### Introduction to Machine Learning for Particle Physics

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- Overview of Machine Learning
- Decision Trees
- Neural Networks
- Example ML analysis using TMVA with CMS data Quark/Gluon Separation
- Obviously, each of these topics could be expanded to several hours of lectures, today is really a brief overview to give a taste of what's possible!

- Machine learning refers to sets of algorithms (techniques) that can "learn" from experience
- Given inputs and expected output can automatically learn to associate patterns in the input to the output and generalize on unseen inputs.
  - Outputs could be (a) real number(s): regression (photon energy)
  - Or it could be a *classification* into one of several classes ( $\gamma \vee e^- \vee \mu^-$ )
- As opposed to traditional algorithms which are explicitly pre-programmed to always act in specific ways
- Example: Does this signal in my detector correspond to a photon or a hadron? Feed the algorithm thousands of (simulated) photons & hadrons, and it will learn to distinguish them
  - More specifically, we will build a parameterized model which gives a "probability"\* for an input datapoint to be photon or hadron, and the algorithms changes the parameters to better classify the photons or hadron examples you feed it

\* Typically, its not actually a probability, but easier to think of it that way

# Classification

- Lets consider classification problems
  - In classification we have some data which could belong to one of several classes
  - We have some well-known data and we want to train a *classifier* which will tell us what class some new, unknown data comes from
  - E.g. classify images of pets into dogs and cats
  - Classify energy depositions in a calo into photons and electrons
  - Based on several indicators (age, height, weight, etc.) say whether someone will get diabetes
- Decisions can be complicated: many input variables with many non-trivial relationships differing by class, modelling this in general is called Multi-Variate Analysis (MVA)
- The goal is to find a *decision boundary*, one one side of the boundary we classify the input as a  $\gamma$ , on the other its a hadron
- We will start with a straightforward technique called Decision Trees



• Decision trees give a path to a result based on some conditions



- Decision trees give a path to a result based on some conditions
- There could be several inputs, with multiple kinds of outputs
  - But always evaluate from top node down
- For true/false boolean inputs, straightforward to enumerate all options
- Write down all the paths through the questions, add a label at the end



- In the case of real valued inputs, we have to be more careful
- We can create left/right branches by asking for a value to be above/below some cut-off
  - We turn a real value variable into a binary decision at each node



• Given a set of data we want to split into red and blue spaces

### Decision Trees with Real Numbers



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# Training Decision Trees: CART algorithm, GINI coefficient

- To train the decision tree, scikit learn (and TMVA?) uses the CART algorithm to minimize the Gini coefficient *G*
- $G = 1 \sum_{k} p_{k}^{2}$  summed over the classification classes k, where  $p_{k}$  is the fraction of data in class k
  - If all the data is in one class then G = 0, if the data is split evenly over n classes then  $G = \frac{n-1}{n}$
  - If the data is more unevenly split, the G value goes closer to 0
- The CART algorithm takes a dataset of *m* datapoints and tries to make a split into two branches left and right, which minimize

$$G = rac{m_{left}}{m}G_{left} + rac{m_{right}}{m}G_{right}$$

- Its trying to minimize the number-of-datapoints-weighted-average Gini-coefficient of the left-right split at each *node*
- After the split, it then applies the same logic again onto each subtree until
  - A stopping parameter condition is met, i.e. the user can say only go to a certain *depth*
  - Each leaf (split dataset) only contains one class
  - It can't find a split which reduces the Gini-coefficient
- A *leaf* is a final decision, i.e. a node with no more splits

### Some Data



 You can see its overfitting, producing high variance changes in decisions based on the exact data, which probably won't generalize to unseen data

- Some synthetic data (i.e. created artificially) we will use to illustrate today's concepts
  - The circles are the datapoints, labelled by colors
- Has 3 categories with 2 variables, lots of overlaps
- Running a decision tree over the data gives a complicated decision boundary
  - The background colors show the decision for each point after training the tree

## Training and Testing



- Without a priori knowledge of the data distribution, we need some way to test if we are overfitting, using the data itself
- We can do this by setting aside some portion of the data, the testing set, and only train on the remaining data, the training set
- If we take 80% of the previous page data and fit the tree, overlaying the output shows clear overtraining (e.g. blue in cut out blocks of green)

### Bias vs Variance



- We can see this particularly clearly by plotting accuracy (fraction of data classified correctly) against the depth of the tree (= complexity)
- As we increase the max depth, the training set gets more accurate, the test set diverges, and becomes less accurate, this is overfitting
   Biss, the difference between our model prediction and the detencints
- Bias: the difference between our model prediction and the datapoints
  - A high bias model has large differences between the datapoints
  - A model which doesn't have enough parameters underfits the data
- Variance, a measure of the fluctuations of the data or model, high variance models are typically fitting the intrinsic noise of the data
  - A high variance model with too many parameters *overfits* the data
- The *bias-variance trade off* is a theorem which tells us that you trade off model bias for variance and vice-versa, best to find a trade-off point between the two regions

### Boosting

- In order to reach that point, we need a *high capacity* model, one which has enough freedom to model our data
- We can use *ensemble classifiers* to increase performance
- In Boosting, we start with a weak classifier (barely better than random chance), and put them together to form a strong classifier
- This is done by weighting the data for each classifier we train
- E.g. start with a depth 1 decision tree, we can weight the misclassified data higher, and the correctly classified data lower
- Train a depth 1 tree on the reweighted data, this gives a different tree, since the gini index will be calculated based on the *weights*, instead of just taking the number of entries in each bucket
- This scheme is AdaBoost, there are also other variations, such as Gradient boosting, but in all the idea is to take a weak classifier and train up an ensemble of strong classifiers

### Boosting



- Left shows a regression task, but the idea is the same, the further the curve is from a point, the higher its weighted in the next tree
- How quickly the weights change are controlled by the learning rate
- After training, the output is taking as a weighted average of the trees, the weight of each tree proportional to the number of correctly classified training datapoints it produced
  - Also produces a smoother decision boundary (average out fluctuations)
- See e.g. chapter 7 of "Hands-On Machine Learning with Scikit-Learn, Keras, and TensorFlow, 2nd Edition" for more details

## Boosting Decision Trees



- When we boost the decision tree learning, we end up with a Boosted Decision Tree (BDT), which really is a *forest* of decision trees
- Various number of depth 1 decision trees trained with AdaBoost on the previously shown dataset
- If the learning rate is too high, the boosting overcorrects too quickly, and we don't get good testing results
- With a lower learning rate, the boosting helps us get a more accurate classifier without overtraining



• The logistic function is defined as  $f(x) = \frac{1}{1+e^{-x}}$ 

- Looks like a classic "turn-on" curve
- "Logistic regression" fits this function from several variables
- Concentrate on the case of two classes (cat/dog or electron/photon), and ask what we want from a classifier output
  - We need to distinguish between the two classes using the output:
  - If the value is 0, it represents the classifier identifying one class (cat)
  - If its near 1, the classifier is identifies the other class (dog)
  - Thus, we need to transform the input variables into 1D, then pass through the logistic function

### Mathematics of Logistic regression

- Setup: we have data from two different classes, which can be described by the same independent variables, and we want to distinguish them based on those independent variables
- We want to build a function such that data from one class goes close to 1, from the other close to 0
- We will build a linear function of the variables, then pass it through the logistic function, and try to minimise the distance of data from 0 (for one class), or 1 (for the other)
- $y_i = f(\vec{\beta} \cdot \vec{x_i}) + \epsilon_i$ ,  $y_i = 0$  if  $x_i$  from class 0, 1 if  $x_i$  from class 1 •  $\vec{\beta} \cdot \vec{x_i} = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k$  and  $f(x) = \frac{1}{1 + e^{-x}}$  the logistic function
- Find  $\beta$  which minimizes a loss function e.g.:  $MSE = \frac{1}{m} \sum_{i} (f(x_i) - y_i)^2$
- We end with the optimized parameters for the intercept and coefficients
  - The intercept sets where the turn occurs
  - Coefficients set how *quickly* the turn on occurs, larger coeff. imply fast turn on (sweep across the logistic function quicker)

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## Illustration: 1D Projection



- $\vec{\beta} \cdot \vec{x}$  is a projection of the data onto a line
- Red and blue are two classes which can be measured in  $(x_1, x_2)$
- We can take the mean of each class (left), form a line between, then project the data onto the line (middle) giving a distribution (right)
  - We have reduced the 2D data into a 1D projection
- After the projection, the logistic rejection chooses a cut point (via  $\beta_0$ ) then sends things below the cut to 0, above to 1
- Here, we see some separation between the classes but a lot of overlap. We can do better

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### Illustration: Better Fit



- Finding the *Fisher discriminant* for our illustrative dataset shows that these two classes are fully separable
- The Logistic Regression will place the cut point between the data and so all red go to 0, blue go to 1 after passing through the logistic function

 ${\tt https://medium.freecodecamp.org/an-illustrative-introduction-to-fishers-linear-discriminant-9484 efee 15 according to the state of the state of$ 

## Some very simple examples for simple logistic regression



- Let's think about using logistic regression to approximate some simple binary functions, i.e. data in 2D, output red or blue, logistic regression will turn on at a boundary line
- OR and AND gates
  - OR is 0 (red) if both input are 0, 1 (blue) otherwise
  - AND is 1 if both inputs are 1, 0 otherwise
- Can we find logistic function approximations for this?
  - That is,  $f(x_1, x_2)$  returns approximately 1 or 0 at the indicated points

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- Can we find logistic function approximations for this?
  - That is,  $f(x_1, x_2)$  returns approximately 1 or 0 at the indicated points
- Yes! Take the projection perpendicular to the line
- and have the logistic turn on at the line

• e.g. 
$$f(x_1, x_2) = \sigma(2x_1 + 2x_2 - 1)$$
 for OR,  
 $f(x_1, x_2) = \sigma(2x_1 + 2x_2 - 3)$  for AND [ $\sigma$  is our logistic function]

## Very simple example with issues for Logistic Regression



- Now consider the XOR gate: 1 if both inputs are the same, 0 otherwise
- The XOR gate can't be generated with a logistic function!
- Try it: no matter what line you draw, can't draw a logistic function that turns on only the blue!

### How to Fix: more logistic curves!



- Can fix by having 2 turn-on curves, one turning on either of the blue points, then summing the result
- $f(x_1, x_2) = \sigma(2x_1 + 2x_2 1) + \sigma(-2x_1 2x_2 + 1)$
- In general, this type of issue (complicated decision boundaries) is why we reach toward multivariate methods such as decision trees and neural networks

# The Feed-Forward Neural Network



• Consider the structure of what we just made

• 
$$y = f(x_1, x_2) = \sigma(-1 + 2x_1 + 2x_2) + \sigma(1 - 2x_1 - 2x_2)$$

#### • Decompose the function into:

- the input layer of  $\hat{x}$ ,
- the hidden layer which calculates  $h_i = \beta_i \cdot x$  then passes if through the activation function  $\sigma$ , ("logistic function" called "sigmoid" in NN)
  - as in logistic, there is an extra  $\beta_0$ , called the *bias*, which controls how big the input into the node must be to activate
- the *output layer* which sums the results of the hidden layer and gives y
  y = 0 + 1 · σ(h<sub>1</sub>) + 1 · σ(h<sub>2</sub>)

## Feed-Forward Neural Network



- In general, we could have several input variables, and output variables
  In the case of classification, we would usually have a final *softmax*
  - applied to  $\hat{y}$ , but could use any *activation*  $\varphi$  here also
    - *softmax* normalizes the output layer, so the outputs add to 1
    - The output layer then acts like a probability for each category

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## Feed-Forward Neural Network



- We can even have several hidden layers
  - The previous layer acts the same as an input layer to the next layer
  - The layers can build up more complex features to discriminate with
- We call each node in the network a neuron

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## Aside: Universal Approximation Thereom

Let  $\varphi : \mathbb{R} \to \mathbb{R}$  be a nonconstant, bounded, and continuous function. Let  $I_m$  denote the *m*-dimensional unit hypercube  $[0, 1]^m$ . The space of real-valued continuous functions on  $I_m$  is denoted by  $C(I_m)$ . Then, given any  $\varepsilon > 0$  and any function  $f \in C(I_m)$ , there exist an integer N, real constants  $v_i, b_i \in \mathbb{R}$  and real vectors  $w_i \in \mathbb{R}^m$  for  $i = 1, \ldots, N$  such that we may define:

$$F(x) = \sum_{i=1}^{N} v_i \varphi\left(w_i^T x + b_i\right)$$

as an approximate realization of the function f; that is,

$$|F(x) - f(x)| < \varepsilon$$

for all  $x \in I_m$ . In other words, functions of the form F(x) are dense in  $C(I_m)$ . This still holds when replacing  $I_m$  with any compact subset of  $\mathbb{R}^m$ .

- In brief: with a hidden layer (of enough nodes), any (sensible) function  $f: \mathbb{R}^m \to \mathbb{R}$  can be approximated by a feed-forward NN
  - Any (sensible) activation  $\varphi$  can work, not just  $\sigma$
- Shows we won't run into the XOR issue with a neural network

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## Training a Neural Network





- What does it mean to train a neural network?
- Consider the XOR network
- There we set by hand, but could try to "train" the network
- Start with random weights and biases, reduce the loss function  $C(x, y|w, b) = \sum_{i} |y_{i}^{true} y(x_{i})|^{2}$  where *i* ranges over our 4 samples  $(x_{i}, y_{i})$  and  $y(x_{i})$  is the network output
  - Start with random weights so that different random features can be extracted by different nodes
  - As before, we train by trying to minimize the mean-squared error
  - There exists an efficient algorithm for doing this with neural networks called *backpropagation* (shown in backup)

# Quark-Gluon Jet Classification



- Quark and gluon jet production has subtly different properties
  - Gluon jets are more radiative so tend to be wider, produce more and softer particles, should tend to  $C_A/C_F = 9/4$  at high  $p_T$
- Proposed uses for discirmination in new physics studies, where more high-energy quark jets are expected than gluon jets
- At CMS, BDTs have been developed for q-g discrimination using variables constructed from PF inputs

• Number of jet consitutents,  $p_T D = \frac{\sqrt{\sum_i p_{T,i}^2}}{\sum_i p_{T,i}}$ , jet ellipse axis lengths

"Quark-gluon Jet Discrimination At CMS", Cornelis for CMS (arxiv:1409.3072)

# Analysis of QG jets from CMS opendata

- We will use some simulated data from the <u>CERN Open Data Portal</u> to try out some of the techniques
  - Already online, so easy to download and analyse
  - But, was obtained using similar techniques to what Prof. Lee taught last week (analysing jets with CMSSW): producing ntuples from the MiniAOD output
- We can use SWAN to setup an environment and access the data
  - You could also work locally in the CMSSW environment you set up last time, but you will not be able to stream the data as I will be showing, you will have to download the file:

http://opendata.cern.ch/record/12100/files/assets/cms/ datascience/JetNtupleProducerTool/JetNTuple\_QCD\_RunII\_13TeV\_ MC/JetNtuple\_RunIISummer16\_13TeV\_MC\_1.root

- SWAN is the "Service for Web based ANalysis", which allows you (if you have a CERN account!) to easily run analyses on CERN's infrastructure, using a jupyter-based environment
  - https://swan.web.cern.ch

• Lets load up swan, create a directory for our analysis, open a notebook and try to load up the file:

For step by step instructions on starting swan, see p21. onward of: https://indico.lip.pt/event/410/contributions/1043/attachments/1002/1149/Lisbon\_TMVA\_Tutorial.pdf

For more information on the various options, see the user guide:

https://root.cern.ch/download/doc/tmva/TMVAUsersGuide.pdf

# Basic plots

- We have a tree with one entry per jet
- We can find light quark (gluon) jets using isPhysUDS (isPhysG)
- Lets draw a basic variable, the  $p_T$  of the jets of the two different categories using the builtin TTree::Draw capabilities

```
tree.SetLineWidth(2)
tree.SetLineColor(rt.kBlue)
tree.Draw("jetPt>>gpt", "isPhysG", "")
tree.SetLineColor(rt.kRed)
tree.Draw("jetPt>>gpt", "isPhysUDS", "same")
rt.gpt.SetTitle(";p_{T} [GeV];# of Jets")
l=rt.TLegend(.6,.7,.9,.9)
l.AddEntry(rt.gpt, "Gluon")
l.AddEntry(rt.qpt, "UDS")
l.Draw()
cvs.Draw() # needed to display the output in Jupyter; don't need this on the command line
```

- The pt spectra are slightly different, we'd need to consider this in a real analysis
- Do the same for the qg separation variables:
  - QG\_mult the multiplicity
  - QG\_axis2 the size of the minor axis
  - QG\_ptD the ptD variable

### Prepare the Data for TMVA

- The Toolkit for MultiVariate Analysis (TMVA) is a library built into ROOT for doing machine learning
- We need to start by initializing TMVA and building a Factory which holds our analysis, and loading the data with a DataLoader

```
# we can use this file later to analyse the results
outputFile = rt.TFile.Open('TMVA output.root', 'recreate')
# we give it a name which it uses in the output, the outputfile, and
# some options (for instance, remove ! in front of Silent to suppress
# output)
factory = rt.TMVA.Factory('TMVAClassification', outputFile,
                          '!V:!Silent:Color:!DrawProgressBar:AnalysisType=Classification')
# create a dataloader and tell it the tree it sould use for signal and background
loader = rt.TMVA.DataLoader('dataset')
# in our case, the same tree holds signal and background, we will tell
# it later how to select the actual signal and background events we
# could also optionally add weights if we had several trees for
# e.a. different background processes
loader.AddSignalTree(tree)
loader.AddBackgroundTree(tree)
# now we define the variables to be used in the analysis, we can also
# give it a name for displaying nicely
loader.AddVariable('QG mult', "multiplicity", "")
loader.AddVariable('QG_axis2', "#sigma_{2}", "")
loader.AddVariable('QG_ptD', "p_{T}D", "")
# finally tell it how to read signal and background and prepare the test/train
```

## Train with TMVA

- Now we can "book" the methods we want TMVA to run, then train them, test them and evaluate them
- We can run as many as we want. Lets do a BDT and a Neural Network, see the user guide for more options
  - In the NN, the HiddenLayers takes a comma separated list of the number of neurons in each layer. N is the number of input variables

```
outputFile.Close()
```

- A good overtraining check is to look at the output distribution for signal and background, comparing testing and training
- A well-trained model should have those distributions match each other
- An overtrained model will have the test distribution performing worse than the training

```
# save pdfs instead of png
rt.TMVA.Config.Instance().fVariablePlotting.fPlotFormat = \
    rt.TMVA.Config.Instance().fVariablePlotting.kPDF
TMVA.mvas("TMVA_output.root", rt.TMVA.kCompareType)
```

```
# TMVA will make a canvas for each machine learning method
# check which is which
[c.GetTitle() for c in rt.gROOT.GetListOfCanvases()]
```

```
# and draw the one of interest
rt.gROOT.GetListOfCanvases().At(0).Draw()
```

## **ROC Curves**

- The Receiver Operating Characteristic (ROC) curve was defined during WW2 for displaying the abilities of radar receiver operators
- Gives the true positive rate (correctly identified signal) versus false positive rate (background incorrectly identified as signal) [TMVA shows 1–FPR]
- The area under the curve (AUC) is often used to summarize a classifier's performance
  - 0.5: completely random classifier
  - 1.0: perfect classifier



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• Note, TMVA puts TPR on the x-axis, and shows 1-FPR on the y-axis, so we want to have the classifier move to the top right, not top left

```
# we can do it through a factory method
cvs = factory.GetROCCurve(loader)
cvs.Draw()
bdt_auc = factory.GetROCIntegral(loader, "BDT")
mlp_auc = factory.GetROCIntegral(loader, "MLP")
print(f"AUCs: BDT {bdt_auc:.3f}, MLP {mlp_auc:.3f}")
# or similar to the way we did the output distributions, using the output file
rt.TMVA.efficiencies("dataset", "TMVA_output.root")
[c.GetTitle() for c in rt.gROOT.GetListOfCanvases()]
rt.c.Draw()
```

• Note that on the command line, we can run rt.TMVA.TMVAGui("outputFile.root") (or TMVA::TMVAGui("outputFile.root") in a root command line) and see a GUI with options to display various results, and some of the GUI options (mvaeffs) haven't been ported to be usable in the notebook (as far as I can tell), and they say they will modernize so sometime in the future all the plots should be drawable with the factory

## TMVA Reader

- When you are happy with your training, you will want to apply it to the data
- To do this, we save out the trained MVA and load it with a Reader
- In python we need to load our variables into an array, so it can interface with the C++ code (which uses pointers)
- The weights are saved after your training in <dataset name>/weights/<factory name>\_<method>

```
from array import array
reader = rt.TMVA.Reader("!Color:!Silent:!V")
mult = array('f', [0.1)
axis2 = array('f', [0.1)
ptD = array('f', [0.1)
reader.AddVariable("QG_mult", mult)
reader.AddVariable("QG_axis2", axis2)
reader.AddVariable("QG_ptD", ptD)
reader.BookMVA("BDT", "dataset/weights/TMVAClassification_BDT.weights.xml")
# set your variables and evaluate (you would do this in an event loop)
ptD[0] = 0.8; mult[0] = 5; axis2[0] = 0.03
reader.EvaluateMVA("BDT")
```

Lets take the second sample in the public data, and run our analyser, saving the output for quark v gluon (of course in real data we wouldn't have that tag...).

```
fname = ("root://eospublic.cern.ch//eos/opendata/cms/datascience/JetNtupleProducerTool/" +
         "JetNTuple_QCD_RunII_13TeV_MC/JetNtuple_RunIISummer16_13TeV_MC_2.root")
f = rt.TFile.Open(fname)
cvs = rt.TCanvas()
hq = rt.TH1F("quark", ";BDT output;Arb. Units", 100, -0.6, 0.6)
hg = rt.TH1F("gluon", ";BDT output;Arb. Units", 100, -0.6, 0.6)
for jet in f.AK4jets.jetTree:
    if not jet.isPhysUDS and not jet.isPhysG: continue
    ptD[0] = jet.QG_ptD
    mult[0] = jet.QG_mult
    axis2[0] = jet.QG axis2
    bdt = reader.EvaluateMVA("BDT")
    if jet.isPhysUDS: hq.Fill(bdt)
    if jet.isPhysG: hg.Fill(bdt)
# DrawNormalized divides each histogram by the total number of entries
# before plotting, so we can see the shape of the distribution and
# ignore the difference in number of light quark v gluon jets
hg.DrawNormalized()
hq.DrawNormalized('same')
cvs.Draw()
```

### Exercises

- Draw all the input variable distributions
- Try changing the parameters of the ML methods, can you improve the performance?
  - For the BDT you could try changing the learning rate of AdaBoost (called AdaBoostBeta by TMVA) and the number of trees
  - For the neural network you could try changing the size and number of hidden layers
- Look through the TMVA tutorials for more examples of how to use TMVA
  - Note, that some of what I showed today is a bit of an older style, which is slowly being replaced. The current version of the tutorials use RDataFrame and RReader for instance, you should look into these, since it will be the future, but most of the material online will be closer to what I showed today (though a lot will be in C++!)
  - There are also options to integrate TMVAs data loading and evaluating facilities with modern libraries like XGBoost for BDTs, Tensorflow for Deep Learning, and so on

# Backup

## Analogy: Steepest descent

Question: How do we actually train these networks?



- A climber is trying to find his way down a mountain in deep fog, how should he proceed?
- One idea is to try to always go downhill the fastest way possible
- So, he figures out which direction has the steepest descent (ie which way is downhill), then takes a step in that direction
- After the step, he checks again, and takes another step
- He keeps proceeding in this manner until he cant go downhill anymore, he's reached the bottom

### Gradient Descent

- From calculus,  $\nabla f(x)$  gives the direction of largest increase of f at x (if its 0, we are at a minimum and done)
- Equivalently,  $-\nabla f(\mathbf{x})$  gives direction of largest decrease, so  $f(\mathbf{x} \gamma \nabla f(\mathbf{x})) < f(\mathbf{x})$  (at least, for some  $\gamma$  small enough)
- We will define a sequence x<sub>i</sub> to find the minimum:
  - Start with some random position  $x_0$
  - Iterate:
    - Find  $x_{n+1} = x_n \gamma_n \nabla f(x_n)$
    - Stop if  $|f({\bf x}_{n+1})-f({\bf x}_n)|<\epsilon$ , i.e. we're not reducing further, so we're close to the minimum
  - Return the final x<sub>n</sub>
- γ<sub>n</sub> can be different for each iteration, extensions to GD keep track of how quickly parameters are changing to update γ also
- $\epsilon$  is the *tolerance*, how close to a minima do we need to be before stopping (again, there are various criteria we could choose here)

## Example function



- Lines are contours of equal value
- Shows how the algorithm picks out different paths depending on starting point

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## Training Neural Networks: Backpropagation

- The algorithm to train neural networks is called backpropagation
- Its essentially a gradient descent implemented taking the network structure into account to speed up evaluation of the partials
- To apply gradient descent, we need a function to minimize, this is our loss function from earlier
  - $L(x_i; \theta) = \sum_i |f(x_i; \theta) y_i|^2$  for inputs  $x_i$  with known output  $y_i$
- We start with the parameters  $\theta$  set to arbitrary values, usually picked from e.g. the unit gaussian
- We run a forward pass through the network and calculate the loss, keeping track of the values at the intermediate nodes
- Using the chain rule, calculate the derivates *for all weights* backward from the loss to the higher layers to the inputs, in a single pass
- Propagate changes based on the gradient  $\Delta \theta_i = -\eta \frac{\partial L}{\partial \theta_i}$
- For more on how backpropagation works: http://neuralnetworksanddeeplearning.com/chap2.html