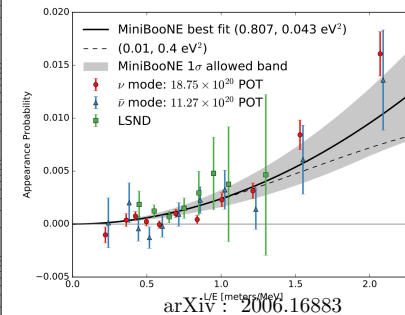
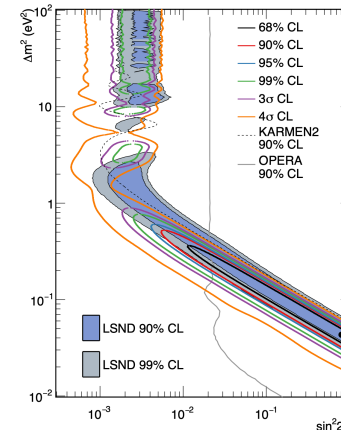
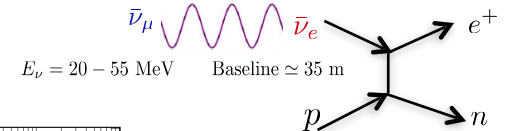
$$\Rightarrow \Delta m^2 \gtrsim 1 \text{ eV}^2$$

$$\bar{R} = 0.84 \pm 0.05 \sim 2.9\sigma \text{ deficit}$$

LSND anomaly (Liquid Scintillator Neutrino Detector)



observed an excess of $\bar{\nu}_e$ events in a $\bar{\nu}_\mu$ beam



If interpreted as ν -Osc.

In the standard 3ν framework $\rightarrow \Delta m_{\text{Atm}}^2$ and Δm_{Sol}^2

$$L^{\text{osc}} = \frac{4\pi E_\nu}{\Delta m^2} \sim \begin{cases} 100 \text{ km} & \text{for } \Delta m_{\text{Sol}}^2 \\ 1 \text{ km} & \text{for } \Delta m_{\text{Atm}}^2 \end{cases}$$

SBL ($1 < \text{km}$) reactor ν -Exps. are **not** sensitive to the solar and atmospheric oscillations

$$\Delta m_{\text{SBL}}^2 \Rightarrow \text{eV mass sterile neutrino}$$

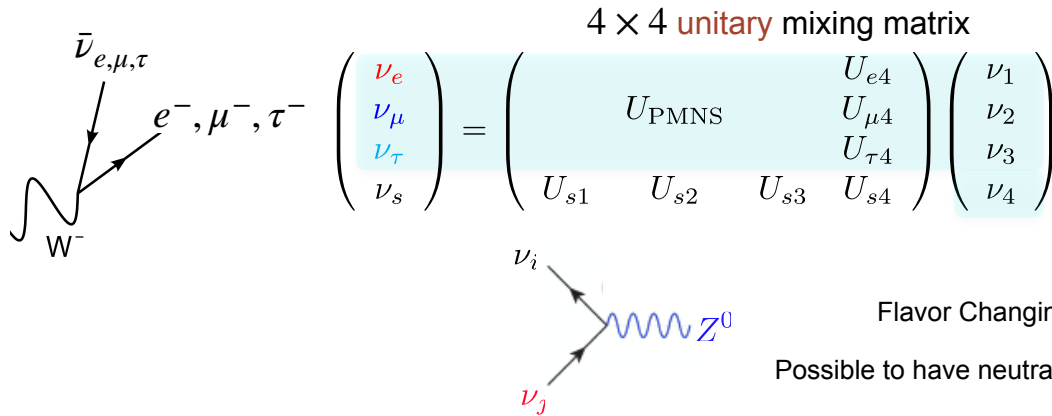
Compatible with $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations provided that $\Delta m^2 \gtrsim 0.1 \text{ eV}^2$
 MiniBooNE has the same L/E_ν as LSND, but much larger E_ν and L

Not consistent with
 3 flavor picture & Large-scale-structure

Conventional model: 3+1 model,,,, for SBL anomalies

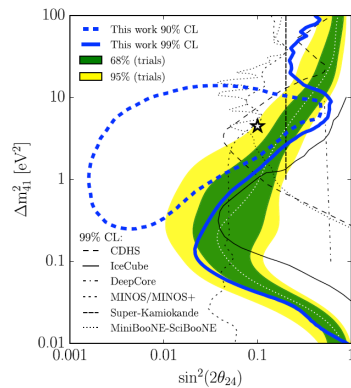
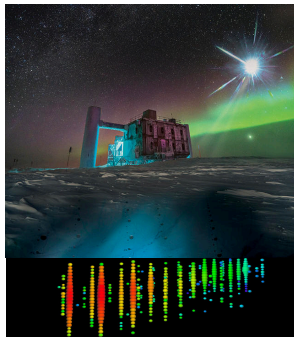
$$\Delta m_{\text{SBL}}^2 \Rightarrow \text{eV } \nu_4 \text{ mass eigenstate}$$

Mohapatra, Nasri, Yu (2005),....

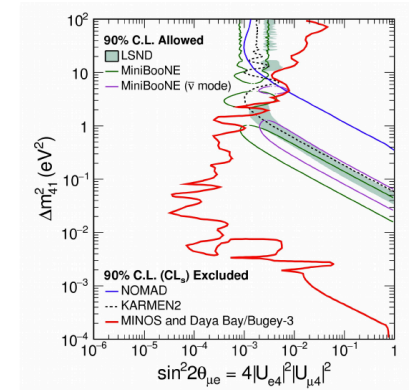
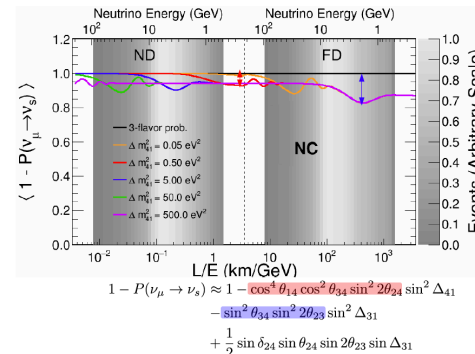


	3ν	3ν + 1 sterile
Mixing angle 3	→	6
CP-violating phase 3	→	6 for Majorana
(1	→	3 for Dirac)

IceCube test of Sterile oscillation
Resonant oscillations due to earth matter effects



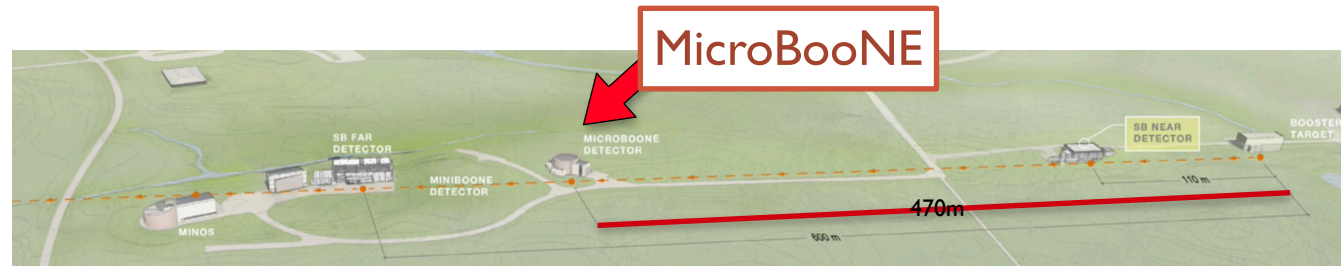
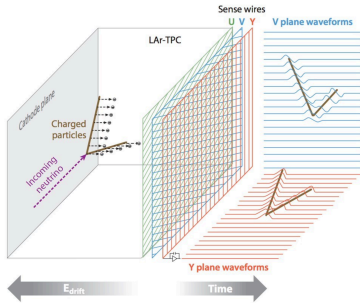
MINOS & MINOS+ (Neutrino 2018)



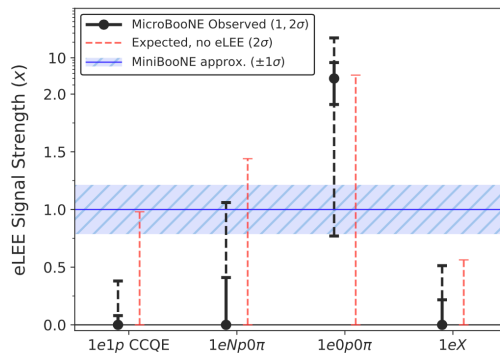
+ MiniBooNE results conflict with
the first data of MicroBooNE

Is the low energy excess at MiniBooNE an electron or photon ?.....

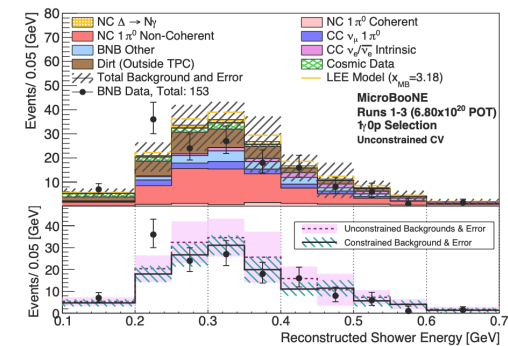
- (i) SBN oscillation at an approximate $L/E = \mathcal{O}(1)$ m/MeV
- (ii) **3D images of ν events** and particle interactions $\Leftarrow$ Liquid argon time projection chamber
- (iii) detailed particle paths and, crucially, **distinguish electrons from photons**



arXiv: 2110.14054



arXiv: 2110.00409



Δ radiative decay:

disfavoring an excess of ν interactions due to the anomalous single-photon as the explanation of the MiniBooNE energy excess

But !! no hint of sterile neutrino.....consistent with what the 3ν SM predicts

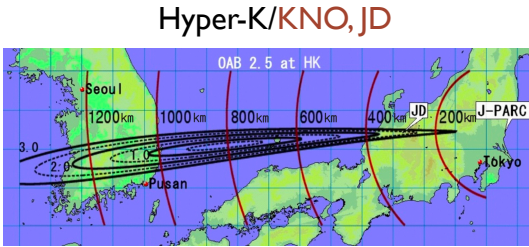
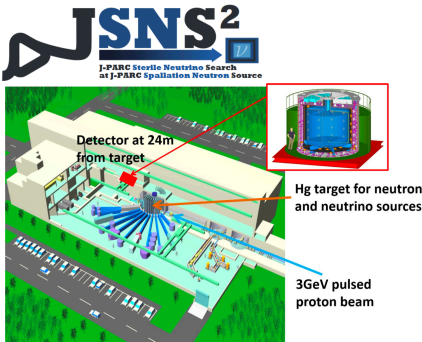
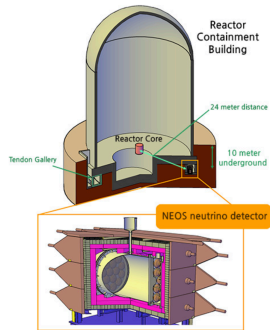
A blow to a theoretical particle 'light sterile' neutrino !!

But !!

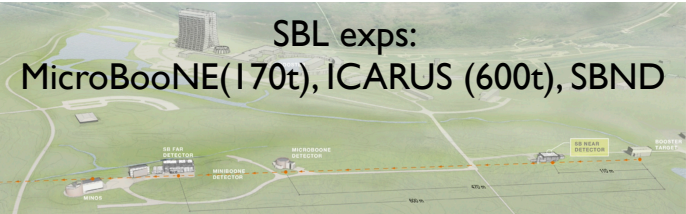
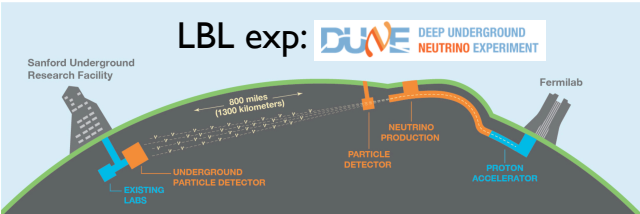
With only half the data from MicroBooNE yet released,
possible explanations still to be considered or tested in future experiments

To search for sterile neutrino signatures !!

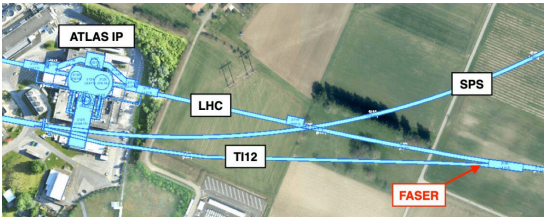
NEOS
DANSS
PROSPECT



The combination of short- and long-baseline neutrino experiments will give insights into the working of neutrinos, including sterile neutrinos



Collider neutrino
at LHC: FASER



detected neutrino candidates produced by LHC

Time for a new theory ?

One of the most basic things we don't know: how many neutrinos are there ?



The discovery of a sterile neutrino has enormous implications for how the universe evolved, how galaxies clump together, how stars explode, and why we live in a matter-dominated universe.

Sterile Neutrinos are one of the candidates of the dark matter.....

- (i) First results of MicroBooNE $\Rightarrow$ Strongly disfavors a sterile neutrino as the sole source of the anomaly observed by MiniBooNE
- (ii) MiniBooNE-like signatures $\Rightarrow$ neither electrons nor the most likely of the photons suspects

have to explain MiniBooNE data

(iii) Additional analyses and future experiments $\Rightarrow$ exotic explanations

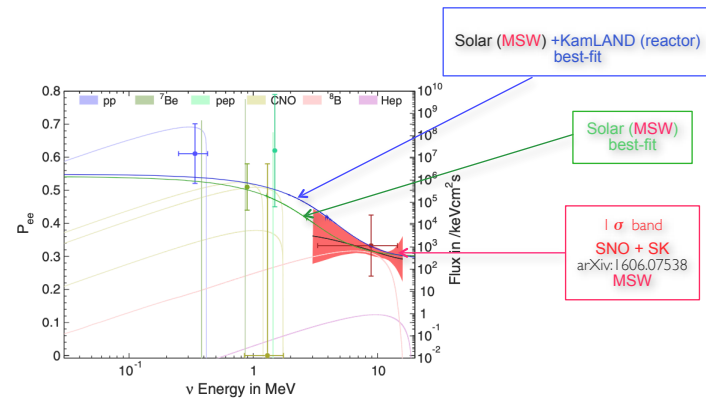
(iv) sterile neutrinos, hiding in even more unexpected ways

Theory development

ex) Sterile ν_s + flavored-axions NPB960.115210

Solar Neutrino Tension

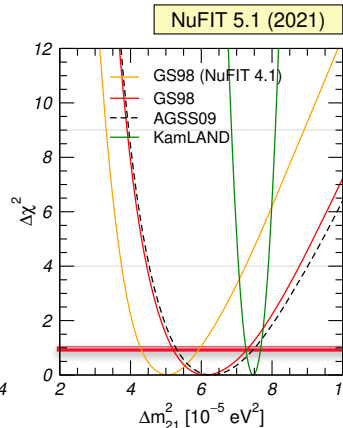
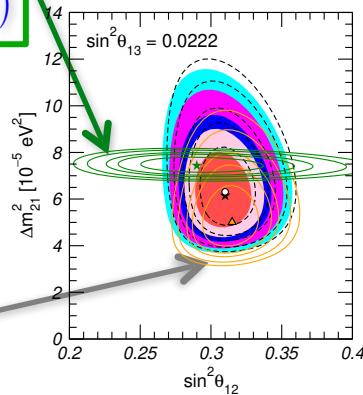
Δm_{21}^2 and $\sin^2 \theta_{12}$ in solar neutrino experiments are **not in complete agreement** with the measurements collected in other types of experiments (such as KamLAND)



Reactor KamLAND
(no MSW effect)

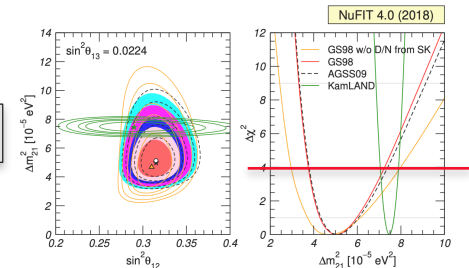
$$P_{\bar{\nu}_e \rightarrow \bar{\nu}_e} \simeq 1 - c_{13}^4 \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

Solar ν (MSW effect)
SNO + SK + Borexino



a statistical fluctuation
in the current data

$\lesssim 1\sigma$ C.L.



A clear discrepancy
for Δm_{21}^2 at $\lesssim 2\sigma$ C.L.

DUNE $\nu_\mu \rightarrow \nu_e + {}^{40}\text{Ar}, \nu_{e,\mu,\tau} + e^-$ and JUNO (reactor neutrino) fixing Δm_{21}^2 and θ_{12}

3D images for each neutrino interaction

Some unknown physical phenomena that only affect solar neutrinos may be at play to explain such discrepancy !!

A possibility: composite Matter effect

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Such discrepancy appearing in the 3ν SF can be removed by the new oscillation effect without significantly modifying the MSW-LMA solution to solar neutrino oscillations

MSW matter effect

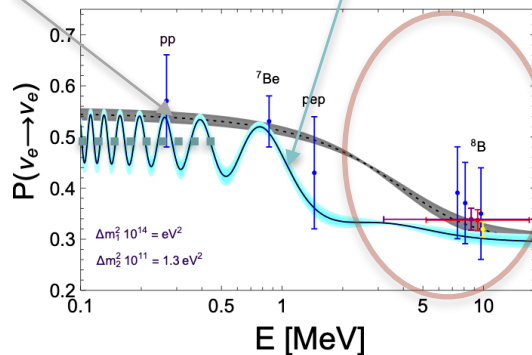
New oscillation effect induced by a sterile neutrino

$$P_{\nu_e \rightarrow \nu_e}^m \simeq \sin^2 \theta_{12} \cos^2 \left(\frac{\Delta m_2^2}{4E} L \right)$$

MSW effect in the 3ν SF

Global 1σ data is used

Composite Matter effect

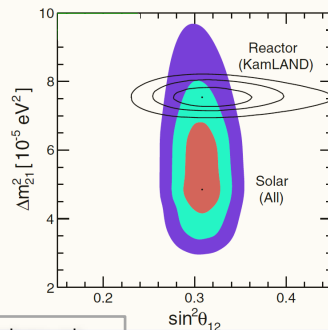


For oscillation length $L_2^{\text{osc}} = \frac{4\pi E}{\Delta m_2^2}$ optimized to L_{es} :

$$L_2^{\text{osc}} = 2.5 \times 10^8 \text{ km} \left(\frac{E}{1 \text{ MeV}} \right) \left(\frac{1.7 \times 10^{-11} \text{ eV}^2}{\Delta m_2^2} \right) \gtrsim L_{es}$$

The data from ^8B neutrino experiments can be well-fitted with that of the KamLAND $\bar{\nu}_e$

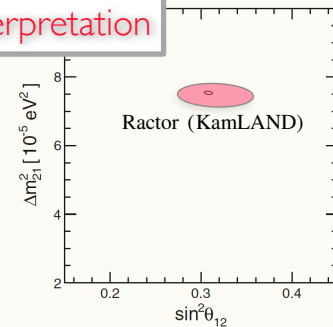
$$0.9 \times 10^{-11} \text{ eV}^2 < \Delta m_2^2 \lesssim 1.7 \times 10^{-11} \text{ eV}^2$$



Background
PRL 123.131803 (2019)
DUNE collaboration

Composite Matter effect

re-interpretation



Solar neutrino solution

A possibility: add N $SU(3) \times SU(2) \times U(1)$ singlets..... ν SM (2016 Ahn, SK Kang, CS Kim)

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N SM gauge singlets R-handed S_R (connects Weyl spinor with itself)

$$\mathcal{L}_{\text{Majorana}} = \mathcal{L}_{\text{SM}} - \bar{\ell}_L Y_\nu \tilde{H} S_R^i - \frac{1}{2} \sum_{i,j=1}^N \bar{S}_R^i M_{ij} S_R^j + \text{h.c.} \dots \equiv \mathcal{L}_{\text{SM}} - \frac{1}{2} (\bar{\nu}_L^c \quad \bar{S}_R \quad \bar{S}_R^c) \mathcal{M}_\nu \begin{pmatrix} \nu_L \\ S_R^c \\ S_R^c \end{pmatrix} + \text{h.c.}$$

higher dimensional operator with (non)-Abelian symmetry (different quantum numbers):

PRD 96 (2017) 015022

flavored-axion framework $\tilde{c}_1 \mathcal{O}_{\mathcal{P}_3}(\mathcal{F})^1 + \mathcal{O}_{\mathcal{P}_4} \sum_{n=0}^{\text{finite}} c_n \left(\frac{\mathcal{F}}{\Lambda} \right)^n$

$\nu_e, \nu_\mu, \nu_\tau, S_{\alpha 1}, S_{\alpha 2}, \dots, S_{\alpha N}, S'_{\beta 1}, S'_{\beta 2}, \dots, S'_{\beta n}$
heavy R-handed singlets S'_R

Low energy effective Neutrinos

Interaction eigenstates

$U^{3 \times 3}_{\text{PMNS}}$

FCNC

Mass eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \vdots \\ S_{\alpha N} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & \dots & U_{eN} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & \dots & U_{\mu N} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & \dots & U_{\tau N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ U_{s\alpha 1} & U_{s\alpha 2} & U_{s\alpha 3} & \dots & U_{s\alpha N} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \vdots \\ S_N \end{pmatrix}$$

$(3+N) \times (3+N)$ unitary matrix

In general,
any arbitrary subset
Not necessarily unitary

To check unitarity,
measure precisely the
 $U^{3 \times 3}_{\text{PMNS}}$ mixing matrix

$$-\mathcal{L}_{av} \supset \frac{1}{2} \bar{\nu}_L^c \mathcal{M}_\nu \nu_L + \frac{1}{2} M_S \bar{S}_R S_R^c + \frac{g}{\sqrt{2}} W_\mu^- \bar{\ell}_L \gamma^\mu \nu_L + \text{h.c.}$$

Flavored-axions

$$+\frac{1}{2} \sum_j \frac{A_j}{f_{aj}} \left\{ p_j \bar{\nu} i \gamma_5 \mathcal{M}_\nu \nu + q_j \bar{S} i \gamma_5 M_S S^c \right\} - \frac{1}{2} \bar{\nu} i \partial \nu - \frac{1}{2} \bar{S} i \partial S$$

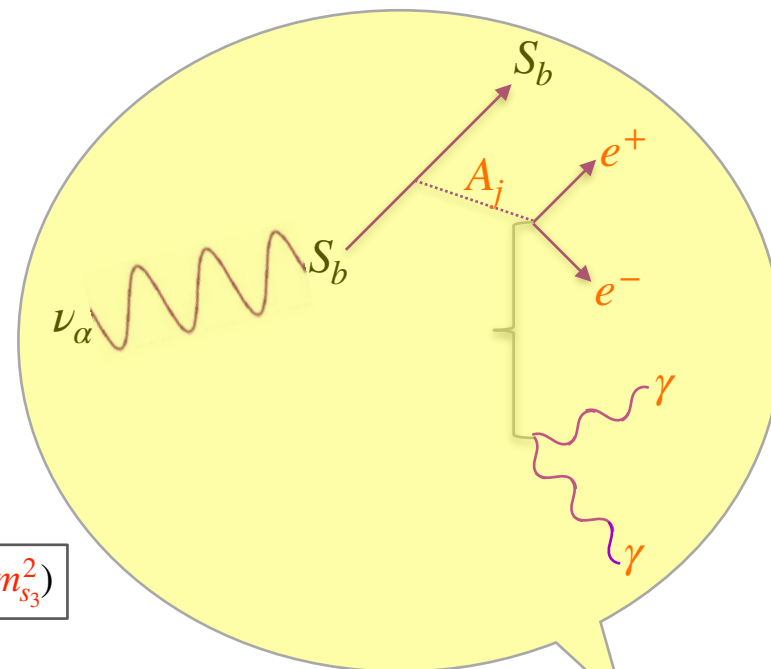
$U(1)$ charge

$$+\frac{\partial_\mu A_j}{2f_j} h_j \bar{e} \gamma^\mu \gamma_5 e + m_e \bar{e} e - \bar{e} i \gamma^\mu D_\mu e$$

$$-\frac{1}{4} g_{j\gamma\gamma} A_j F_{\mu\nu} \tilde{F}^{\mu\nu} + m_{A_j}^2 f_j^2 \left\{ 1 - \cos\left(\frac{A_j}{f_j}\right) \right\} - \frac{1}{2} (\partial_\mu A_j)^2$$



A potential to the SBL anomalies
+ MicroBooNE data
+ Solar neutrino tension



 $\nu_\alpha \rightarrow \nu_\beta$ $U_{\text{PMNS}}, \Delta m_\nu^2$ $\theta_k, \Delta m_k^2$	$\nu_\alpha \rightarrow S_b$ $U_R, U_{\text{PMNS}}, \Delta m_\nu^2, \Delta m_S^2$ $\theta_k, \Delta m_k^2$
$S_a \rightarrow \nu_\beta$	$U_R, \Delta m_S^2 \quad \theta_k, \Delta m_k^2$ $S_a \rightarrow S_b$

6X6 mixing
new parametrization

$\text{diag}(m_{\nu_1}^2, m_{\nu_2}^2, m_{\nu_3}^2, m_{s_1}^2, m_{s_2}^2, m_{s_3}^2)$

$$P_{\nu_a \rightarrow \nu_b}(W_\nu, \Delta m^2; L, E) = \left| \left(W_\nu^* e^{-i \frac{\hat{M}_\nu^2}{2E} L} W_\nu^T \right)_{ab} \right|^2 =$$

$a, b = e, \mu, \tau, s_1, s_2, s_3$

Conclusion

The upcoming high-precision neutrino oscillation experiments are expected to determine the neutrino mass ordering, mixing angles, and CP violation at high C.L. and to provide a rigorous test of the 3- ν oscillation framework at various baselines (L) and energies (E)

Results of the MicroBooNE experiment, while not confirming the existence of a sterile neutrino, provide a path forward to explore physics beyond the SM

Additional analysis, future experiments, and theory developments

MiniBooNE-like signatures $\Rightarrow$ neither electrons nor the most likely of the photons suspects

have to explain MiniBooNE data