

# Particle accelerators

October 19, 2021 and Sehwook Lee

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- If you have any questions on particle detectors, don't hesitate to send me email
- References
  - Calorimetry: Energy Measurement in Particle Physics (2nd Edition), Richard Wigmans, Oxford University Press
  - Calorimetry for Collider Physics, an Introduction, Richard Wigmans, Springer
  - Particle Physics Experiment at High Energy Colliders, John Hauptman, Wiley-VCH
  - Classical Electrodynamics, John David Jackson, Wiley
  - The Classical Theory of Fields, L.D. Landau
  - Classical Electromagnetic Radiation, M.A. Healed and J.B. Marion
  - Principles of Optics, Max Born

**All text and figures in slides are from prof.  
Richard Wigmans' note for Nuclear and  
Particle Physics lectures offered at TTU in  
Spring 2013**

# Introduction

- After the pioneering work by Rutherford revealed the existence of the atomic nucleus, it became clear that scattering experiments with higher energy probes could provide invaluable insight into the structure of that nucleus
- The more energy a particle had, the more deeply it could probe within the nucleus
- The details one may hope to reveal in a scattering experiment are of the order of the wavelength of the scattered particles

$$\lambda = \frac{h}{p} = \frac{hc}{cp}$$

- The momentum of Rutherford  $\alpha$  particles, which had a kinetic energy of 4.8 MeV,  $hc = 1240 \text{ MeV}\cdot\text{fm}$

$$cp = \sqrt{E_{\alpha}^2 - m_{\alpha}^2} = \sqrt{3730.8^2 - 3726^2} = 190 \text{ MeV}$$

- de Broglie wavelength was  $\sim 7 \text{ fm}$ , which happens to be the radius of a nucleus in their gold foil target
- Applications of particle accelerators
  - the study of the subatomic structure of matter
  - hospitals: diagnostic purposes, treating tumors
  - the electronics industry: etch sub-nm structures on to silicon wafers
  - determine the composition of samples at the ppb level
  - converting long-lived radioactive waste into stable or shorter-lived nuclides
  - serving condensed matter research community (DESY, SLAC, Cornell, ...)

# Cockcroft-Walton

## The beginning: Electrostatic accelerators

- Cockroft and Walton voltage multiplier
- The voltage achievable is about 0.5 - 1 million volts
- It is limited by electrical discharge in the air between anode and cathode
- The kinetic energy gained by the ions or electrons is equal to  $q\Delta V$
- A lithium target with 400 keV protons was bombarded and the following reaction was observed
$${}^1_1\text{H} + {}^7_3\text{Li} \rightarrow {}^4_2\text{He} + {}^4_2\text{He}$$
- In 1951, the physics Nobel prize for this pioneering work

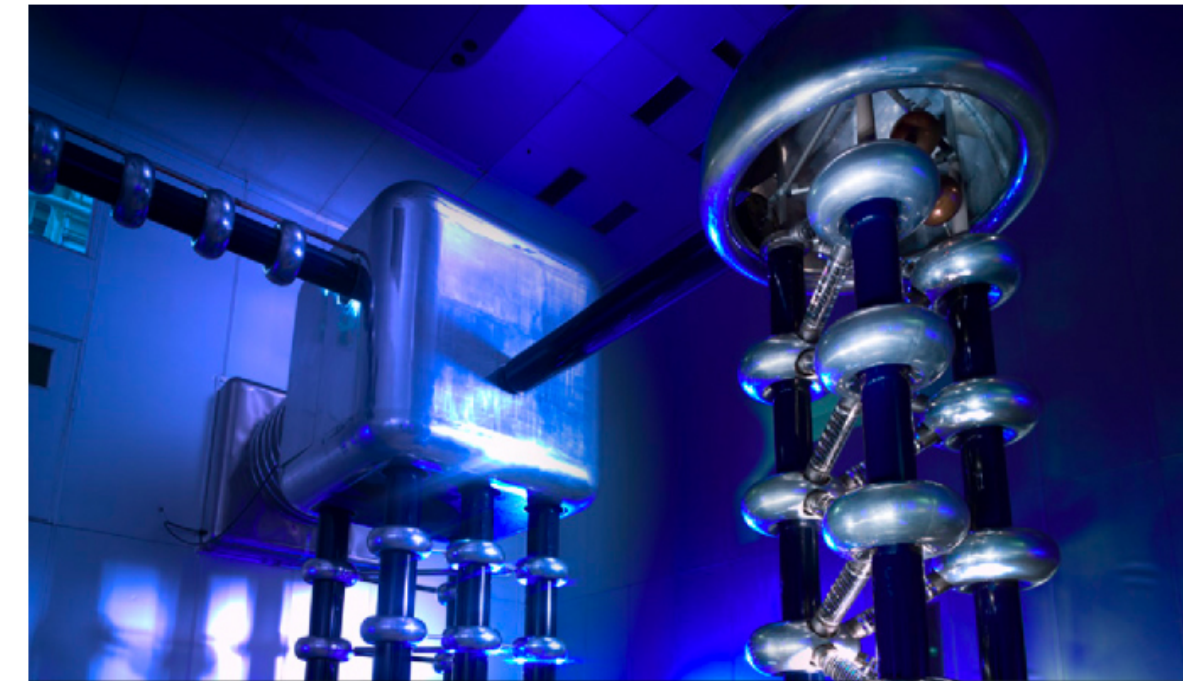
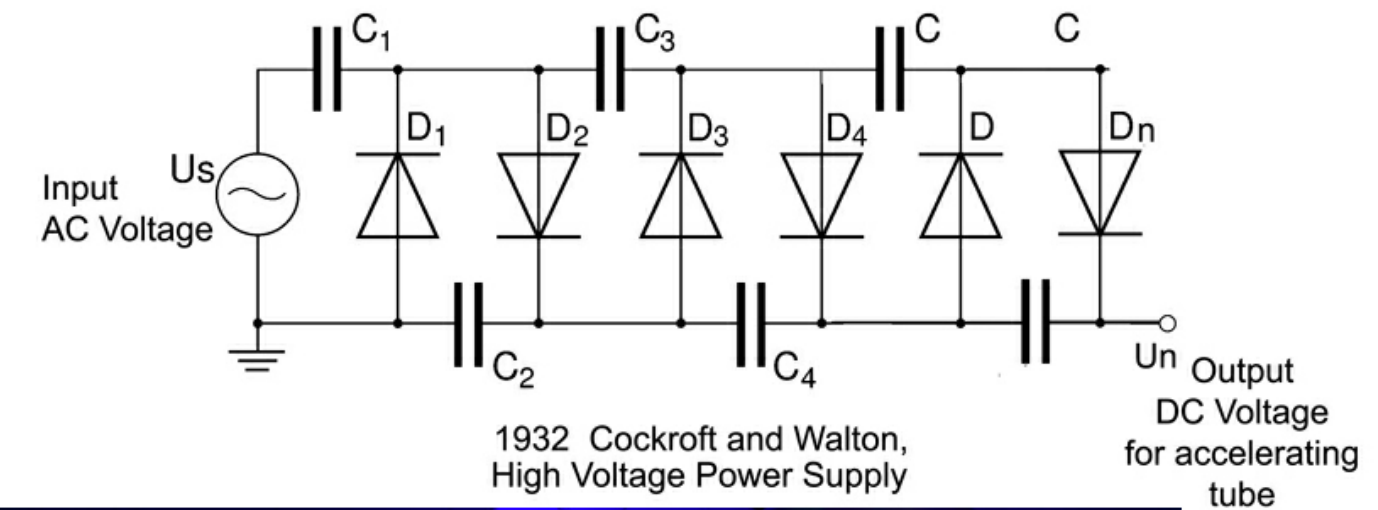


Figure 10.1: The Cockcroft-Walton generator that forms part of the accelerator complex at Fermilab, and the principle of a Cockcroft-Walton voltage multiplier.



# Vandegraaff Accelerator

## The beginning: Electrostatic accelerators

- Robert VandeGraaff used the principle that electric charge resides on the outermost surface of a conducting sphere
- In a VandeGraaff accelerator, electric charge is carried on a conveyor belt into a large metallic dome, the dome may be charged up to a voltage as high as 12 MV
- The ions are accelerated from the ion source to the target
- The entire apparatus is usually enclosed in a pressurized vessel that contains an inert gas, in order to avoid any discharges to the surrounding environment

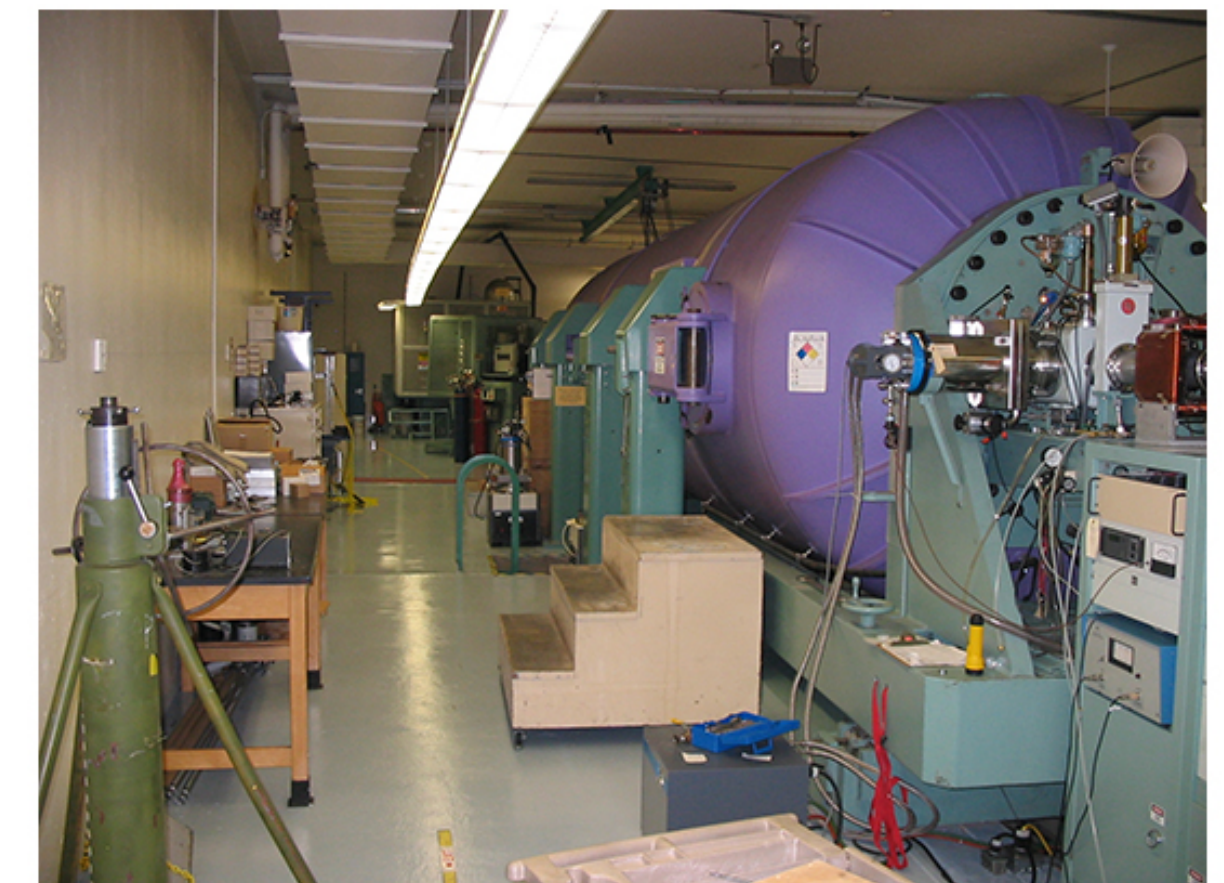
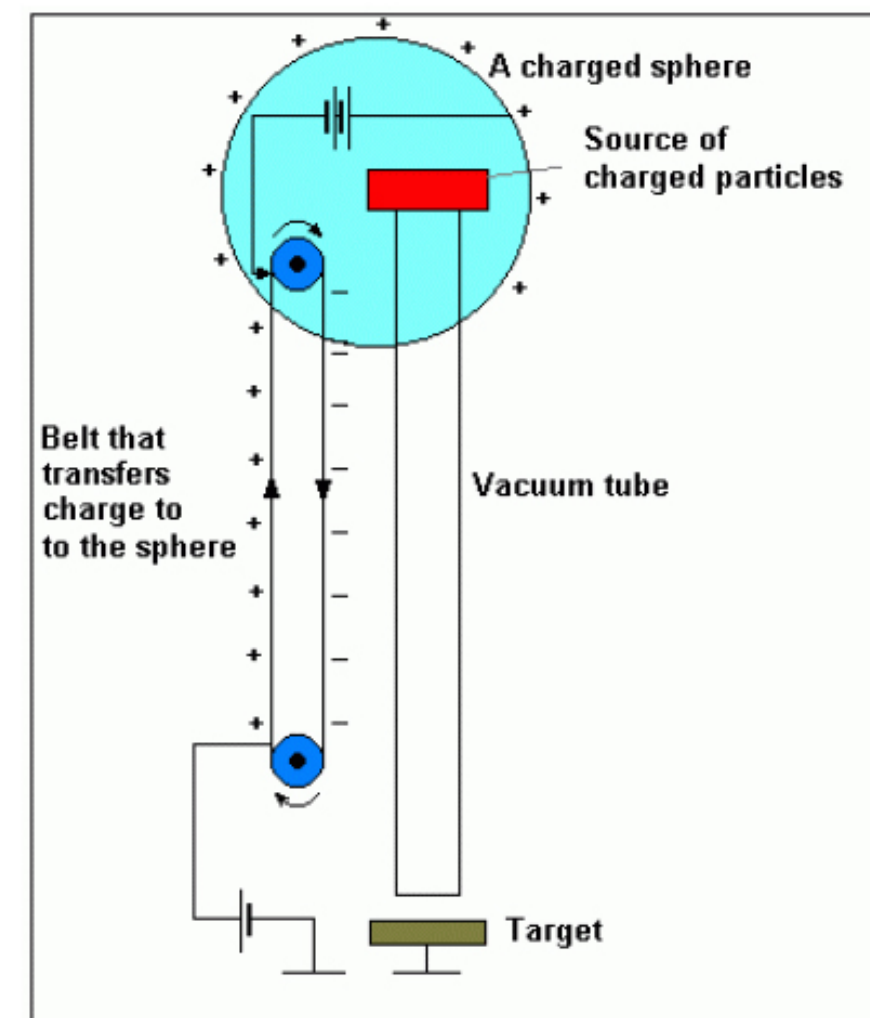


Figure 10.2: The 7 MV VandeGraaff accelerator at Kansas State University and the operating principle of this machine.



# Cyclotron

## Resonance accelerators

- E.O. Lawrence (1932)
- two hollow evacuated D-shaped metal chambers, which are connected to an AC high-voltage power supply
- The entire system is placed inside a strong magnetic field oriented perpendicular to the plane of the D's
- The entire process of cyclotron works because the time it takes the particles to describe a half circle inside the D does not change when the particle's energy increases
- Lorentz force acts as the centripetal force that keeps the particles in to a circular orbit:

$$F_L = Bqv = F_c = \frac{mv^2}{r} \rightarrow \frac{v}{r} = \frac{Bq}{m}$$

- The time to describe a half circle in the D's is

$$t = \frac{\pi r}{v} = \pi \frac{m}{Bq}$$

- Time only depends on the mass and charge of the particle, and the strength of the magnetic field
- The frequency at which this field changes direction is chosen correctly:

$$\omega = 2\pi f = \frac{v}{r} = \frac{Bq}{m}, \quad f = \frac{Bq}{2\pi m}$$

- Relativistic effects limit the performance of cyclotron

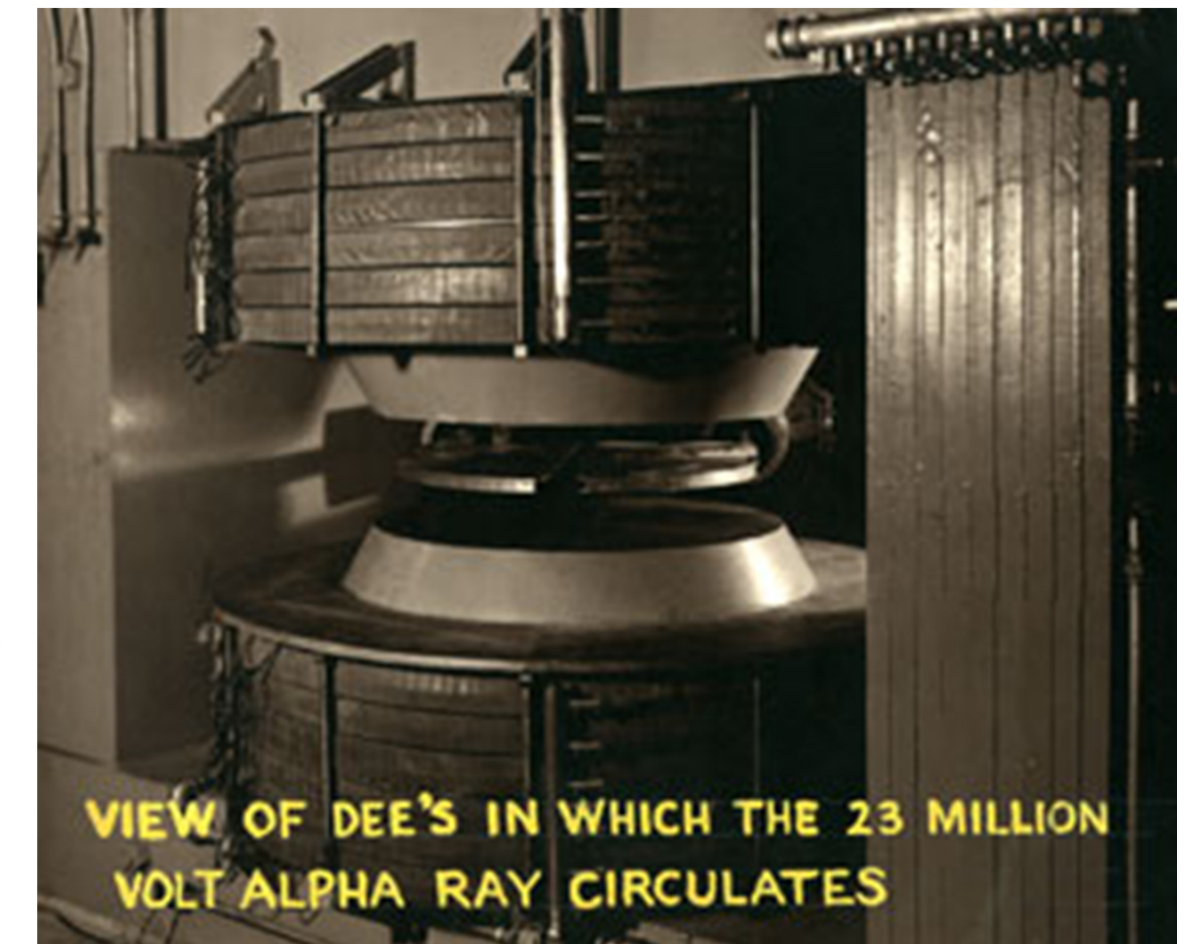
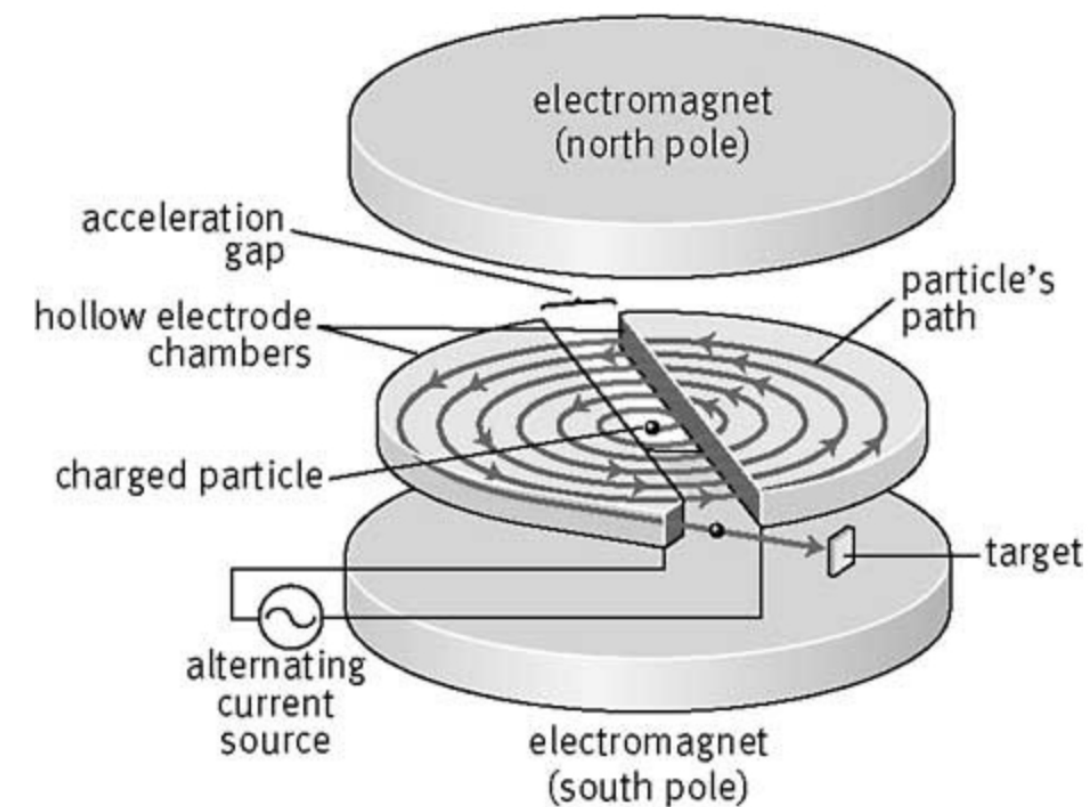


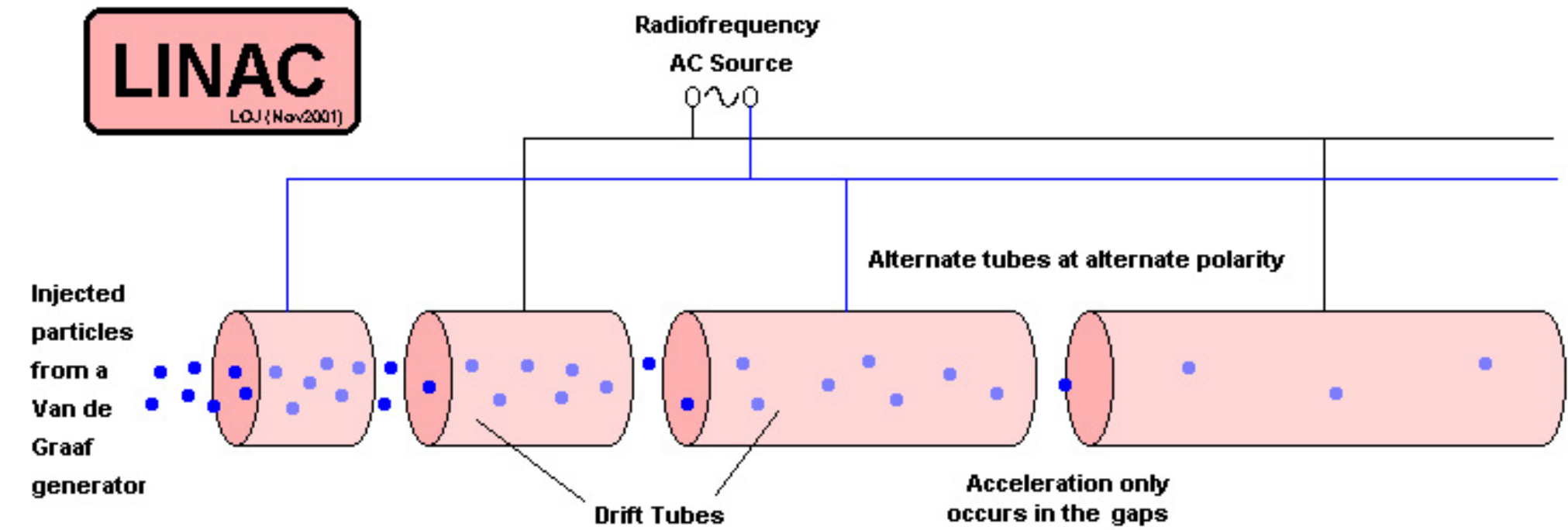
Figure 10.3: The cyclotron at Indiana University and the schematic structure of this device.



# Linear accelerators

## Resonance accelerators

- The particles are accelerated in a straight line, and travel through a series of hollow metal electrodes (called drift tubes) from the ion source to the target area
- Many linacs are in fact used as injectors (or pre-accelerators) for higher-energy machines
- The electrodes are located in a large vacuum vessel and connected successively to alternate terminals of a radiofrequency oscillator, as shown in Figure 10.4
- Unlike cyclotrons, the energy is not limited by relativistic effects
- Linacs can also be used to accelerate electrons
- A major difference with ion linacs is the fact that the electrons lose a lot of energy in the form of synchrotron radiation when they are accelerated
- The energy lost in this process is supplied by powerful microwave fields that travel in step with the particles. These fields are provided by klystron amplifiers
- Many hospitals have a linac, with an energy typically ranging from 4 - 25 MeV. These machines provide both beams of electrons and X rays. The world's largest linac was built at SLAC (Stanford), where electrons and positrons were accelerated to 50 GeV



10.4: The linear proton accelerator J-PARC in Japan and a schematic of such



# Synchrocyclotrons

## Synchronous accelerators

- In synchrocyclotrons, the frequency of the oscillator that drives the AC high-voltage supply is gradually decreased as the accelerated particles move outward
- In this way, the particles remain in phase with the accelerating electric field
- The limitations of this technique are mainly financial, due to the size of the magnet that is needed
- Example:
  - we want to accelerate protons to a momentum of 10 GeV/c with such as machine
  - If conventional magnets are used ( $B_{\max} \sim 2 \text{ T}$ ), then that implies a maximum radius of 16.5 m, since  $p_{\max} = Bqr_{\max} = 10 \text{ GeV}/c = 5.3 \cdot 10^{-18} \text{ kg}\cdot\text{m/s}$
  - In such a machine, the magnet disks would each have a volume of  $\pi r^2 d$ , with  $d$  being the thickness
  - This thickness would have to be at least 1 m for a magnet of this size
  - This would mean that each disk would have a mass of almost 7,000 tons and cost tens of millions of dollars

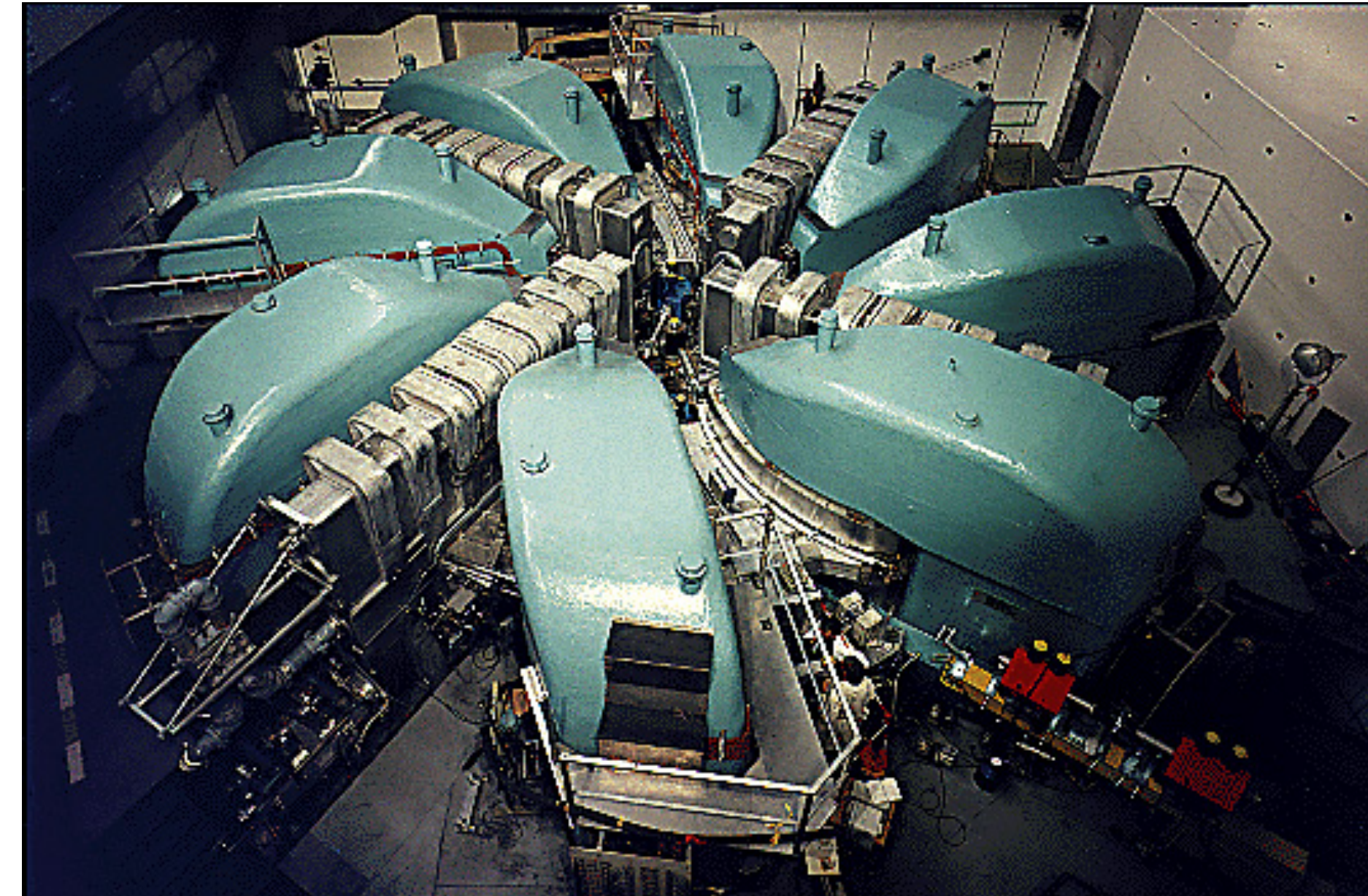


Figure 10.5: The 580 MeV synchrocyclotron at PSI (Villigen, Switzerland).



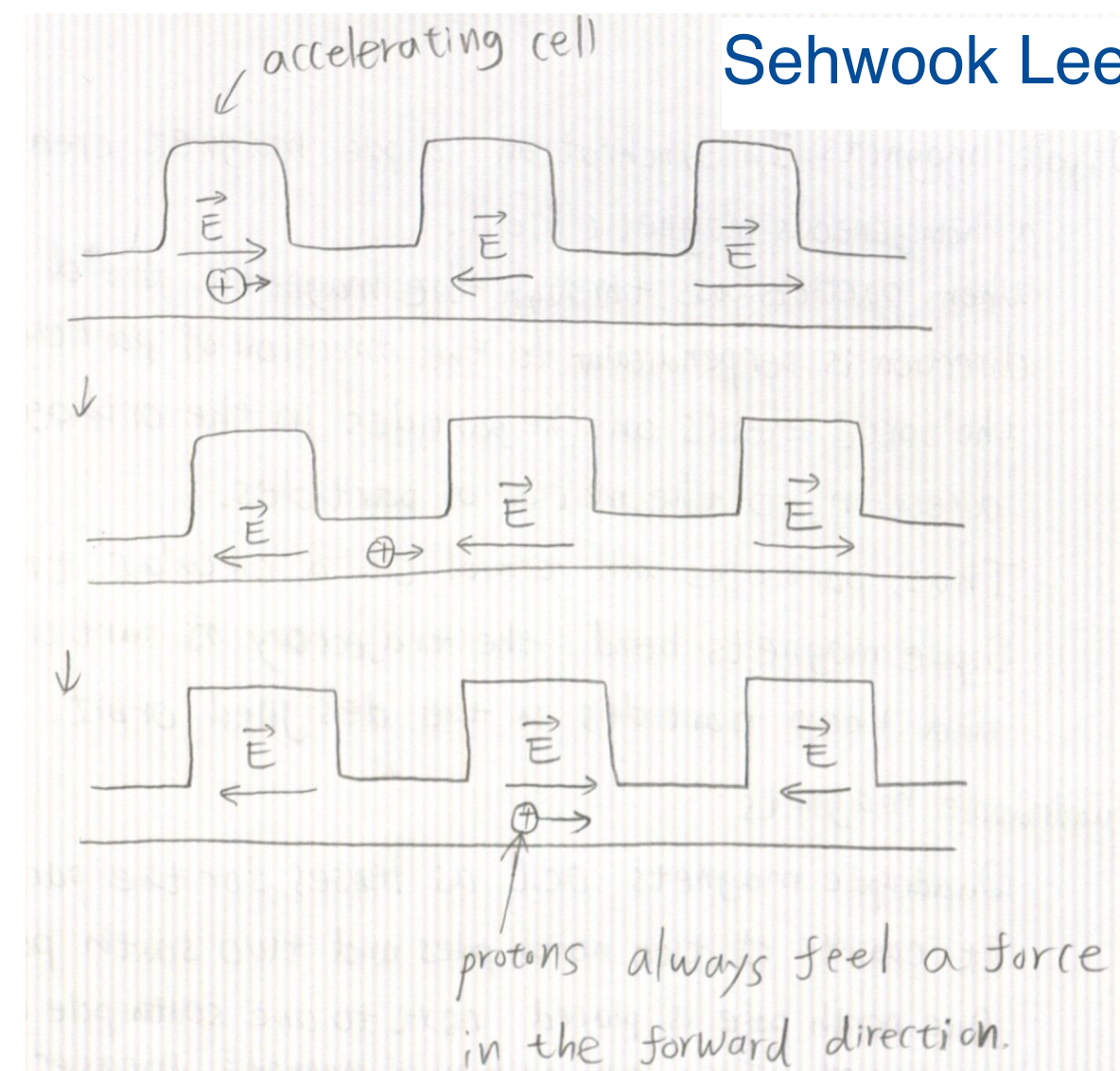
# Synchrotron

## Synchronous accelerators

- The particles run around in a narrow vacuum tube, and are kept in orbit by the magnetic field
- Their momentum increases proportionally with  $B$
- The particles gain energy when they pass through RF cavities, which are installed at several places along the ring
- Every time the particles pass through the RF cavities, they gain somewhere between 0.1 and 10 MeV from the electric fields, depending on the number of cavities in the ring



Figure 10.6: Aerial view of the synchrotron at Brookhaven National Laboratory (NY).



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PHY862 "Accelerator Systems"  
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# Synchrotron

## Synchronous accelerators

- For example:
  - CERN's Proton Synchrotron (PS) has a circumference of 628 m
  - It takes relativistic particles  $2.1 \mu\text{s}$  to complete an orbit in this machine
  - This synchrotron ramps up the energy of the protons to 25 GeV in 0.6 seconds
  - During this time, the particles complete 280,000 orbits and, therefore, they gain about 100 keV per orbit

# Synchrotron

## Synchronous accelerators

- The maximum momentum achievable in a circular accelerator depends on the machine parameters:

$$p_{\max} = Bqr$$

- Since  $1 \text{ GeV}/c = 1.6 \times 10^{-10} / 3.0 \times 10^8 = 5.3 \times 10^{-19} \text{ kg}\cdot\text{m}/\text{s}$  and  $q = 1.6 \times 10^{-19} \text{ C}$ , the relation is (p: GeV/c, R: m, B: T)

$$R = \frac{p}{0.33B}$$

- The CERN Super Proton Synchrotron:
  - a circumference of 6.9 km, which means  $R = 1,100 \text{ m}$ . Its magnets provide a field of up to 1.25 Tesla.
  - it can accelerate protons to a maximum momentum of  $1,100 \times 0.33 \times 1.25 \approx 450 \text{ GeV}/c$
- Can you do this for LHC?, How strong magnet do we need to accelerator protons up to 8 TeV/c?



# Synchrotron

## Synchronous accelerators

- The situation is completely different for synchrotrons that are used to accelerate electrons
- The synchrotron radiation is the main limiting factor

- The energy loss due to synchrotron radiation scales with  $\gamma^4$

$$P \text{ (energy radiated per second)} = \frac{2}{3}(\alpha\hbar c)c\frac{\gamma^4}{R^2}$$

- The energy loss (in MeV) of an electron with energy  $E$  (in GeV) that describes a circular orbit with radius  $r$  (in m)

$$\Delta E = 8.85 \cdot 10^{-2} \frac{E^4}{r}$$

- For example,
  - Before the Large Hadron Collider came into operation, it served as an electron synchrotron annex storage ring. The maximum energy to which electrons were accelerated in this machine amounted to 104.5 GeV
  - the electrons lost  $8.85 \times 10^{-2} \times 104.5^4 / 4,243 \approx 2,500$  MeV or 2.5 GeV per orbit in the form of synchrotron radiation

# Tricks to make synchrotrons work

## Synchrotron

- Vacuum requirements
  - The first condition for a long life time of the beam (in storage rings life times of the order of one day are quite common) is to avoid collisions with rest gas
  - The level of vacuum is needed to achieve a life time of one day
    - The mean free path of particles traveling through a certain medium is given by  $(\sigma n)^{-1}$ .
    - In this particular case, that mean free path should be equal to one light day, i.e.  $86,400 \times 3 \times 10^{10} = 2.6 \times 10^{15}$  cm.
    - If the cross section for collisions with the rest gas is 1 barn ( $10^{-24}$  cm<sup>2</sup>), this limits the density of rest gas atoms  $n$  to  $3.8 \times 10^8$  atoms/cm<sup>3</sup>.
    - At atmospheric pressure, air has a mass of 1.3 g/liter =  $1.3 \times 10^{-3}$  g/cm<sup>3</sup>.
    - Since air consists predominantly of nitrogen, 14 g corresponds to  $6.02 \times 10^{23}$  atoms and, therefore, at atmospheric pressure (760 torr), air contains  $5.6 \times 10^{19}$  atoms/cm<sup>3</sup>.
    - A pressure of 1 torr thus corresponds to a density of  $7.3 \times 10^{16}$  atoms/cm<sup>3</sup>, and the maximum density quoted above thus implies a maximum pressure of  $5 \times 10^{-9}$  torr



# Tricks to make synchrotrons work

## Synchrotron

- Phase stability
  - Maintaining particles in their nominal orbits for long periods of time
  - For example, cyclotron:
    - The electric field in the gap between the D's varies with time as indicated in Figure 10.7
    - There is always some finite spread in the time of arrival of the individual beam particles into the region where they experience the accelerating force
    - Let us label as “synchronous”, or “in time”, the particle that arrives at the time (or phase)  $\tau$  in Fig. 10.7 (particle 1).
    - Particle 2 arrives a bit earlier. It sees a larger voltage, is therefore more accelerated, describes a somewhat larger orbit and will thus arrive closer in time to particle 1 at the next passage in between the D's.
    - Particle 3 experiences the reverse effect. It arrives later than particle 1, is therefore less accelerated, described a somewhat smaller orbit and thus arrives closer in time to particle 1 at the next acceleration step.
    - This restoring effect is the reason why particles in such accelerators are apparently accelerated in “bunches”, centered on the synchronous particle

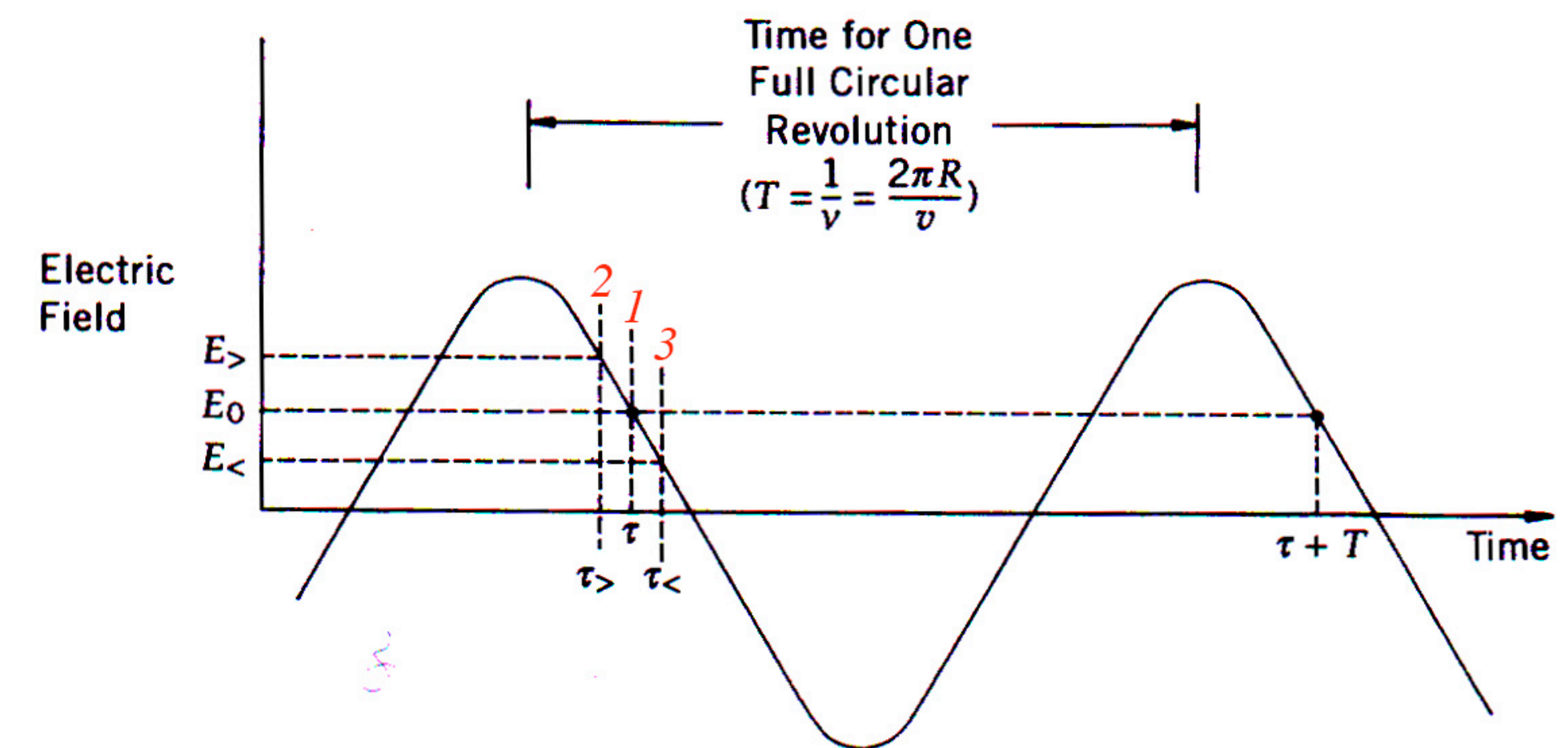


Figure 10.7: The time variation of the electric field in the gap between the D's of a (synchro)cyclotron.

# Tricks to make synchrotrons work

## Synchrotron

- Phase stability
  - For example, synchrotron:
  - In the vertical plane, the fringe fields of the magnets are responsible for a restoring effect (Figure 10.8).
  - The Lorentz force vector,  $F = qv \times B$  is oriented perpendicular to  $B$ , and therefore the fringe field forces the particles back to the median plane of the vacuum tube in which they describe their orbits

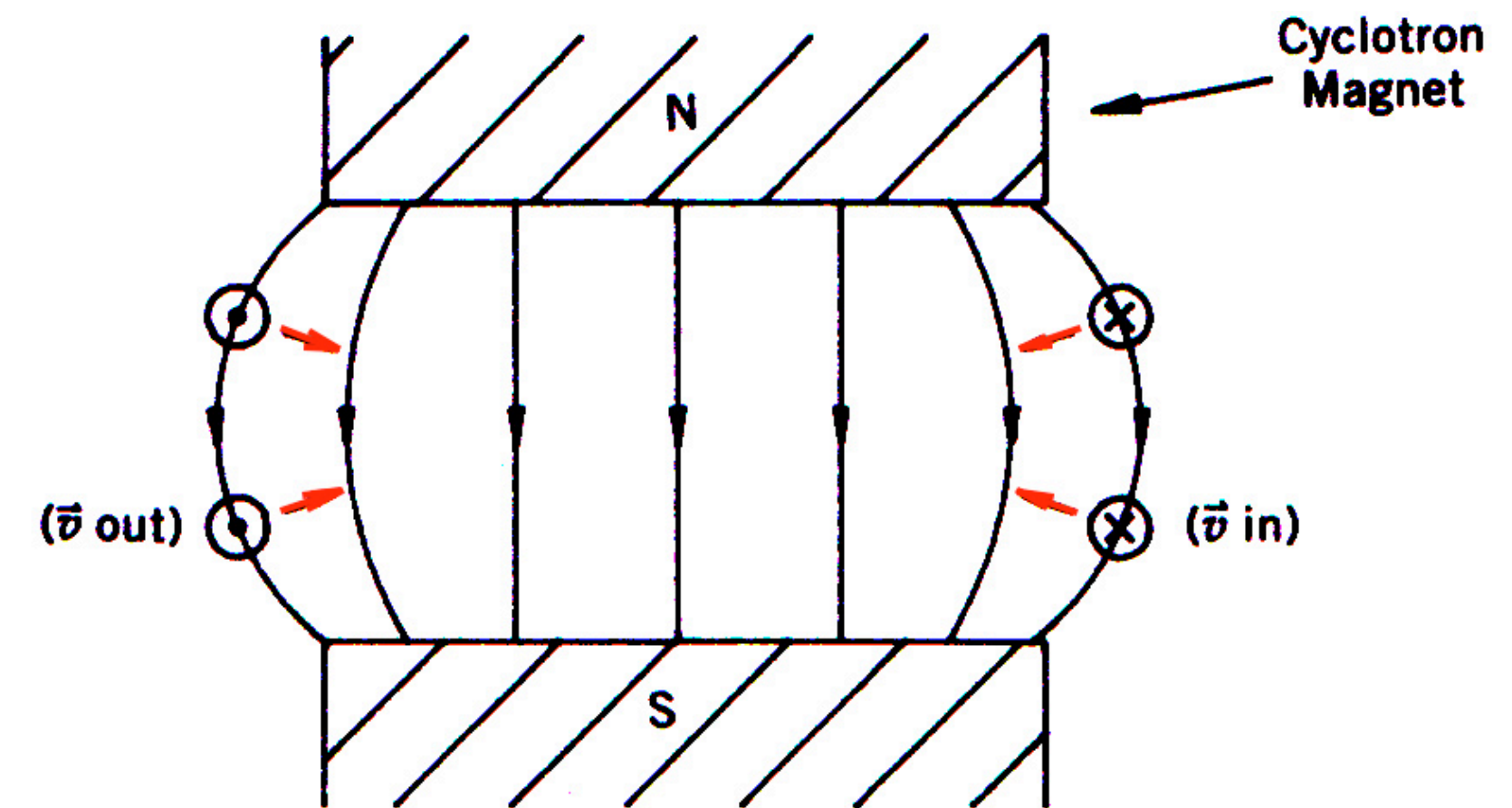


Figure 10.8: The magnetic field lines in a (synchro)cyclotron and the effect of the Lorentz force exerted on a positively charged particle circulating near the edge of the magnet.



# Tricks to make synchrotrons work

## Synchrotron

- Strong focusing
  - magnetic quadrupoles could be made to work as lenses for the particle beams
  - Figure 10.9 shows how such a quadrupole functions
  - It shows the direction of the Lorentz force experienced by a positively charged particle that enters the magnet perpendicular to the plane of the paper (your screen) in the vicinity of each of the four compartments of the quadrupole
  - Magnetic quadrupoles thus always focus the particle beam in one plane, and defocus it in the perpendicular plane. For that reason, quadrupoles always appear in pairs in particle beam lines

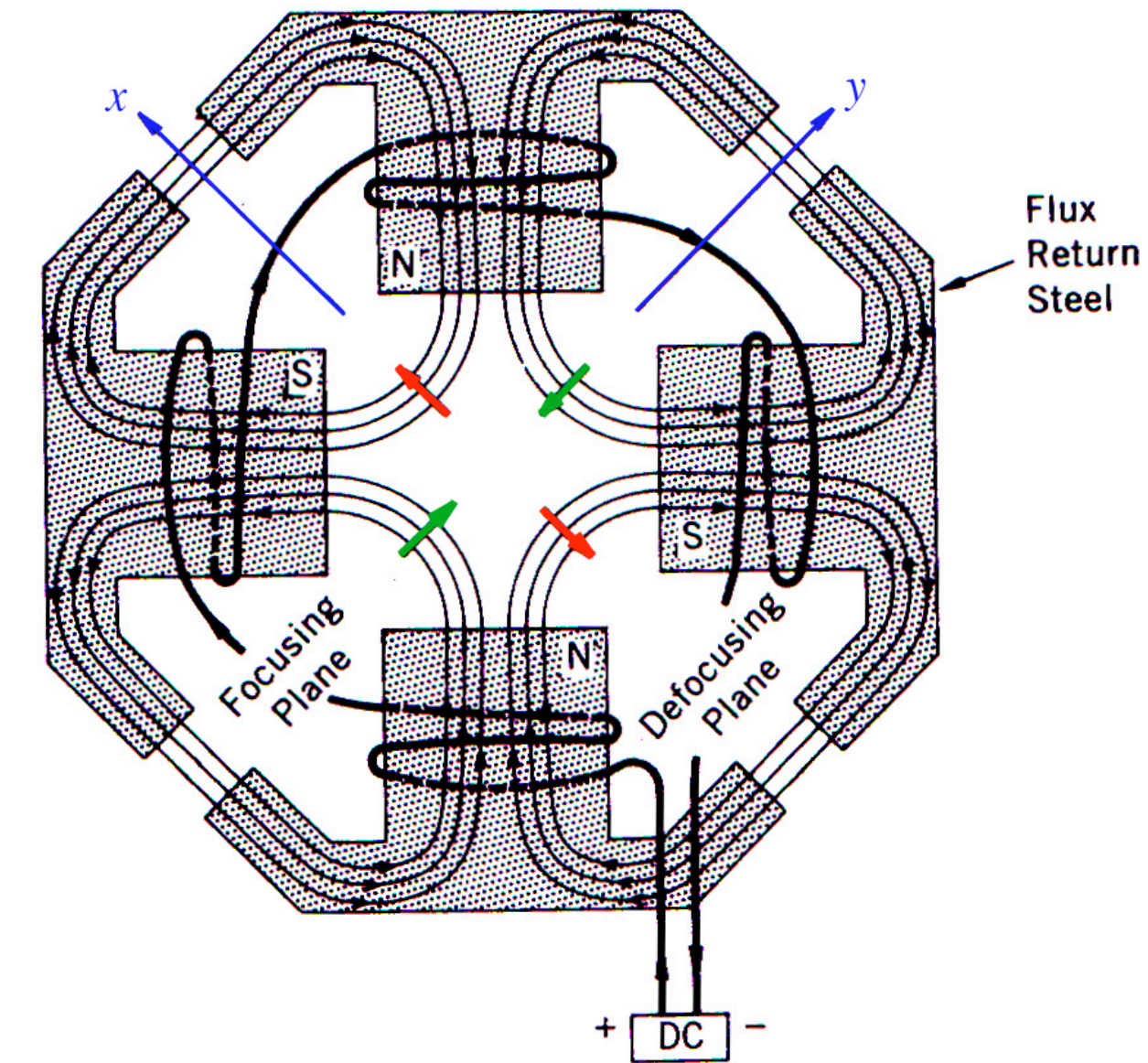


Figure 10.9: The magnetic field lines in a magnetic quadrupole and the effect of the Lorentz force exerted on a positively charged particle that enters this quadrupole at various positions perpendicular to the plane of the paper.